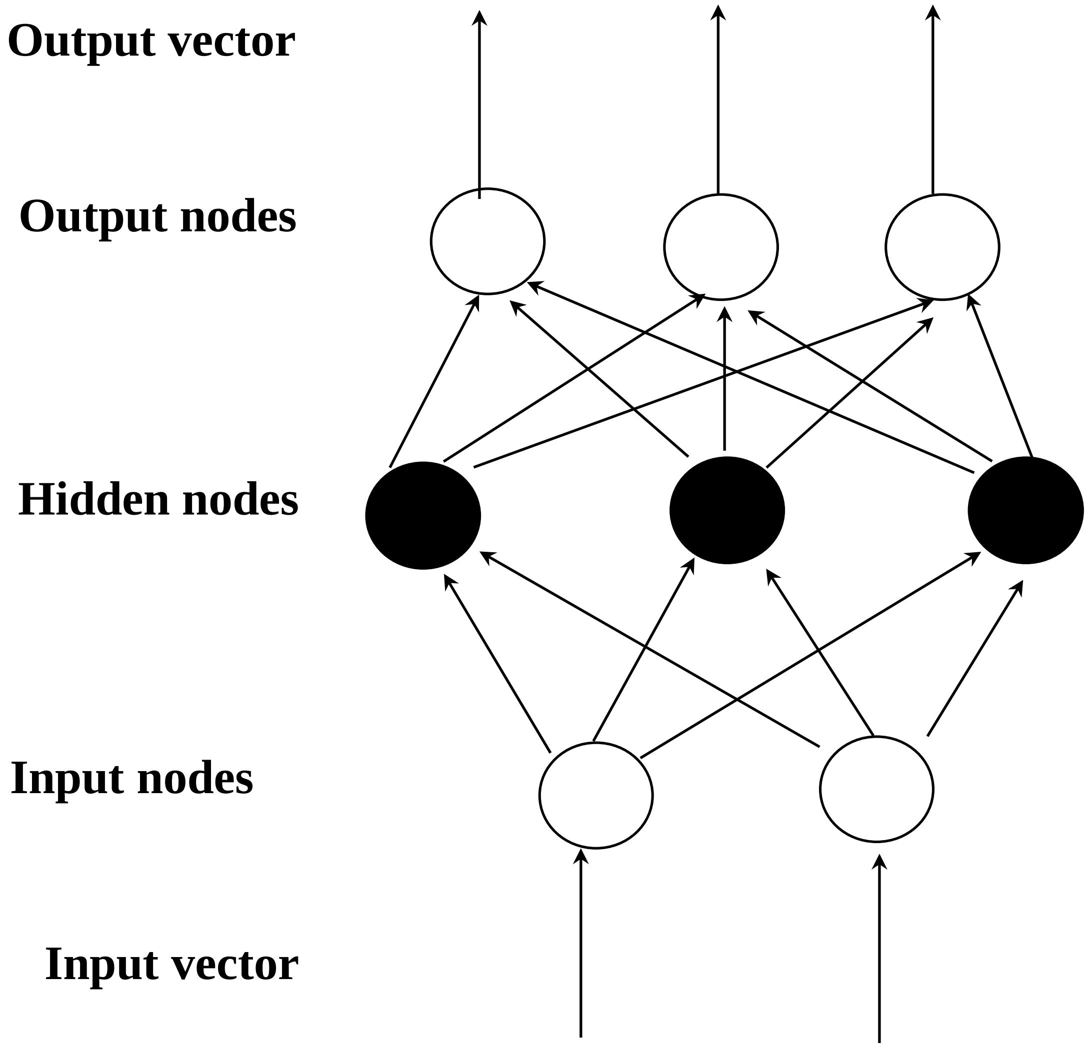
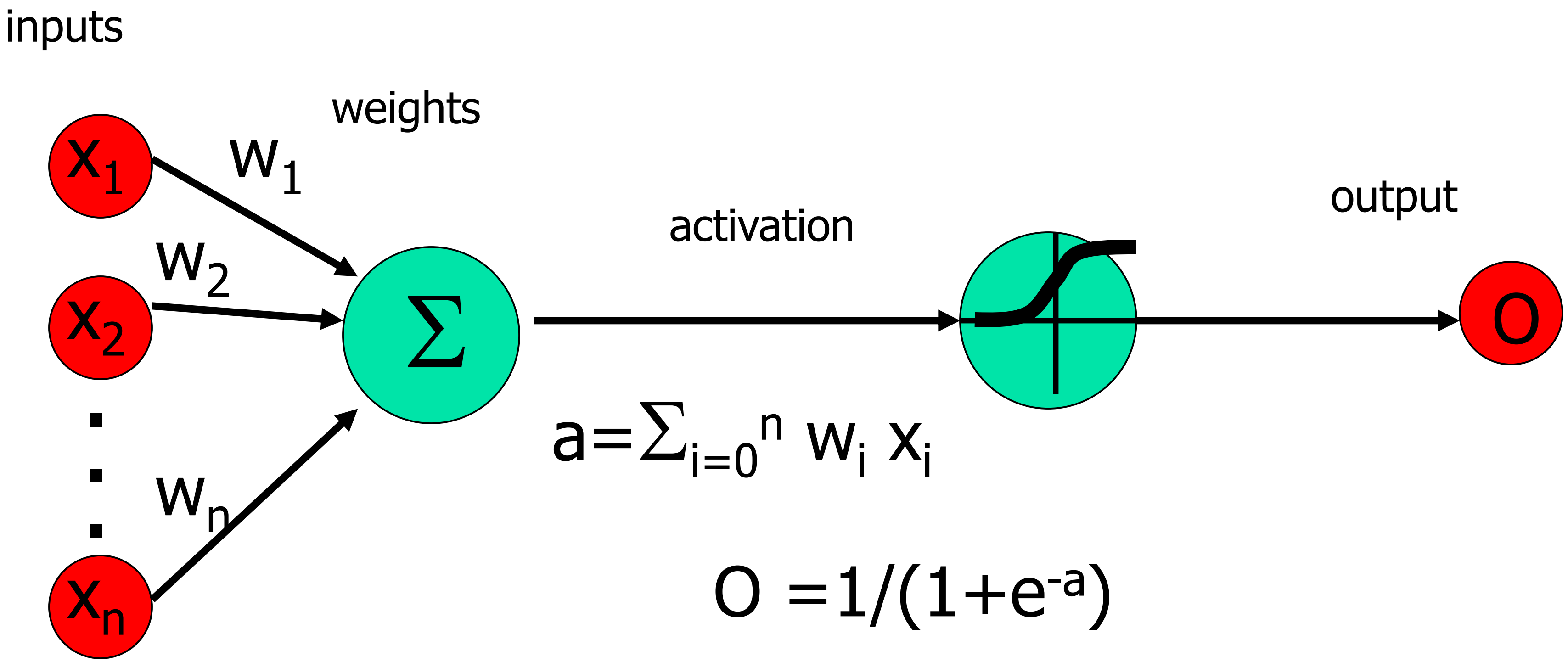
Part 2. Multi Layer Networks



Sigmoid-Function for Continuous Output



$$a=\Sigma_{i=0}^{n} w_i x_i$$

$$O = 1/(1+e^{-a})$$

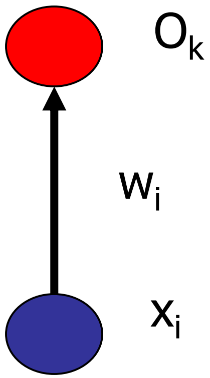
Output between 0 and 1 (when a = negative infinity, O = 0; when a= positive infinity, O=1.

Gradient Descent Learning Rule

- For each training example X,
  - Let O be the output (bewteen 0 and 1)
  - Let T be the correct target value
- Continuous output O
  - $a = w_1 x_1 + \ldots + w_n x_n + \theta$
  - $O = 1/(1+e^{-a})$
- Train the w_i's such that they minimize the squared error
  - $E[w_1,\ldots,w_n] = \frac{1}{2} \sum_{k \in D} (T_k - O_k)^2$

  where D is the set of training examples

Explanation: Gradient Descent Learning Rule



$$\Delta w_i = \alpha \; \underline{O_k (1-O_k)} \; (T_k - O_k) \; x_i^k$$

learning rate

derivative of
activation function

error δ_k of
post-synaptic neuron

activation of
pre-synaptic neuron

Backpropagation Algorithm (Han, Figure 9.5)

- Initialize each w_i to some small random value
- Until the termination condition is met, Do
  - For each training example <$(x_1,\ldots x_n)$,t> Do
    - Input the instance $(x_1,\ldots,x_n)$ to the network and compute the network outputs O_k
    - For each output unit k
      - $Err_k=O_k(1-O_k)(t_k-O_k)$
    - For each hidden unit h
      - $Err_h=O_h(1-O_h) \sum_k w_{h,k} Err_k$
    - For each network weight $w_{i,j}$ Do
    - $w_{i,j}=w_{i,j}+\Delta w_{i,j}$ where

      $\Delta w_{i,j}= \alpha\, Err_{j*} O_{i,}$
    - $\theta_j=\theta_j+\Delta\theta_j$ where

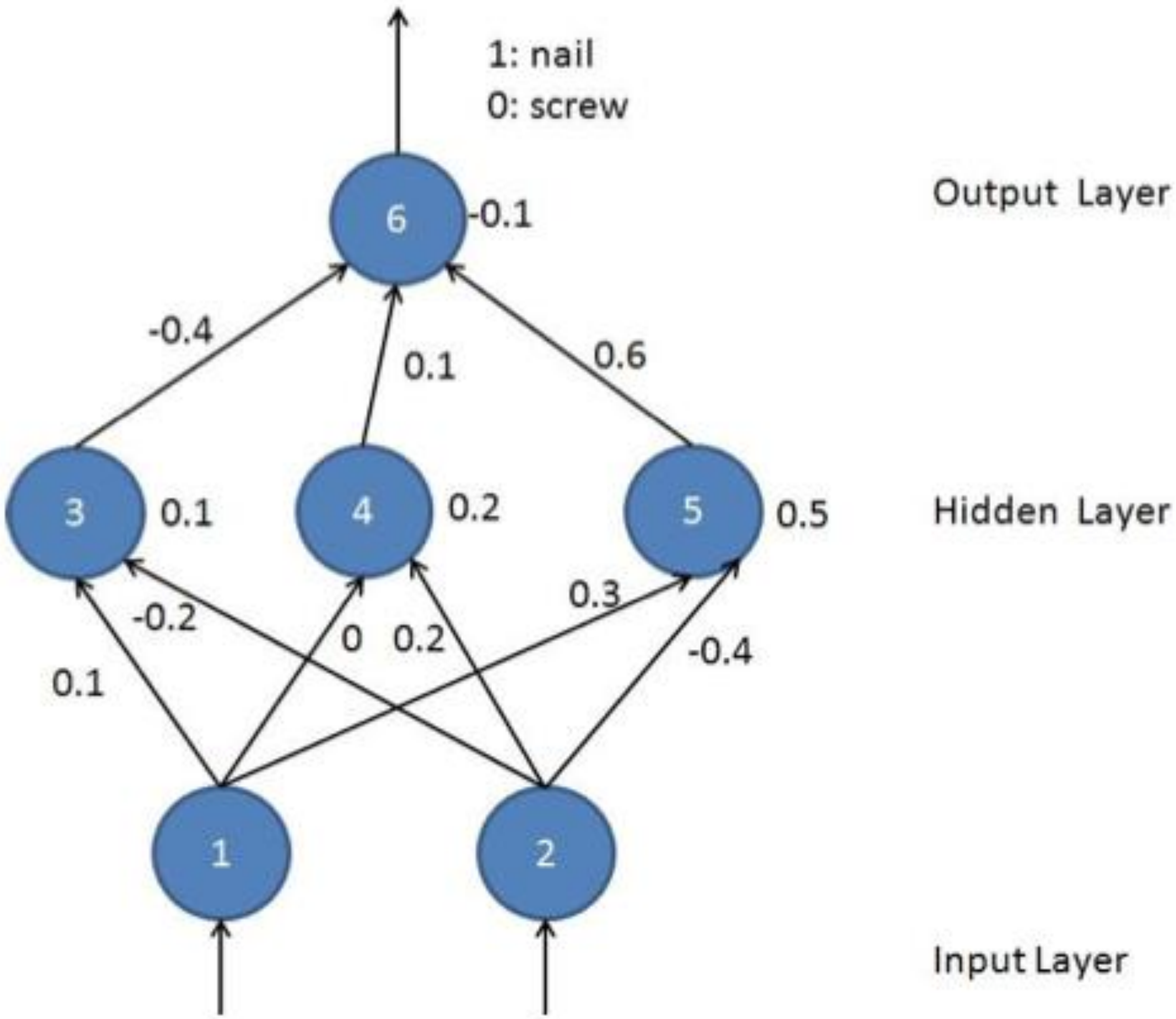
      $\Delta\theta_j = \alpha\, Err_{j,}$

α: is learning rate, set by the user;

Multilayer Neural Network

- Given the following neural network with initialized weights as in the picture(next page), we are trying to distinguish between nails and screws and an example of training tuples is as follows:
  - T1{0.6, 0.1, nail}
  - T2{0.2, 0.3, screw}
- Let the learning rate (l) be 0.1. Do the forward propagation of the signals in the network using T1 as input, then perform the back propagation of the error. Show the changes of the weights. Given the new updated weights with T1, use T2 as input, show whether the predication is correct or not.

Multilayer Neural Network



Multilayer Neural Network

- Answer:
- First, use T1 as input and then perform the back propagation.
  - At Unit 3:
    - $a_3 = x_1 w_{13} + x_2 w_{23} + \theta_3 = 0.14$
    - $o_3 = \dfrac{1}{1+e^{-a}} = 0.535$
  - Similarly, at Unit 4,5,6:
    - $a_4 = 0.22$, $o_4 = 0.555$
    - $a_5 = 0.64$, $o_5 = 0.655$
    - $a_6 = 0.1345$, $o_6 = 0.534$

Multilayer Neural Network

- Now go back, perform the back propagation, starts at Unit 6:
  - $Err_6 = o_6(1- o_6)\ (t- o_6) = 0.534 * (1-0.534)*(1-0.534) = 0.116$
  - $\Delta w_{36} = (l)\ Err_6 O_3 = 0.1 * 0.116 * 0.535 = 0.0062$
  - $w_{36} = w_{36} + \Delta w_{36} = -0.394$
  - $\Delta w_{46} = (l)\ Err_6 O_4 = 0.1 * 0.116 * 0.555 = 0.0064$
  - $w_{46} = w_{46} + \Delta w_{46} = 0.1064$
  - $\Delta w_{56} = (l)\ Err_6 O_5 = 0.1 * 0.116 * 0.655 = 0.0076$
  - $w_{56} = w_{56} + \Delta w_{56} = 0.6076$
  - $\theta_6 = \theta_6 + (l)\ Err_6 = -0.1 + 0.1 * 0.116 = -0.0884$

Multilayer Neural Network

- Continue back propagation:
- *Error at Unit 3:*

  Err_3 $= o_3 (1- o_3) (w_{36} Err_6) = 0.535 * (1-0.535) * (-0.394*0.116) = -0. 0114$

  $w_{13} = w_{13} + \Delta w_{13} = w_{13} + (l) Err_3 X_1 = 0.1 + 0.1*(-0.0114) * 0.6 = 0.09932$

  $w_{23} = w_{23} + \Delta w_{23} = w_{23} + (l) Err_3 X_2 = -0.2 + 0.1*(-0.0114) * 0.1 = -0.2001154$

  $\theta_3 = \theta_3 + (l) Err_3 = 0.1 + 0.1 * (-0.0114) = 0.09886$
- *Error at Unit 4:*

  Err_4 $= o_4 (1- o_4) (w_{46} Err_6) = 0.555 * (1-0.555) * (-0.1064*0.116) = 0.003$

  $w_{14} = w_{14} + \Delta w_{14} = w_{14} + (l) Err_4 X_1 = 0 + 0.1*(-0.003) * 0.6 = 0.00018$

  $w_{24} = w_{24} + \Delta w_{24} = w_{24} + (l) Err_4 X_2 = 0.2 + 0.1*(-0.003) * 0.1 = 0.20003$

  $\theta_4 = \theta_4 + (l) Err_4 = 0.2 + 0.1 * (0.003) = 0.2003$
- *Error at Unit 5:*

  Err_5 $= o_5 (1- o_5) (w_{56} Err_6) = 0.655 * (1-0.655) * (-0. 6076*0.116) = 0.016$

  $w_{15} = w_{15} + \Delta w_{15} = w_{15} + (l) Err_5 X_1 = 0.3 + 0.1* 0.016 * 0.6 = 0.30096$

  $w_{25} = w_{25} + \Delta w_{25} = w_{25} + (l) Err_5 X_2 = -0.4 + 0.1*0.016 * 0.1 = -0.39984$

  $\theta_5= \theta_5 + (l) Err_5 = 0.5 + 0.1 * 0.016 = 0.5016$

Multilayer Neural Network

- After T1, the updated values are as follows:

w_{13}	w_{14}	w_{15}	w_{23}	w_{24}	w_{25}	w_{36}	w_{46}	w_{56}
0.09932	0.00018	0.30096	-0.2001154	0.20003	-0.39984	-0.394	0.1064	0.6076

θ_3	θ_4	θ_5	θ_6
0.09886	0.2003	0.5016	-0.0884

- Now, with the updated values, use T2 as input:
  - **At Unit 3:**
  - $a_3 = x_1 w_{13} + x_2 w_{23} + \theta_3 = 0.0586898$
  - $o_3 = \frac{1}{1+e^{-a}} = 0.515$

Multilayer Neural Network

- Similarly,
  - $a_4 = 0.260345$, $o_4 = 0.565$
  - $a_5 = 0.441852$, $o_5 = 0.6087$
- At Unit 6:
  - $a_6 = x_3 w_{36} + x_4 w_{46} + x_5 w_{56} + \theta_6 = 0.13865$
  - $o_6 = \dfrac{1}{1+e^{-a}} = 0.5348$
- Since O_6 is closer to 1, so the prediction should be nail, different from given “screw”.
- So this predication is NOT correct.