Impact of Image Processing Operations on MR Noise Distributions

Paul Summers[*], Albert C. S. Chung[#], J. Alison Noble[#]
[*]Clinical Neurosciences, King's College London, SE5 8AF, U. K.
[#]Department of Engineering Science, Oxford University, Oxford, OX1 3PJ, U. K.

Algorithms relying on noise distribution models have been applied to MR data for such purposes as spectral peak extraction, image segmentation and restoration.[1-5] It is noted that mapping from complex (vector image) values to a real-valued intensity scale using modulus or phase operators modifies the noise characteristics.[6,7] In discussions however, we have found that the behaviour of noise under image operations is not fully appreciated. We have examined the impact of some *non-linear* operators commonly used to manipulate and display of MR images.

The noise associated with the acquired MR signal is generally taken to be additive, uncorrelated and Gaussian with zero (or low value) mean and of comparable variance for each quadrature channel.[8,9] A similar distribution thus holds for each component of the complex-valued image formed by Fourier transformation because of its linearity. This is a characteristic of linear operators (including: averaging, scaling and offset correction). They may affect the mean and variance of the noise, but do not alter the shape of the noise distribution itself.[10,11]

Non-linear operators do not necessarily share this distribution preserving behaviour. In particular, the shape of the noise distribution can become dependent on the local signal value.[5] The modulus operator for instance progresses from a Rayleigh to a Gaussian noise distribution with increasing signal to noise ratio. A rather different trend is seen with the phase operator. Here, a uniform probability distribution exists at negligible signal levels which, again, becomes more Gaussian with increasing signal levels.

Some further operators of interest in MR are square logarithm. For a signal z = Re(z) + i*Im(z), and assuming that the real and imaginary components are corrupted by independent and uncorrelated Gaussian noise (with equal standard deviation □, and different means, foir which square root of the square sum of means is A), the summary probability density functions for the noise distributions arising from these operators are:

$$f(|z|^2) = \frac{1}{2\sigma^2}\left(\frac{|z|}{A}\right)^{\left(\frac{k-2}{2}\right)} e^{\frac{-(|z|^2+A^2)}{2\sigma^2}} I_{\frac{k}{2}-1}\left(\frac{|z|A}{\sigma^2}\right) u(|z|)$$

$$f(\ln|z|) = 2|z|^2 f(|z|^2)$$

where |z| is modulus of z, k is the number of real and imaginary components (In general, k=2*number of orthogonal image decoding directions), I is the modified Bessel function of the first kind, u(z) is a unit step function.

Methods - T1 and T2/PD wtd spin echo, phase contrast MRA and time of flight MRA sequence images were acquired on a 1.5T scanner. Conventional modulus and phase contrast "speed" images were used to form squared and logartihm images. The noise models differ from Gaussian most substantially when SNR is low. Therefore, to compare the fit of these new MR noise models and standard Gaussian noise each model was fitted to histograms obtained from regions of interest chosen in background air.

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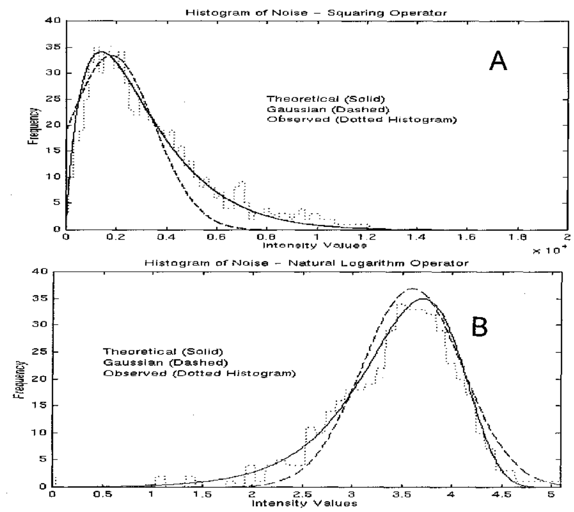
Results - Examples of theoretical and Gaussian model noise fits to image results (dotted histogram) following A) squaring operator and B) logarithm operator.



Mean Squared Error: Theoretical & Gaussian noise Fitting

	Gaussian	Theoretical
Squaring Operator	9.4473	2.5740
Natural Log Operator	8.7271	3.6529

Discussion - It is clear that the square and ln operators lead to non-Gaussian noise distributions in regions of low signal intensity. The use of appropriate noise models as described here will therefore be a useful inclusion in any model-based processing. Filtering, and coherent noise may account for some of the discrepancy with observed noise.

These particular operators are of interest in quite distinct areas of MR. The squaring operator has been suggested as a means of maximising contrast in phase contrast MR angiograms.[13] Thus, preceding steps would typically involve complex subtraction of two or more images. Although these linear operations retain the Gaussian noise distribution they serve to reduce the signal intensity for most (non-moving) parts of the image. It would then be the case that squaring transforms the Gaussian noise distribution into a Maxwell distribution given here for air and static tissue.

The logarithm operation may be used to linearise data for curve-fitting when deriving relaxation times. The results here highlight the change in behaviour of noise for low signal intensities, implying that appropriate noise models are needed for fitting to heavily decayed sample points.

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