On the Progression of Knowledge in the Situation Calculus

Yongmei Liu

Department of Computer Science
Sun Yat-sen University
Guangzhou, China

Joint work with Ximing Wen

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Litmus test example [Moore 1985]

The initial KB:
\{acid, \neg red, K(\neg red)\}
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How should we update the KB?

Intuitively, the new KB should entail $K(\text{acid})$
Main problem

How to update the knowledge base (KB) after an action is executed?

The KB includes both objective and subjective knowledge

1. Whether the new KB is definable in first-order modal logic?
2. Whether the new KB is finite?
3. Whether the new KB is computable?
Our main work

1. Definition of knowledge progression

2. Progression wrt. physical actions
   - this reduces to forgetting in first-order modal logic
   - progression wrt local-effect physical actions

3. Progression wrt. sensing actions

4. Extension to the multi-agent case
Sensing actions of the form $\text{sense}_\psi(\vec{x})$

A special fluent $K(s', s)$, meaning that situation $s'$ is accessible from situation $s$.

Knowing $\phi$ at situation $s$ is represented as:

$$\text{Knows}(\phi, s) \overset{\text{def}}{=} \forall s'. K(s', s) \supset \phi[s'].$$
Basic action theories with knowledge

\[ \mathcal{D} = \Sigma \cup \mathcal{K}_{\text{Init}} \cup \mathcal{D}_{\text{una}} \cup \mathcal{D}_{\text{ap}} \cup \mathcal{D}_{\text{ss}} \cup \mathcal{D}_{S_0}, \text{ where} \]

- \( \mathcal{D}_{\text{ss}} \): successor state axioms (SSA), e.g.,
  
  \[
  \begin{align*}
  \text{red}(\text{do}(a, s)) & \equiv a = \text{dip} \land \text{acid}(s) \lor \text{red}(s), \\
  \text{acid}(\text{do}(a, s)) & \equiv \text{acid}(s) \land a \neq \text{dilute}
  \end{align*}
  \]

- \( \mathcal{D} \) includes the SSA for the \( K \) fluent [Scherl & Levesque 2003]:
  
  \[
  K(s', \text{do}(a, s)) \equiv \exists s^*. K(s^*, s) \land s' = \text{do}(a, s^*) \land \\
  \land \forall \vec{x}[a = \text{sense}_\psi(\vec{x}) \supset \psi(\vec{x}, s^*) \equiv \psi(\vec{x}, s)].
  \]

A consequence of the SSA for \( K \) is that the agent knows her successor state axioms for ordinary fluents

- \( \mathcal{D}_{S_0} \): the initial KB, a finite set of sentences about \( S_0 \), e.g.
  
  \[
  \text{acid}(S_0), \quad \text{Knows}(\neg \text{red}, S_0)
  \]
An intuitive definition

Let $\alpha$ be a ground action, and let $S_\alpha$ denote $do(\alpha, S_0)$. A progression of $D_{S_0}$ wrt $\alpha$ is a set of sentences $D_{S_\alpha}$ s.t.

1. Just as $D_{S_0}$ is about $S_0$ only, $D_{S_\alpha}$ should be about $S_\alpha$ only;

2. the old theory $D$ and the new theory $D' = (D - D_{S_0}) \cup D_{S_\alpha}$ should be equivalent wrt the possible futures of $S_\alpha$. 

Progression [Lin & Reiter 1997]
A formal definition (A variant from [Lin & Reiter 1997])

$\mathcal{D}_{S_\alpha}$ is a progression of $\mathcal{D}_{S_0}$ wrt $\alpha$, written $\mathcal{D}_{S_0} \xrightarrow{\alpha} \mathcal{D}_{S_\alpha}$, if

1. $\mathcal{D}_{S_\alpha}$ is about $S_\alpha$ only, $\mathcal{D} \models \mathcal{D}_{S_\alpha}$, and
2. for every $M \models \mathcal{D}'$, there is $M' \models \mathcal{D}$ such that $M \sim_{S_\alpha} M'$.

How to define $M \sim_{S_\alpha} M'$ in the presence of knowledge?

We adapt the concept of bisimulation from modal logic.
Bisimulation

$M \sim_{S_\alpha} M'$ if there is a relation $Z$ between their situations s.t. the denotations of $S_\alpha$ are $Z$-related, and whenever $w Z w'$,

1. $w$ and $w'$ agree on all ordinary fluents;

2. The forth condition: for all $v$ accessible (by $K$) from $w$, there is $v'$ accessible from $w'$ s.t. $v Z v'$;

3. The back condition: for all $v'$ accessible from $w'$, there is $v$ accessible from $w$ s.t. $v Z v'$.

**Proposition.** If $M \sim_{S_\alpha} M'$, then for any formula $\phi$ about $S_\alpha$ and the futures of $S_\alpha$, $M \models \phi$ iff $M' \models \phi$. 

Fig. 2.3. The forth condition.
1. Definition of knowledge progression

2. Progression wrt. physical actions
   - this reduces to forgetting in first-order modal logic
   - progression wrt local-effect physical actions

3. Progression wrt. sensing actions

4. Extension to the multi-agent case
The modal formula $K\phi$ means knowing $\phi$.

**Theorem**

Let $D_{S_0}$ be $\phi[S_0]$ where $\phi$ is in FO modal logic, and $\alpha$ a physical action. Let $k\text{forget}(\phi \land KD^*_s[\alpha, S_0], \vec{F}) \iff \psi$, where $\psi$ is in second-order modal logic. Then $D_{S_0} \models^\alpha \psi[S_{\alpha}]$.

- $D_{ss}[^\alpha, S_0]$ denotes the instantiation of $D_{ss}$ wrt $\alpha$ and $S_0$
- $\psi^*$ denotes the result obtained from $\psi$ by replacing $F(\vec{t}, S_0)$ with $F(\vec{t})$ and $F(\vec{t}, S_{\alpha})$ with $F'(\vec{t})$

So for physical actions, progression of knowledge reduces to forgetting predicates in first-order modal logic.

They defined forgetting based on the notion of bisimulation.

Their definition coincides with:
- the semantic definition of formula $\exists V \phi$ in [French 2005] and
- the notion of uniform interpolation in [Ghilardi et al. 2006].

A result in [Ghilardi et al. 2006] shows that propositional S5 is closed under forgetting.
Forgetting atoms in first-order modal logic

**Definition**

φ is in ∃-DNF if it is of the form \( \exists \vec{x}(\phi_1 \lor \ldots \lor \phi_n) \), where each \( \phi_i \) is of the form \( \alpha \land K \beta \land \bigwedge_i \forall \vec{x}_i M \gamma_i \), called an extended term, where \( \alpha, \beta, \) and \( \gamma_i \)'s are all objective, and \( M \gamma \) abbreviates for \( \neg K \neg \gamma \), and means possibly \( \gamma \)

∃-DNF goes beyond formulas without quantifying-in

**Proposition.** \( k\text{forget}(\alpha \land K \beta \land \bigwedge_i \forall \vec{x}_i M \gamma_i, \mu) \iff \text{forget}(\alpha \land \beta, \mu) \land K \text{forget}(\beta, \mu) \land \bigwedge_i \forall \vec{x}_i M \text{forget}(\gamma_i \land \beta, \mu) \).

**Theorem**

Let \( \phi \) be in ∃-DNF. Then the result of forgetting an atom in \( \phi \) is definable in first-order modal logic and computable.
Forgetting predicates in first-order modal logic

**Theorem**

*If a sentence $\phi$ entails knowing that the truth values of predicates $P$ and $Q$ are different at only a finite number of certain instances, then forgetting $Q$ in $\phi$ can be achieved by forgetting the $Q$ atoms of these instances in $\phi$ and then replacing $Q$ by $P$ in the result.*
Local-effect actions: if an action $A(\vec{c})$ changes the truth value of an atom $F(\vec{a}, s)$, then $\vec{a}$ is contained in $\vec{c}$.

For action $A(\vec{c})$, let $\Omega = \{ F(\vec{a}) \mid \vec{a} \text{ is contained in } \vec{c} \}$.

**Theorem**

Let $\alpha$ be a local-effect physical action. Let $\mathcal{D}_{S_0}$ be $\phi[S_0]$ where $\phi$ is in $\exists$-DNF. Then there is $\psi$ in FO modal logic s.t. $\kappa\text{forget}(\phi \land K\mathcal{D}_{ss}^*[\Omega], \Omega) \iff \psi$, and $\mathcal{D}_{S_0} \models \alpha \psi(\vec{F}/\vec{F}')[S_\alpha]$.

In this case, progression of knowledge is definable in first-order modal logic.
Example: Progression wrt “dip”

- $D_{S_0} = \{acid(S_0), K(\neg red, S_0)\}$.
- $S_1 = do(dip, S_0)$.
- $\Omega = \{red\}$, $D^*_{ss}[\Omega]$ includes:
  
  \[ red' \equiv acid \lor red \]

- $k_{forget}(\phi, red)$, where
  \[ \phi = acid \land K(\neg red) \land K(red' \equiv acid \lor red) \]

- $D_{S_1} =$
  
  \[ \{acid(S_1), red(S_1), K(acid \equiv red, S_1)\} \]
Outline

1. Definition of knowledge progression
2. Progression wrt. physical actions
3. Progression wrt. sensing actions
4. Extension to the multi-agent case
The modal formula $W\phi$ abbreviates for $K\phi \lor K\neg\phi$

**Theorem**

Assume that $\mathcal{D}_{S_0}$ does not contain negative knowledge. Let $\alpha$ be $\text{sense}_\psi(\vec{c})$, where $\psi(\vec{x})$ is an objective formula. Then $\mathcal{D}_{S_0} \models \alpha \Rightarrow \mathcal{D}_{S_0}(S_0/S_\alpha) \cup \{W\psi(\vec{c})[S_\alpha]\}$.

A formula does not contain negative knowledge if no knowledge atom appears in the scope of an odd number of negation operators.

In the general case, without the assumption, we do not yet know how to do progression wrt sensing actions.
Example: Progression wrt “sense”

\[
D_{S_1} = \{acid(S_1), red(S_1), K(acid \equiv red, S_1)\}
\]

\[
S_2 = do(sense_{red}, S_1)
\]

\[
D_{S_2} = \{acid(S_2), red(S_2), K(red \equiv acid, S_2), W(red, S_2)\}
\]

\[
D_{S_2} \Leftrightarrow \{K(red, S_2), K(acid, S_2)\}
\]
Outline

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The modal formula \( C\phi \) means that \( \phi \) is common knowledge.

By introducing a second-order axiom describing the transitive closure of the union of the \( K_i \) fluent, we can represent common knowledge in the SitCalc.

We only consider public actions whose occurrence is common knowledge.
Theorem

Let $D_{S_0}$ be $\phi[S_0]$ where $\phi$ is in FO modal logic, and $\alpha$ a public physical action. Let $k\text{forget}(\phi \land CD^*_{ss}[\alpha, S_0], \vec{F}) \Leftrightarrow \psi$, where $\psi$ is in SO modal logic. Then $D_{S_0} \Rightarrow \psi[S_\alpha]$.

Theorem

Assume that $D_{S_0}$ does not contain negative knowledge. Let $\psi(\vec{x})$ be an objective formula, and $\alpha$ a public sensing action $\text{sense}_{i, \psi}(\vec{c})$. Then $D_{S_0} \Rightarrow D_{S_0}(S_0/S_\alpha) \cup \{CW_i\psi(\vec{c})[S_\alpha]\}$.
Conclusions

1. Adapted the concept of bisimulation and extended Lin & Reiter’s notion of progression to accommodate knowledge.

2. Showed for physical actions, progression of knowledge reduces to forgetting predicates in FO modal logic.

3. Showed for local-effect physical actions, when the initial KB is in $\exists$-DNF, prog’n of knowledge is definable in FO modal logic.

4. Showed for sensing actions, when the initial KB does not contain negative knowledge, prog’n can be simply achieved.

5. Initial results on extension to the multi-agent case.
Future work

1. An in-depth study of forgetting in FO modal logic
2. Progression wrt sensing actions in the general case
3. Progression in the general multi-agent case