$$\Rightarrow \qquad \frac{f_n}{n} = \lim_{R\to\infty} \frac{1}{2i\pi} \int_\Gamma I(s)ds + O(n^\alpha)$$