Dynamic Indexability and the Optimality of B-trees and Hash Tables

Ke Yi

Hong Kong University of Science and Technology

Dynamic Indexability and Lower Bounds for Dynamic One-Dimensional Range Query Indexes, *PODS* '09

Dynamic External Hashing: The Limit of Buffering, with Zhewei Wei and Qin Zhang, *SPAA* '09

+ some latest development

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  - Dow Jones, S & P, Hang Seng

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- An index is a list of academic publications and their citations
- An index (search engine) is an inverted list from keywords to web pages
- An index (database) is a (disk-based) data structure that improves the speed of data retrieval operations (queries) on a database table.

Hash Table and B-tree

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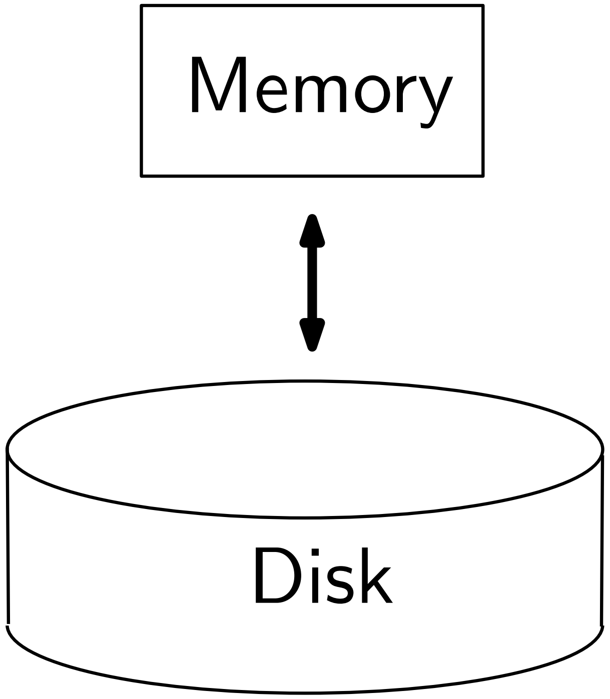
External memory model (I/O model):

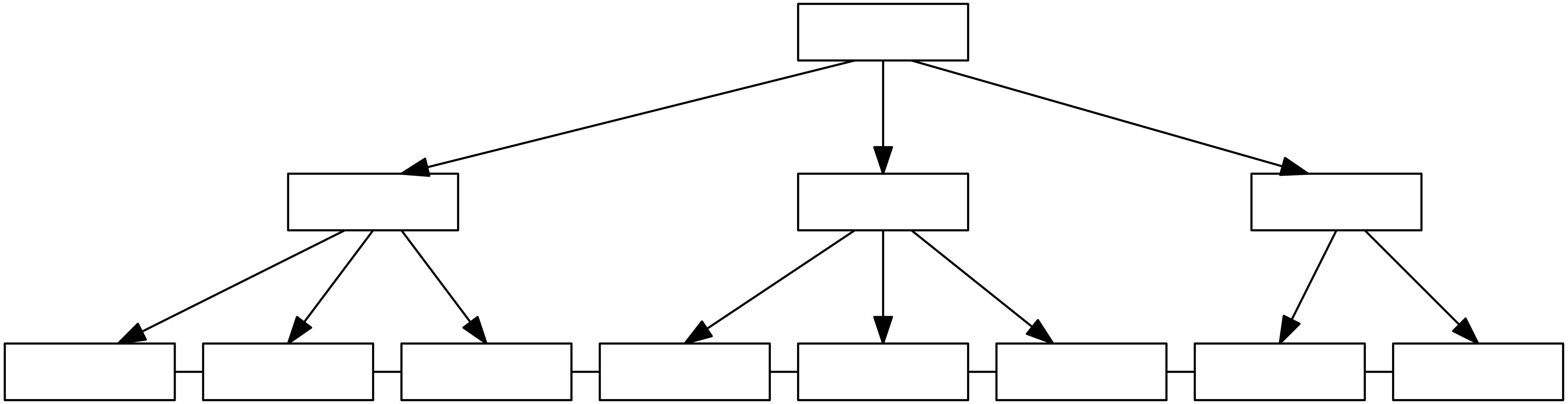


Memory of size M

Each I/O reads/writes a block

Disk partitioned into blocks of size B

The B-tree



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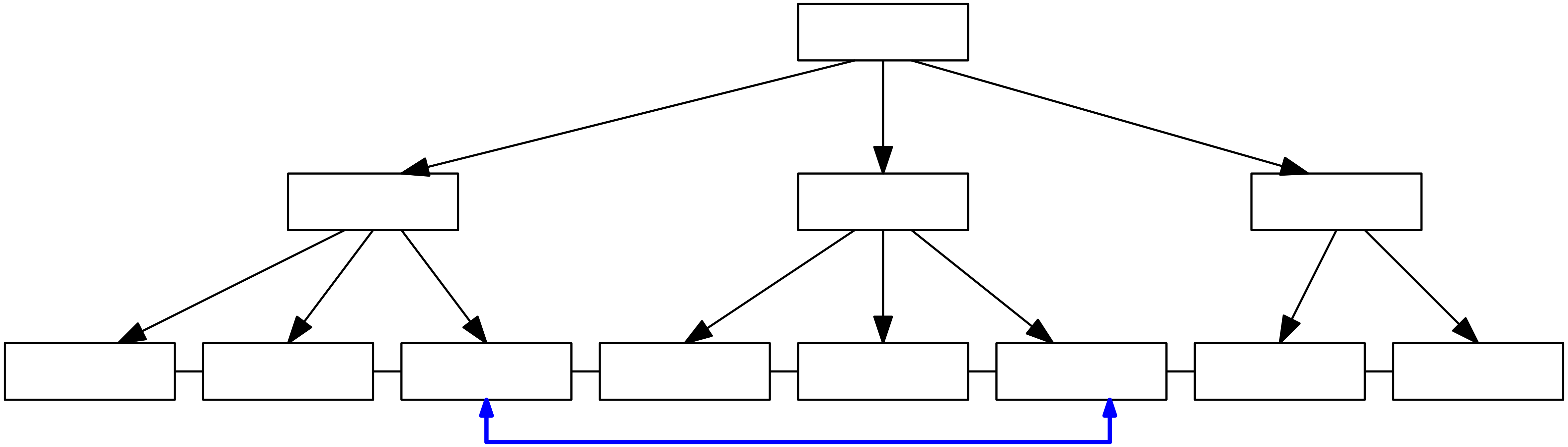
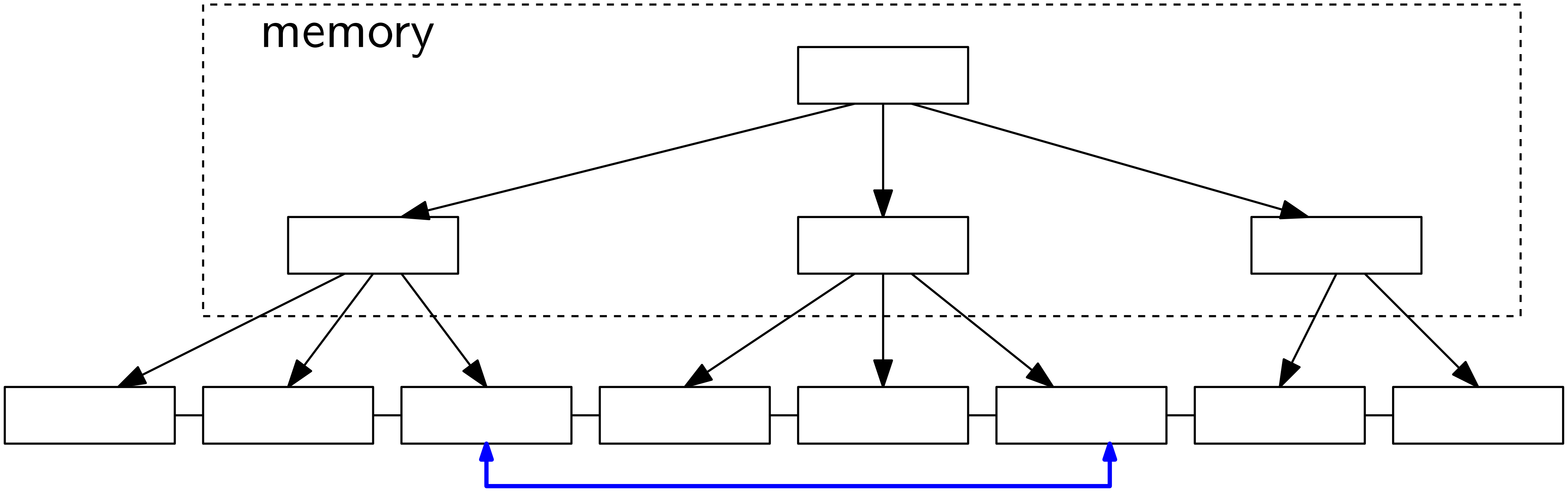


A range query in $O(\log_B N + K/B)$ I/Os

K: output size

The B-tree



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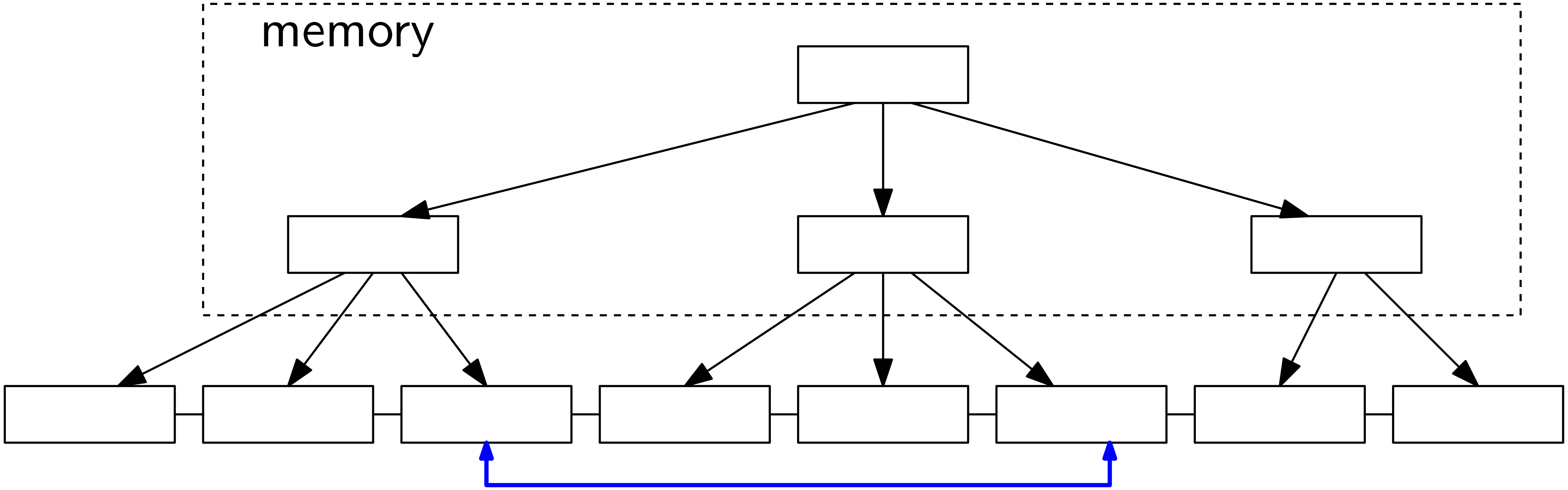

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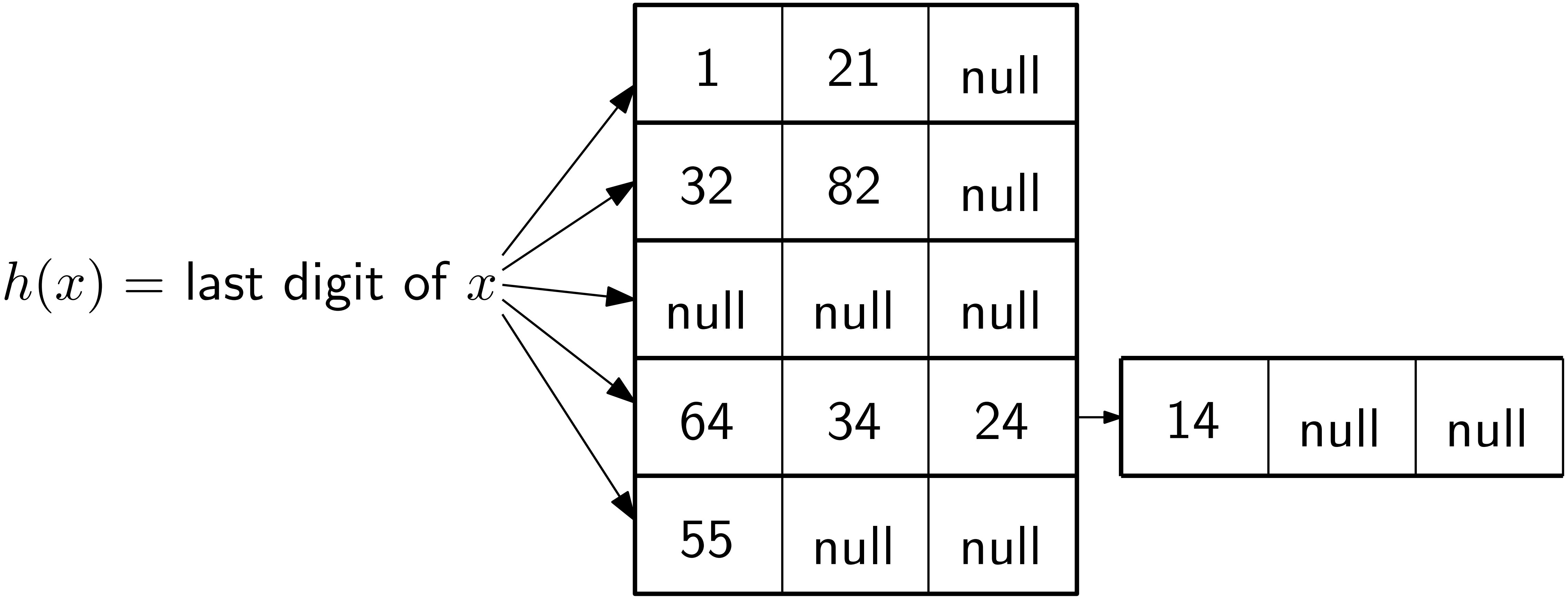
The height of B-tree never goes beyond 5 (e.g., if $B = 100$, then a B-tree with 5 levels stores $n = 10$ billion records). We will assume $\log_B \frac{N}{M} = O(1)$.

External Hashing



$h(x) =$ last digit of x

1	21	null
32	82	null
null	null	null
64	34	24
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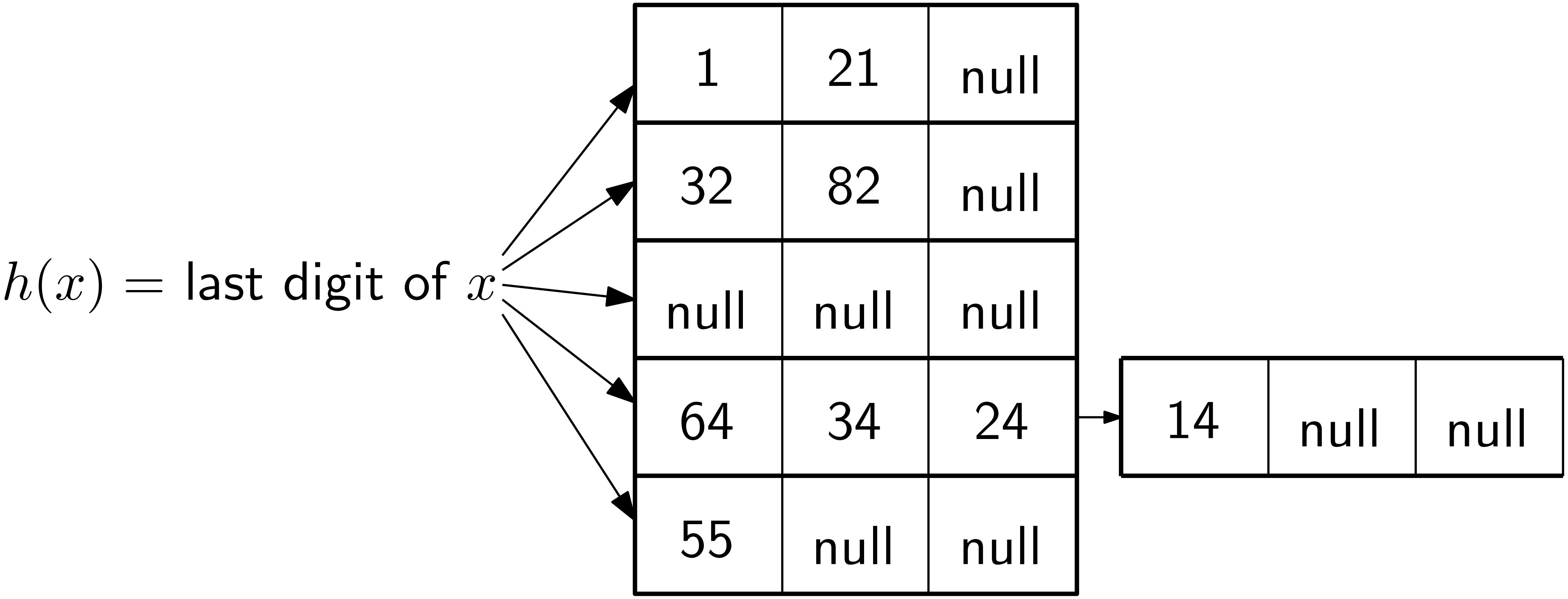

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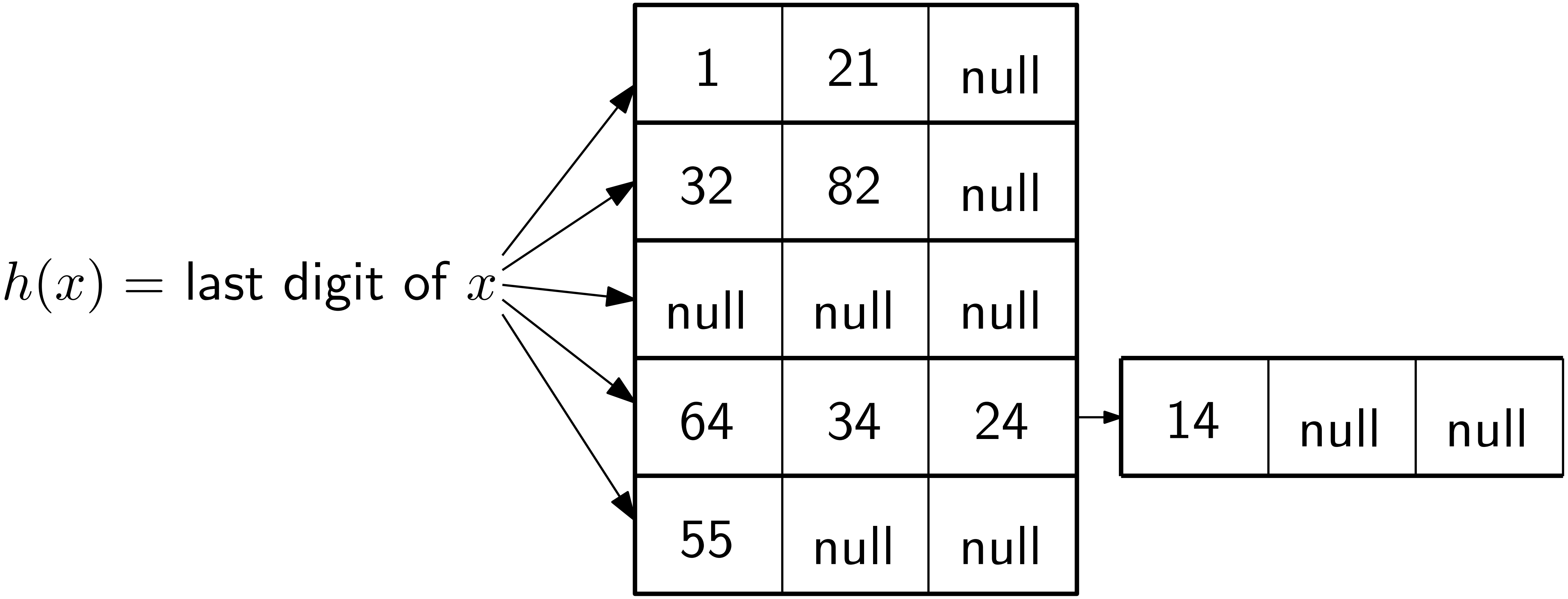

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Expected average cost of a successful (or unsuccessful) lookup is $1 + 1/2^{\Omega(B)}$ disk accesses, provided the load factor is less than a constant smaller than 1 [Knuth, 1973]

Exact Numbers Calculated by Knuth

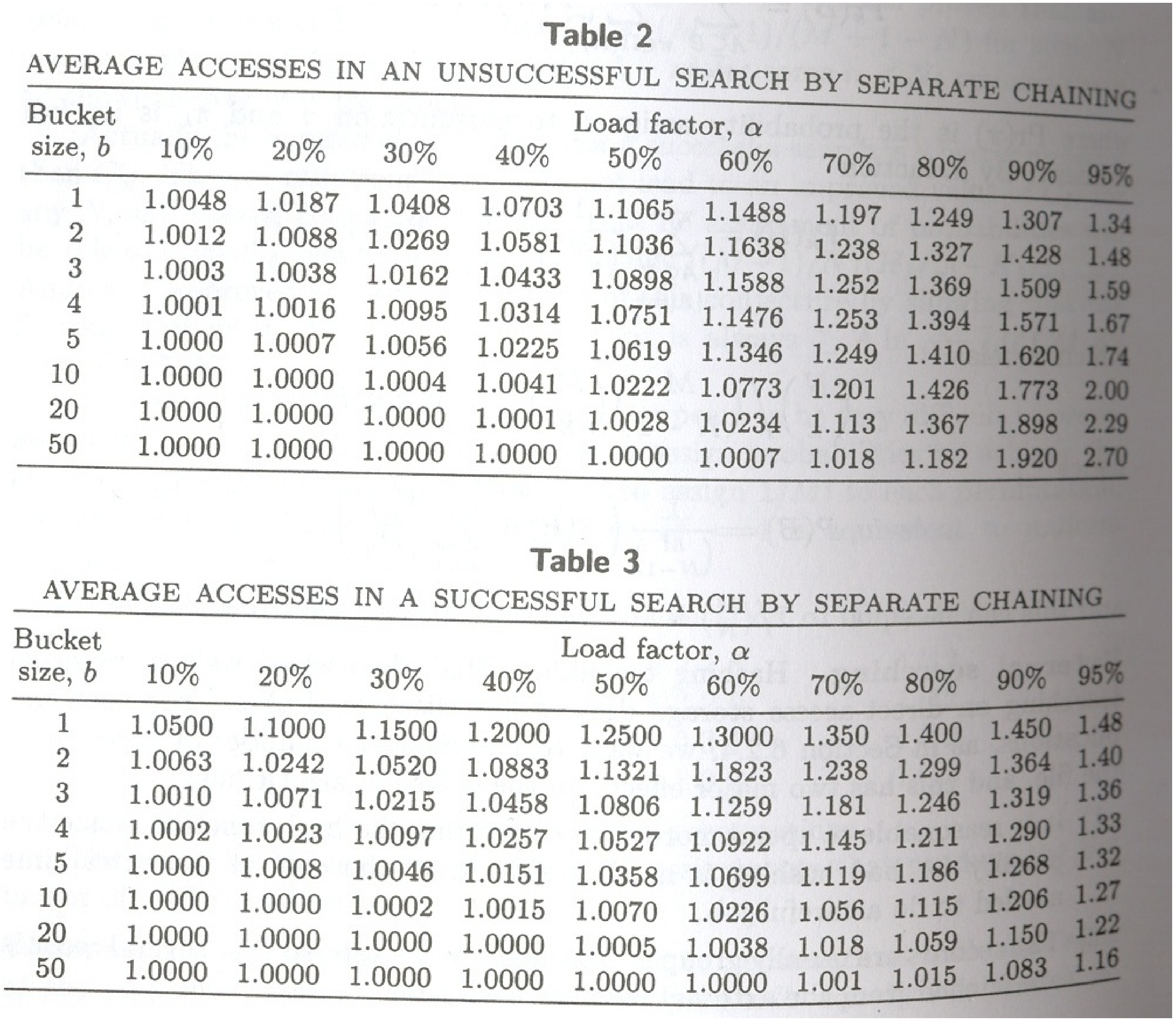

Table 2

AVERAGE ACCESSES IN AN UNSUCCESSFUL SEARCH BY SEPARATE CHAINING

Bucket size, b	Load factor, α 10%	20%	30%	40%	50%	60%	70%	80%	90%	95%
1	1.0048	1.0187	1.0408	1.0703	1.1065	1.1488	1.197	1.249	1.307	1.34
2	1.0012	1.0088	1.0269	1.0581	1.1036	1.1638	1.238	1.327	1.428	1.48
3	1.0003	1.0038	1.0162	1.0433	1.0898	1.1588	1.252	1.369	1.509	1.59
4	1.0001	1.0016	1.0095	1.0314	1.0751	1.1476	1.253	1.394	1.571	1.67
5	1.0000	1.0007	1.0056	1.0225	1.0619	1.1346	1.249	1.410	1.620	1.74
10	1.0000	1.0000	1.0004	1.0041	1.0222	1.0773	1.201	1.426	1.773	2.00
20	1.0000	1.0000	1.0000	1.0001	1.0028	1.0234	1.113	1.367	1.898	2.29
50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0007	1.018	1.182	1.920	2.70

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The Art of Computer Programming, volume 3, 1998, page 542

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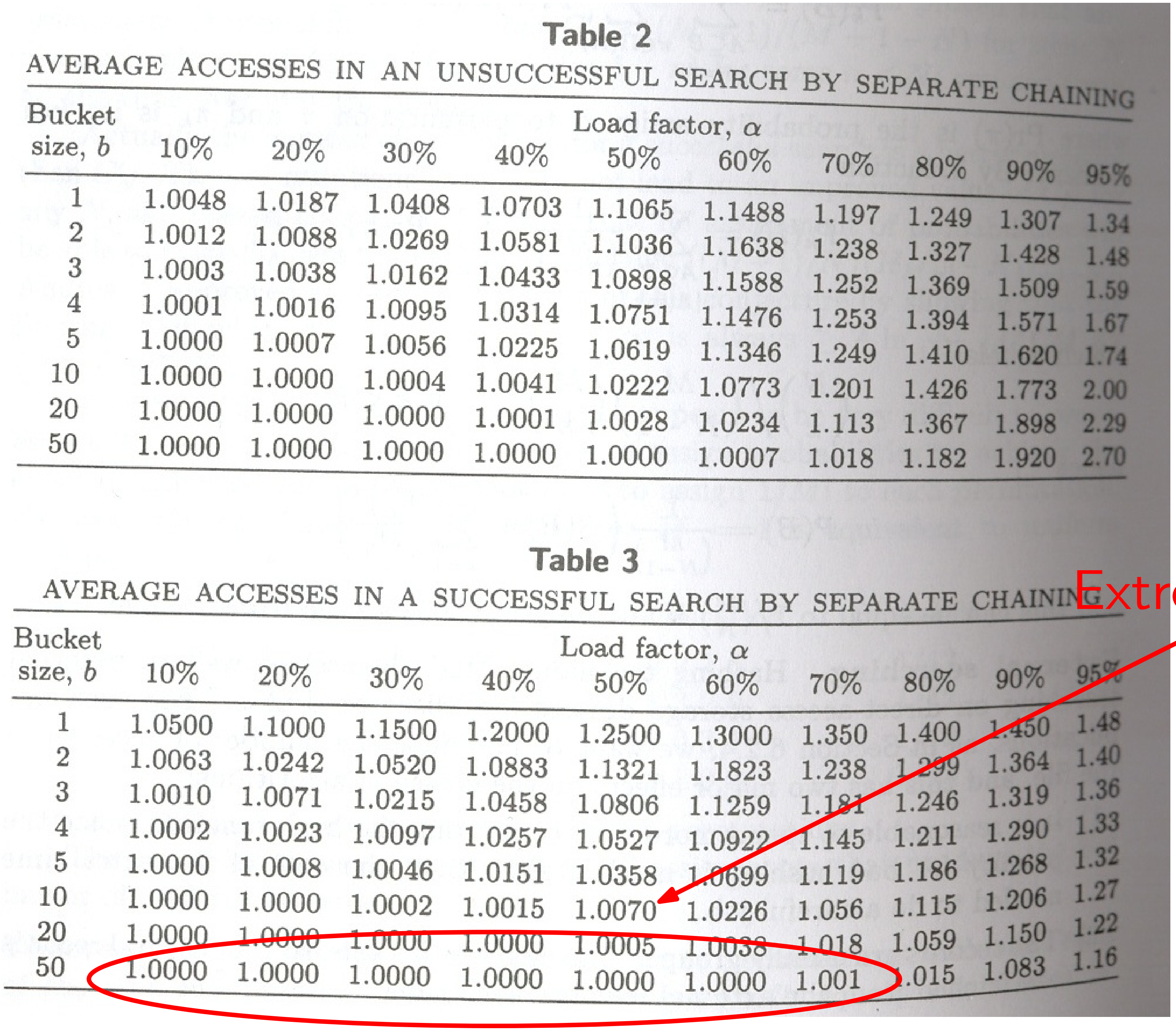

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Extremely close to ideal

The Art of Computer Programming, volume 3, 1998, page 542

Now Let's Go Dynamic

- Focus on insertions first: Both the B-tree and hash table do a search first, then insert into the appropriate block
  - B-tree: Split blocks when necessary
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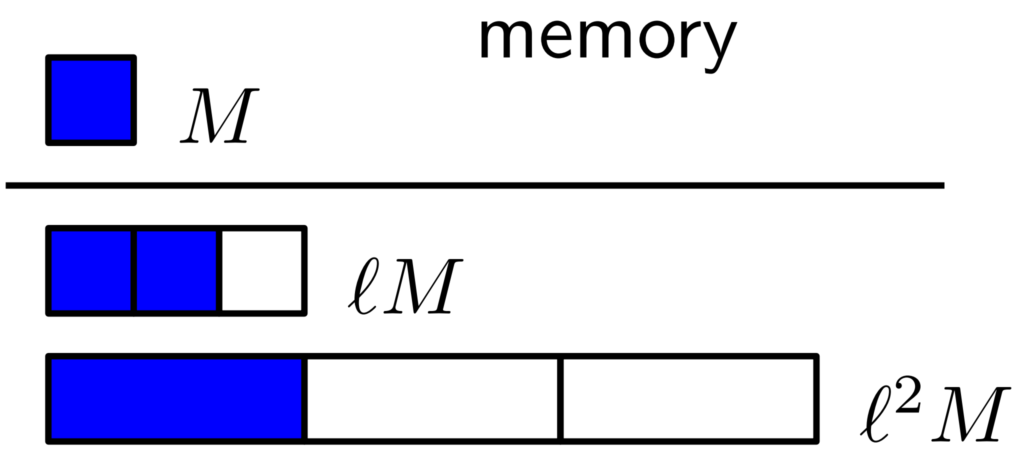
- Cannot hope for lower than 1 I/O per insertion only if the changes must be committed to disk right away (necessary?)
  - Otherwise we probably can lower the amortized insertion cost by buffering, like numerous problems in external memory, e.g. stack, priority queue,... All of them support an insertion in $O(1/B)$ I/Os — the best possible

Dynamic B-trees

Dynamic Hash Tables

Dynamic B-trees for Fast Insertions

- *LSM-tree* [O'Neil, Cheng, Gawlick, O'Neil, Acta Informatica'96]: Logarithmic method + B-tree



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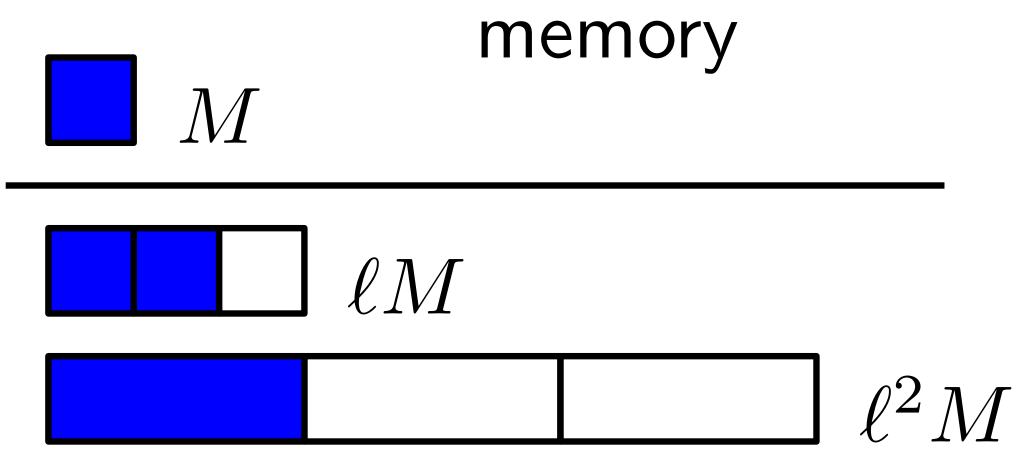

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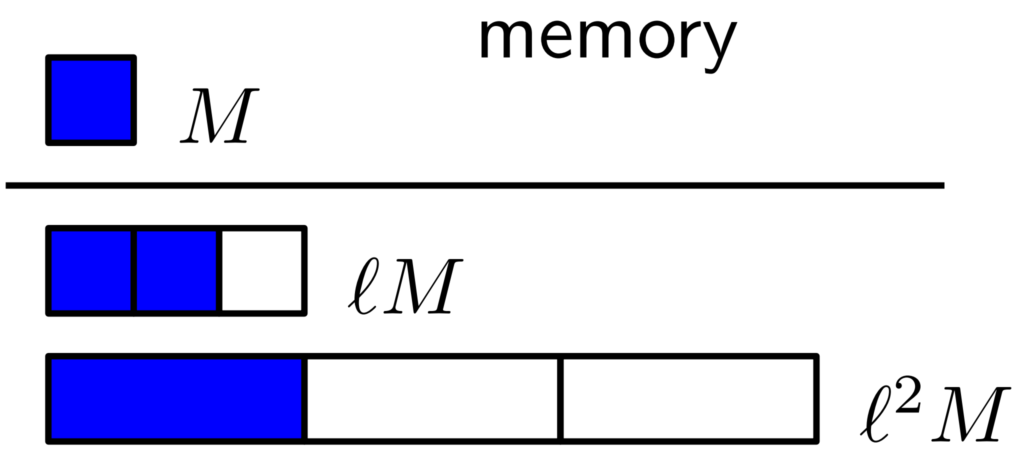

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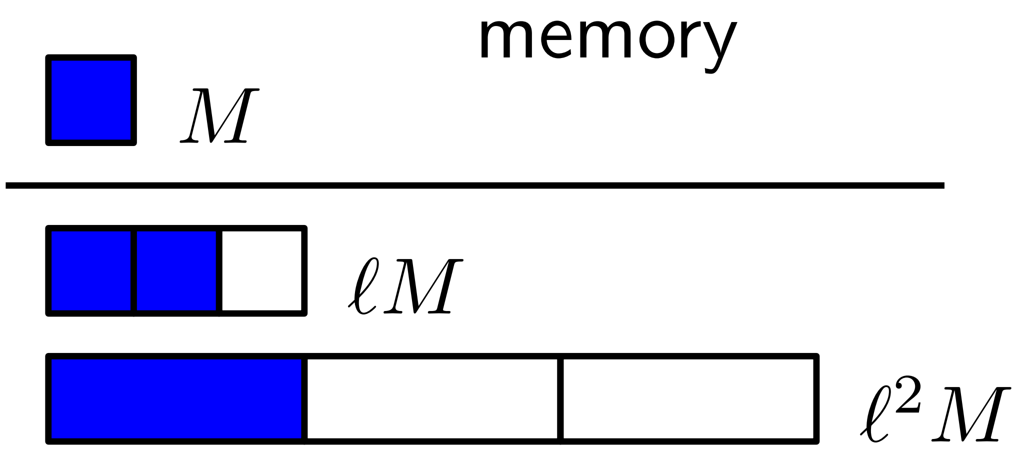

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  - Insertion: $O(\frac{1}{B} \log_\ell \frac{N}{M})$
  - Query: $O(\ell \log_\ell \frac{N}{M})$
- Usually ℓ is set to be a constant, then they both have $O(\frac{1}{B} \log \frac{N}{M})$ insertion and $O(\log \frac{N}{M})$ query

More Dynamic B-trees

- Buffer-tree (buffered-repository tree) [Arge, WADS'95; Buchsbaum, Goldwasser, Venkatasubramanian, Westbrook, SODA'00]
- Streaming B-tree [Bender, Farach-Colton, Fineman, Fogel, Kuszmaul, Nelson, SPAA'07]
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- Cache-oblivious model [Demaine, Fineman, Iacono, Langerman, Munro, SODA'10]

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  - Deletions? Standard trick: inserting "delete signals"
- No better solutions known ...

Compare with the rich results in RAM!

- Range reporting
  - $O(\sqrt{\log N / \log\log N})$ insertion and query [Andersson, Thorup, JACM'07]
  - $O(\log N / \log\log N)$ insertion and $O(\log\log N)$ query [Mortensen, Pagh, Pătrașcu, STOC'05]
  - Other results that depend on the word size w
- Predecessor
  - $\Theta(\sqrt{\log N / \log\log N})$ insertion and query [Andersson, Thorup, JACM'07]
- Partial-sum
  - $\Theta(\log N)$ insertion query [Pătrașcu, Demaine, SODA'04]

Are the EM and DB people just dumb?

Our Main Result

For any dynamic range query index with a query cost of q and an amortized insertion cost of u, the following tradeoff holds

$$\begin{cases} q \cdot \log(uB/q) = \Omega(\log B), & \text{for } q < \alpha \log B, \alpha \text{ is any constant;} \\ uB \cdot \log q = \Omega(\log B), & \text{for all } q. \end{cases}$$

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Assuming $\log_B \frac{N}{M} = O(1)$, all the bounds are tight!

Current upper bounds:

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$\frac{1}{B} \log \frac{N}{M}$

Can't be true for $B = o(\sqrt{\log n \log\log n})$, since the *exponential tree* achieves $u = q = O(\sqrt{\log n / \log\log n})$ [Andersson, Thorup, JACM'07]. ($n = N/M$)

The real question

How large does B need to be for buffer-tree to be optimal for range reporting?

Known: somewhere between $\Omega(\sqrt{\log n \log\log n})$ and $O(n^{\epsilon})$

Lower Bound Model: Dynamic Indexability

- *Indexability*: [Hellerstein, Koutsoupias, Papadimitriou, PODS’97, JACM’02]

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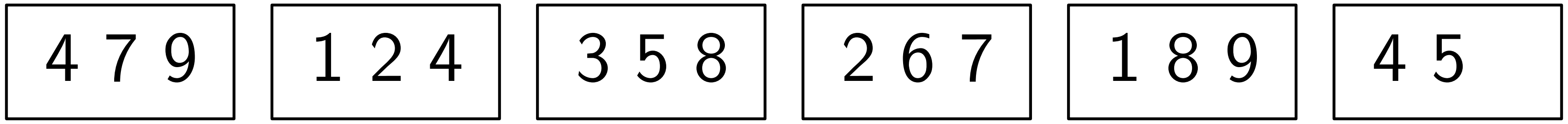

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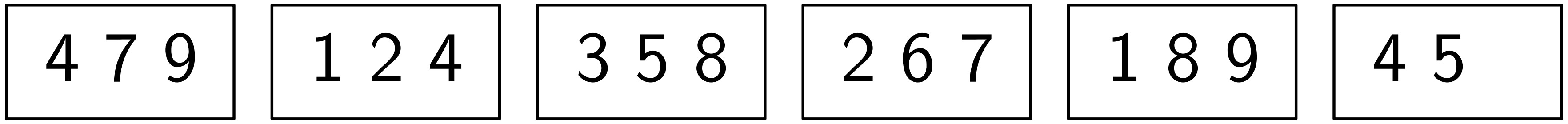

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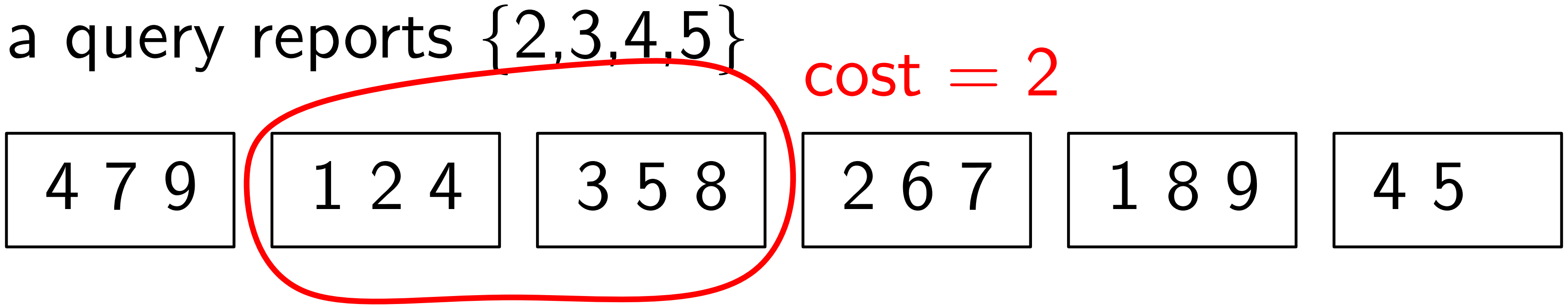

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  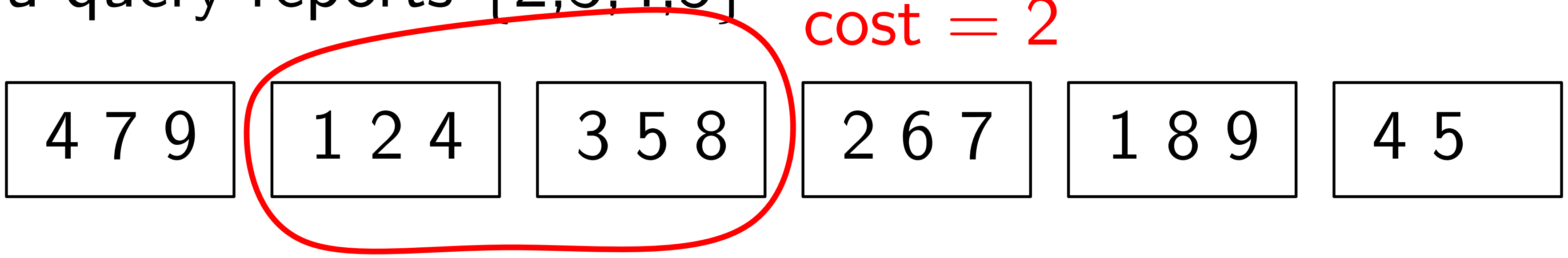

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- Similar in spirit to popular lower bound models: cell probe model, semigroup model

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- 2D range queries: $s/N \cdot \log q = \Omega(\log(N/B))$ [Hellerstein, Koutsoupias, Papadimitriou, PODS'97], [Koutsoupias, Taylor, PODS'98], [Arge, Samoladas, Vitter, PODS'99]

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  - Adding dynamization makes it much more interesting!

Dynamic Indexability

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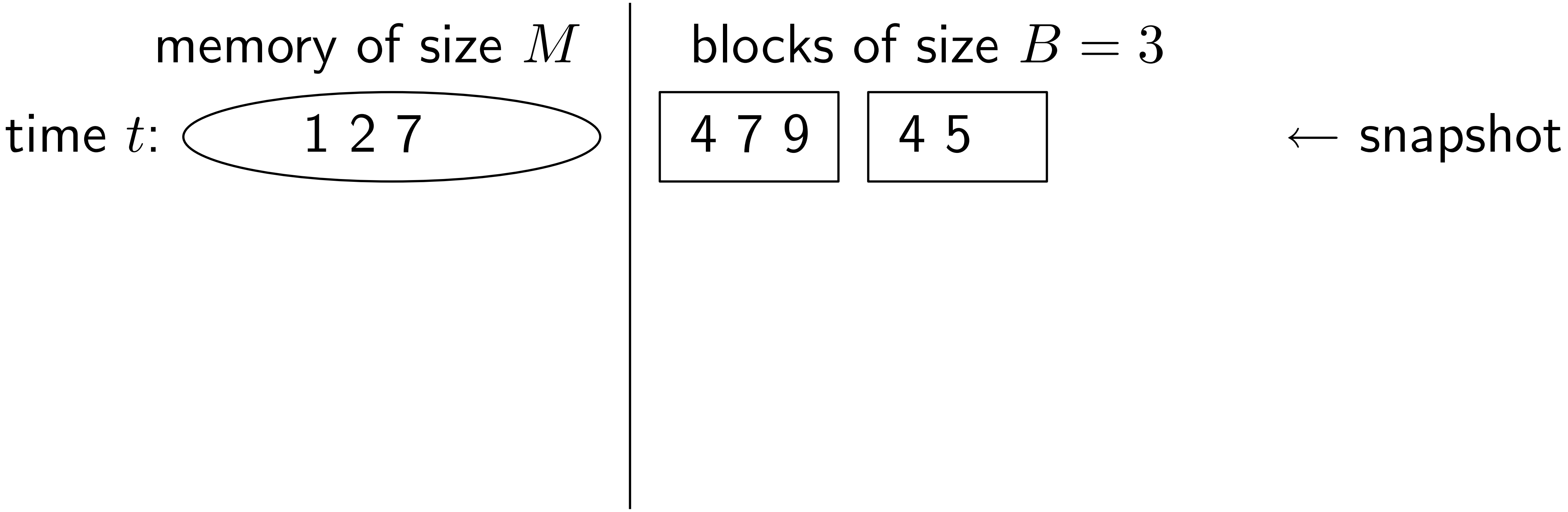

memory of size M

blocks of size $B = 3$

time t: 1 2 7

4 7 9

4 5

← snapshot

Dynamic Indexability

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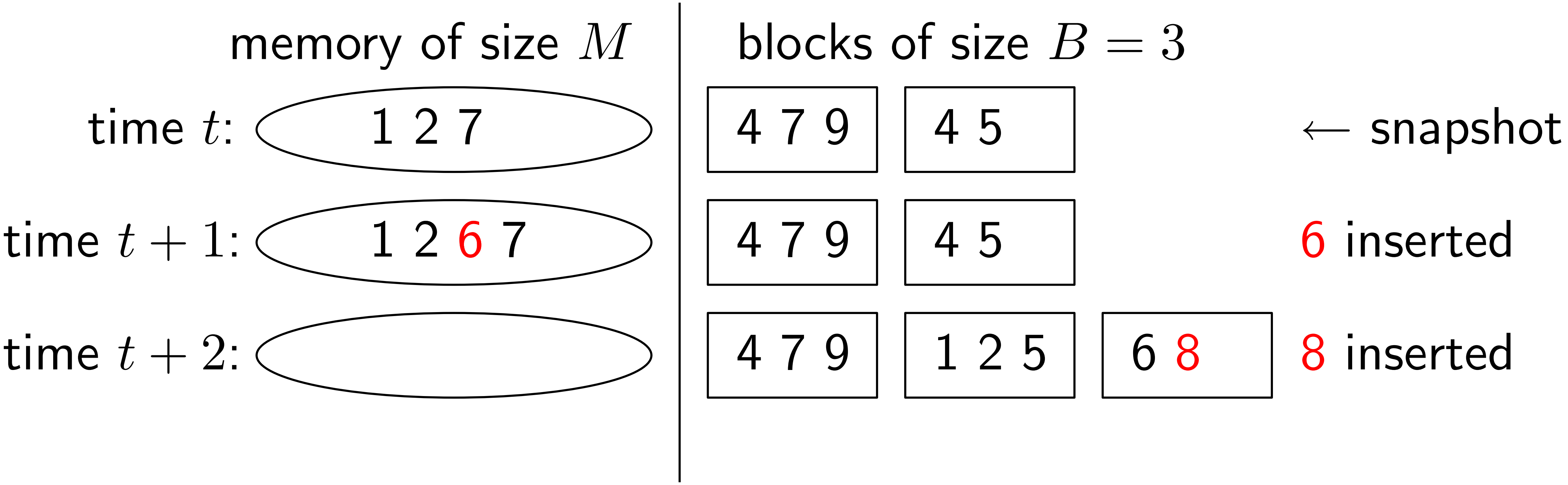

	memory of size M	blocks of size $B = 3$	
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Dynamic Indexability

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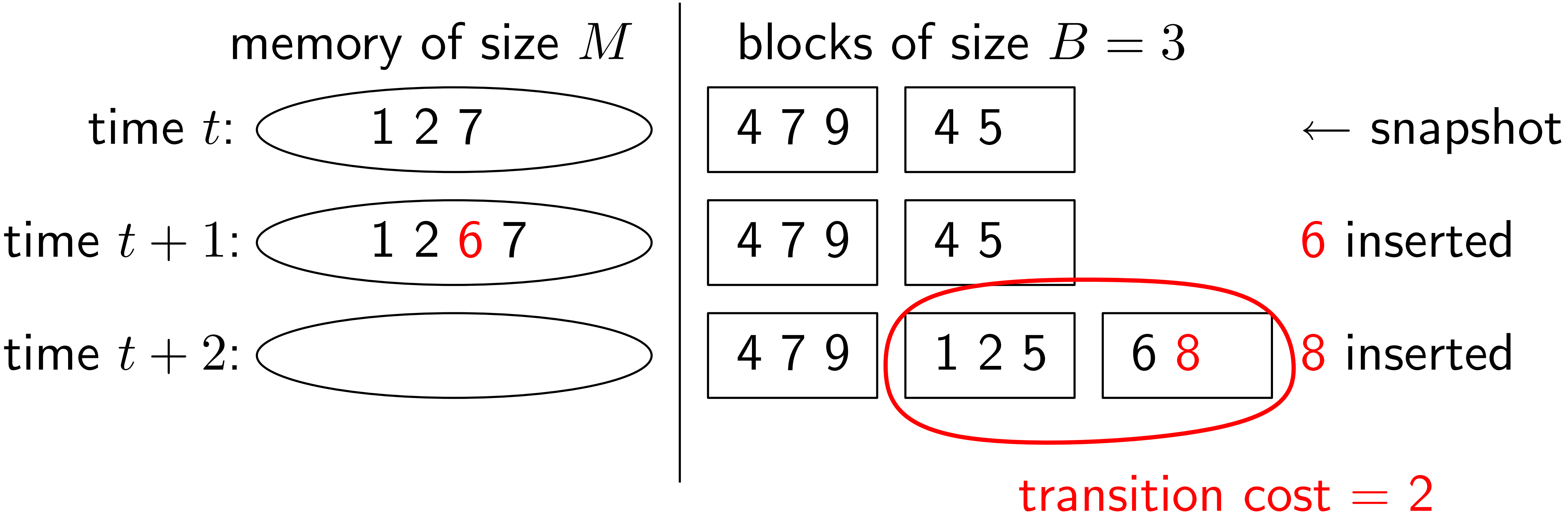

memory of size M

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transition cost $= 2$

Dynamic Indexability

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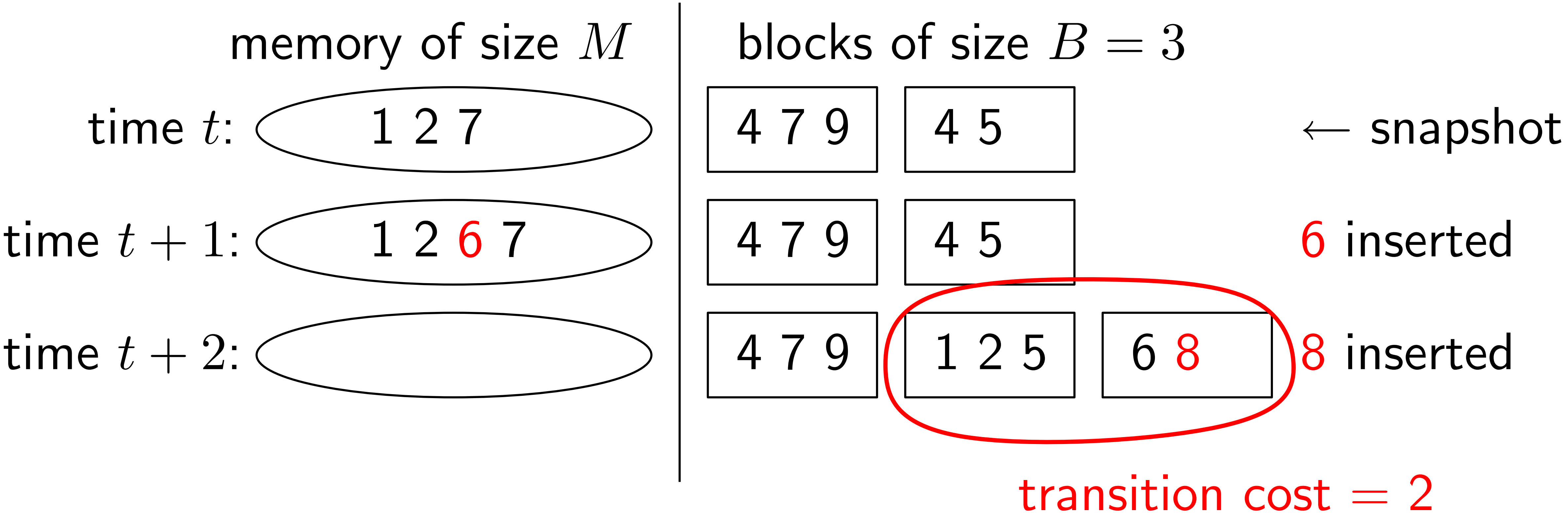

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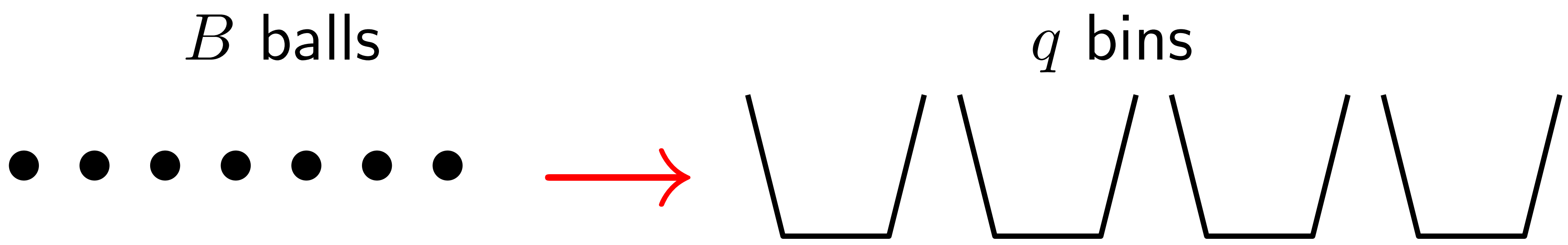
transition cost = 2

- Update cost: u = amortized transition cost per insertion

The Ball-Shuffling Problem



B balls

q bins

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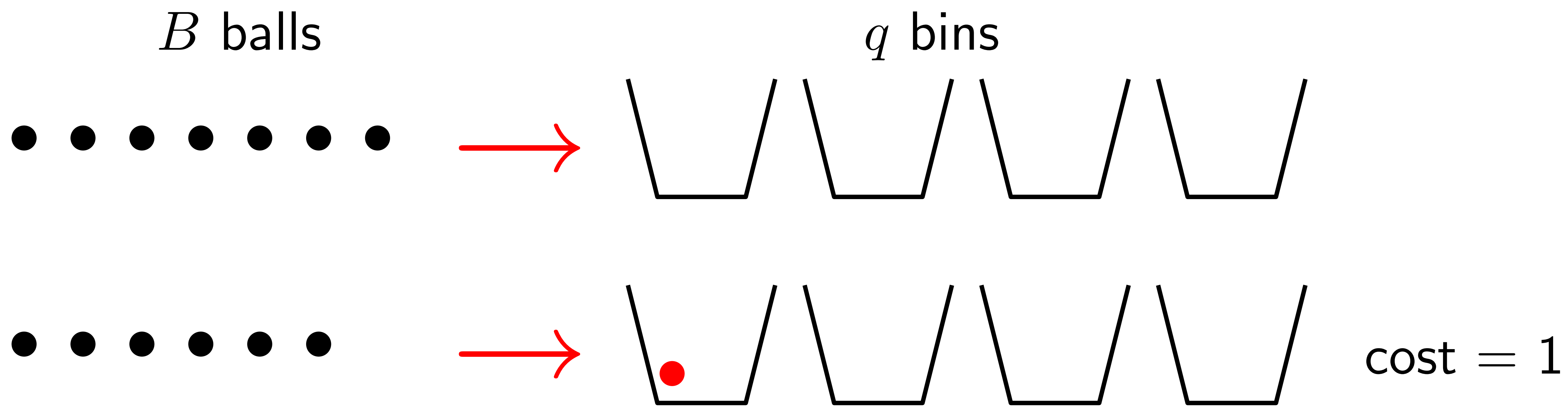

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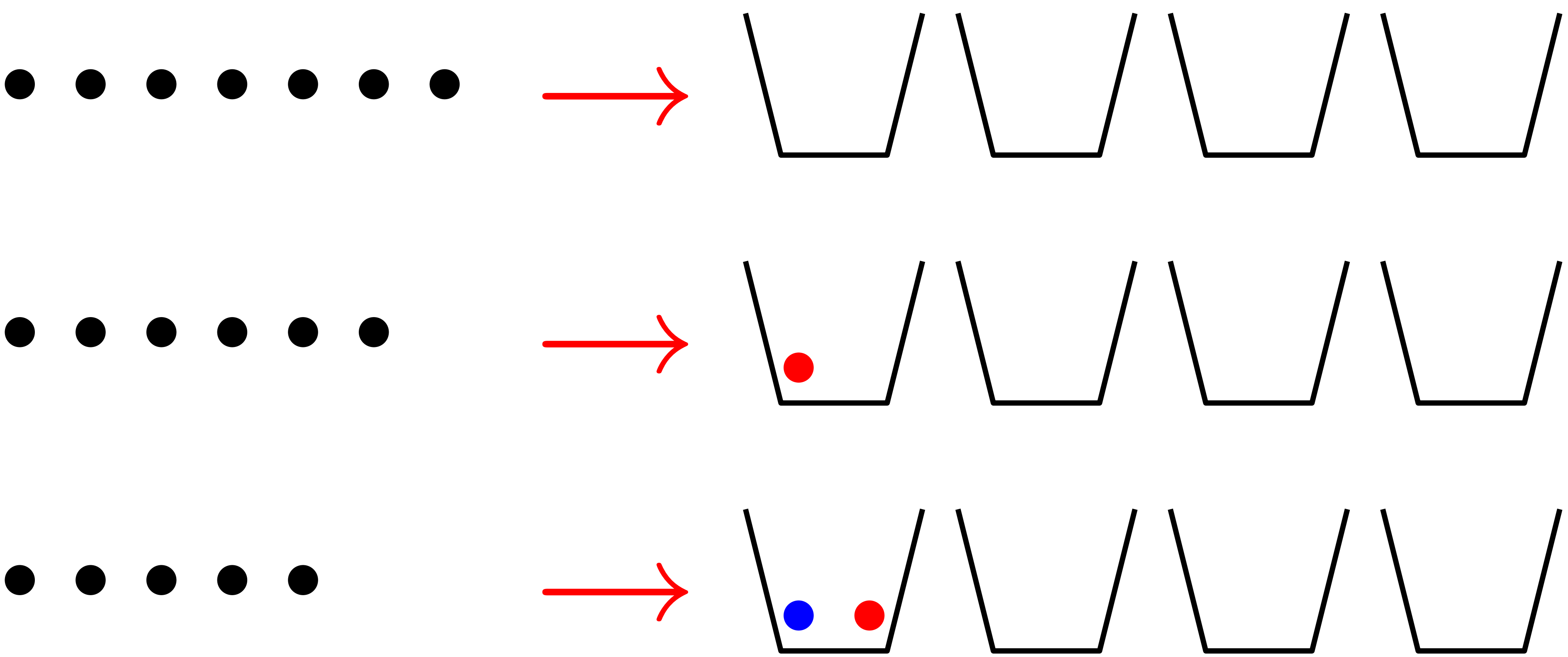

B balls

q bins

cost $= 1$

cost $= 2$

cost of putting the ball directly into a bin $=$ $\#$ balls in the bin $+ 1$

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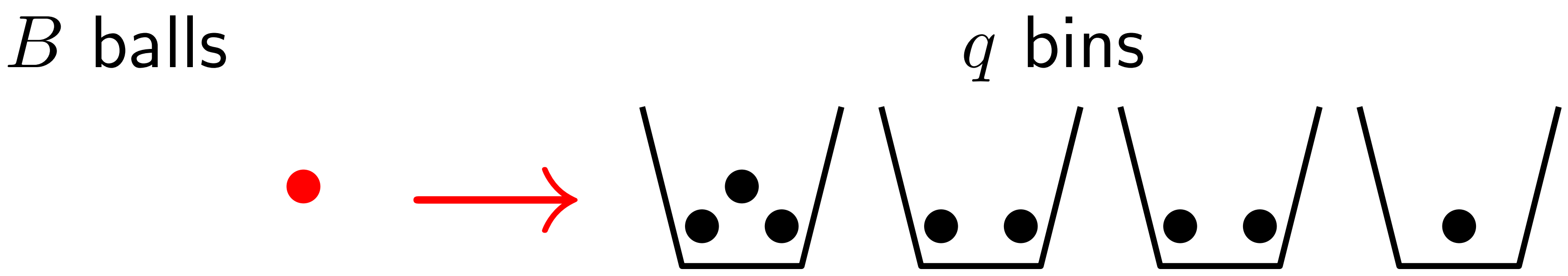

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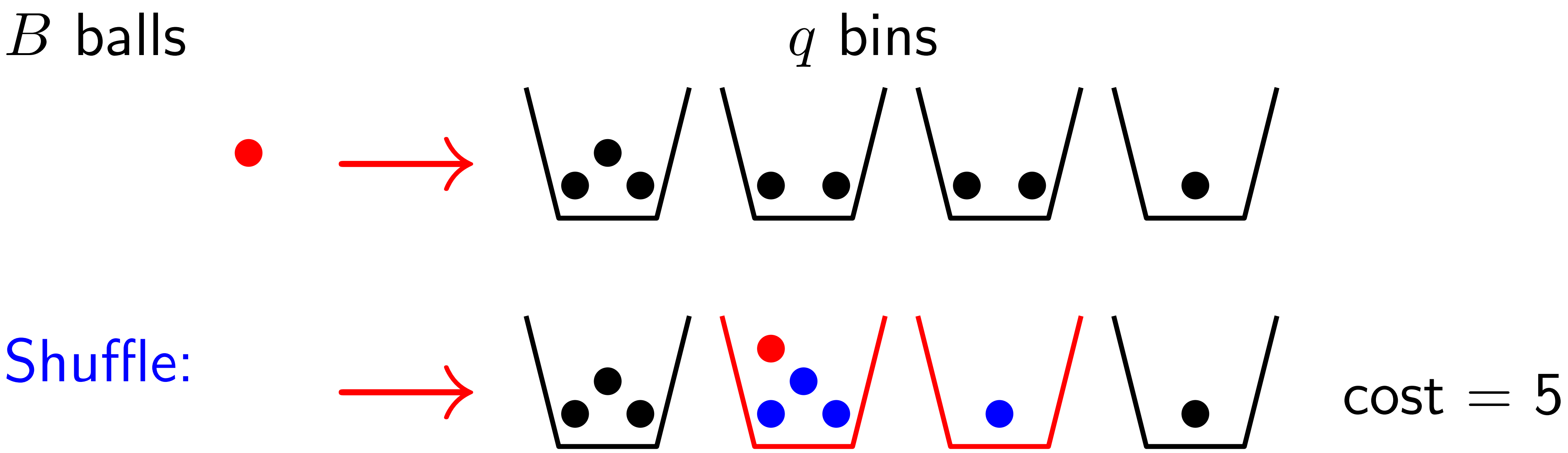

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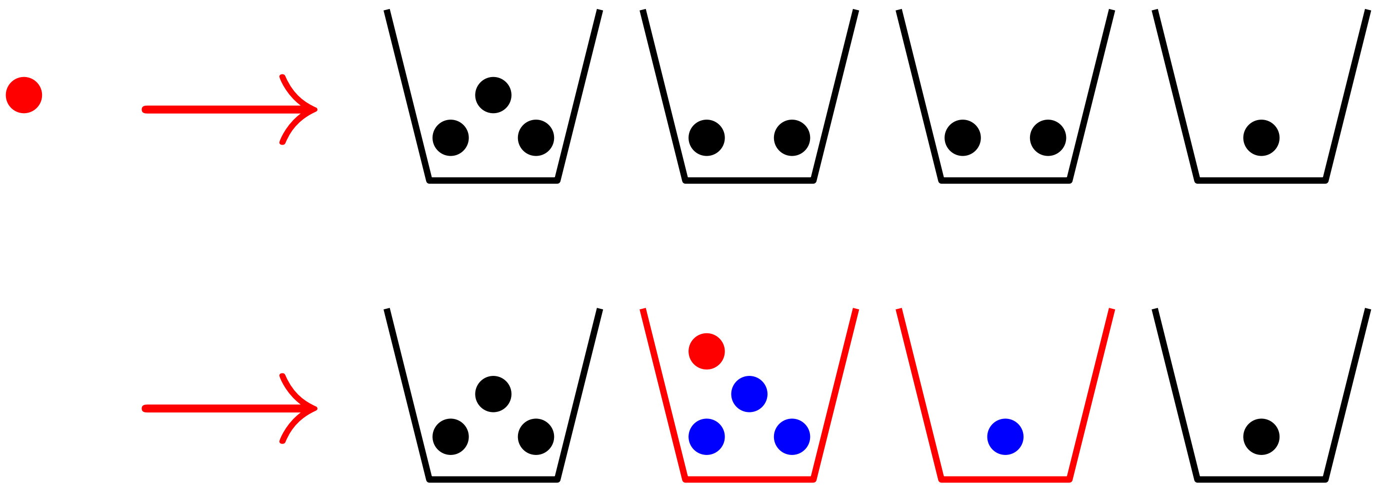

Shuffle:

cost $= 5$

Cost of shuffling $=$ # balls in the involved bins

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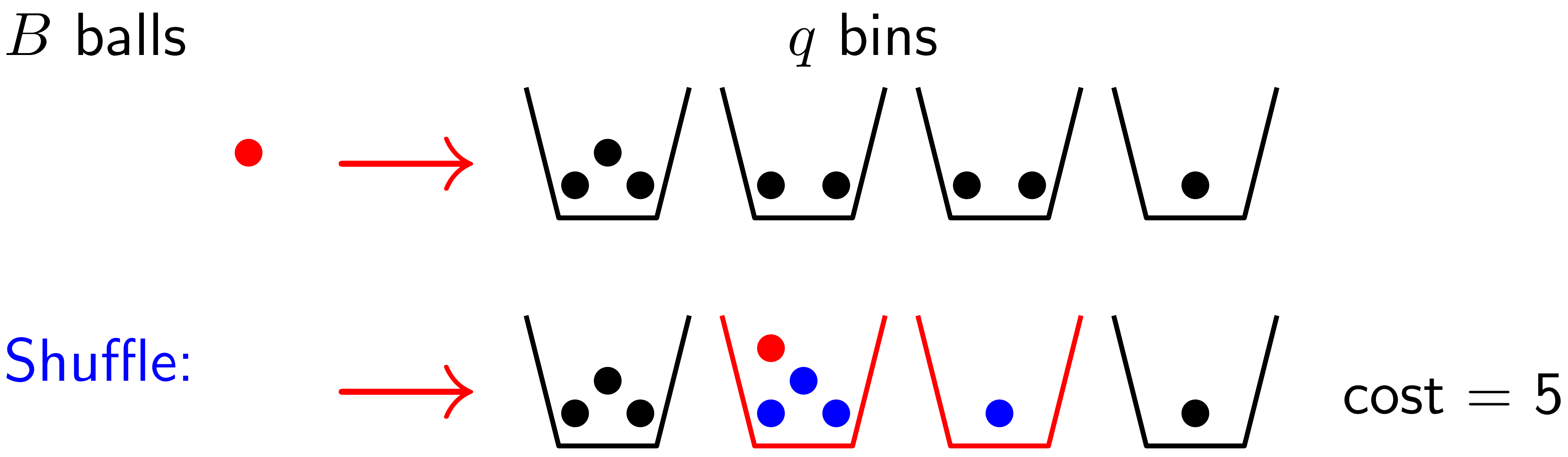

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Putting a ball directly into a bin is a special shuffle

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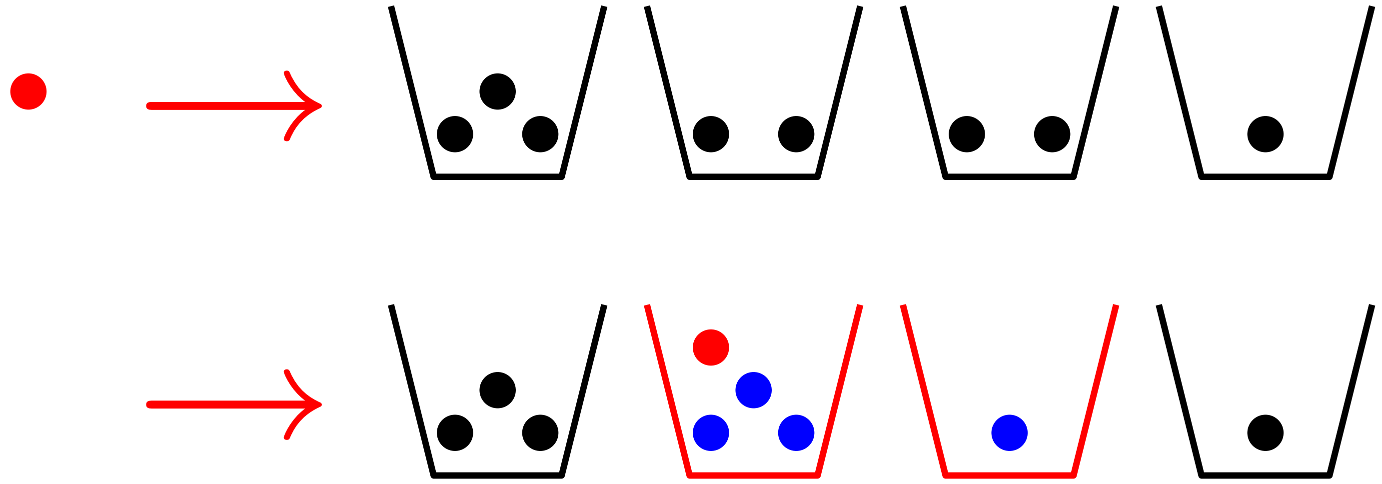

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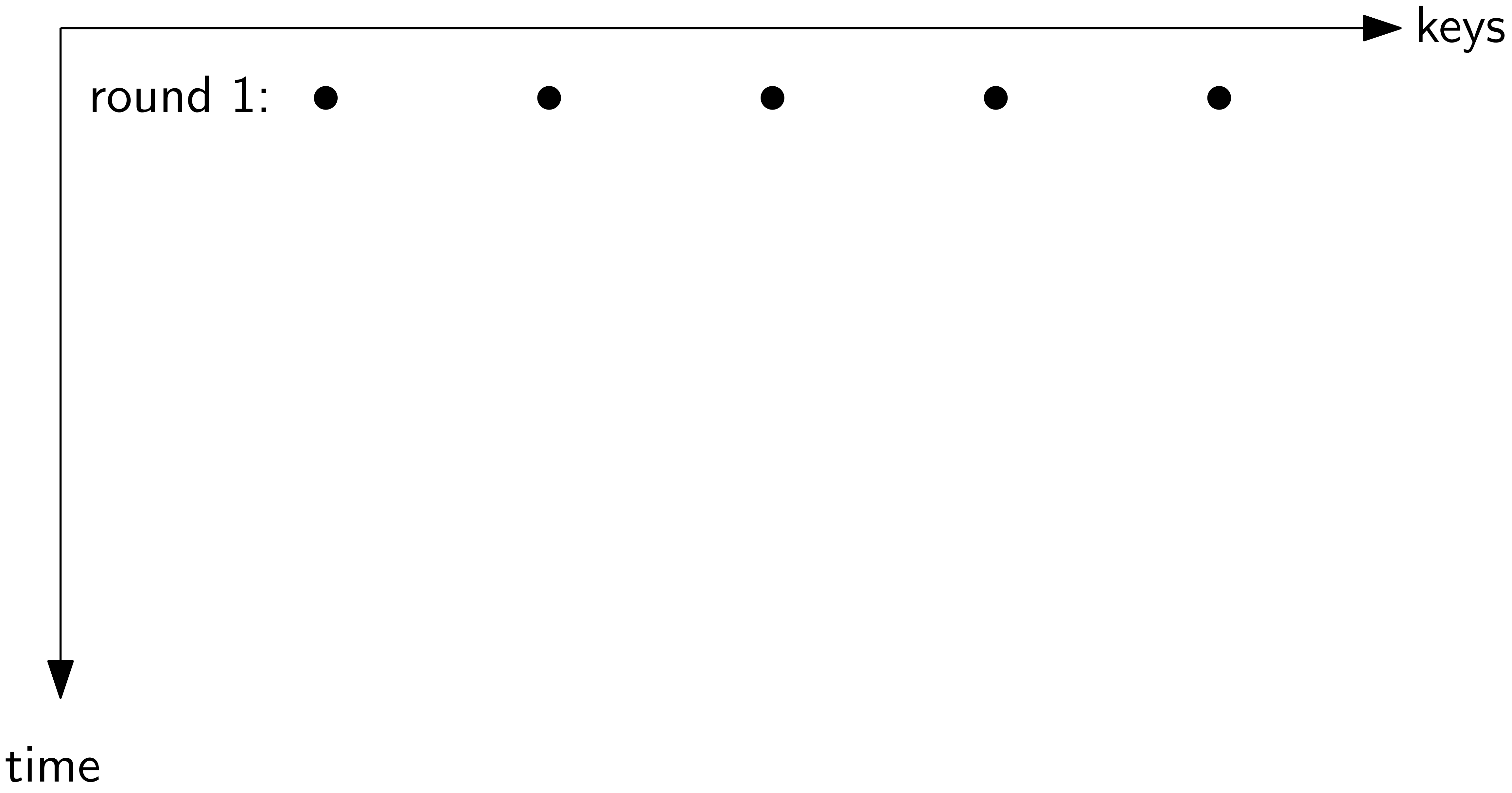
Goal: Accommodating all B balls using q bins with minimum cost

The Workload Construction



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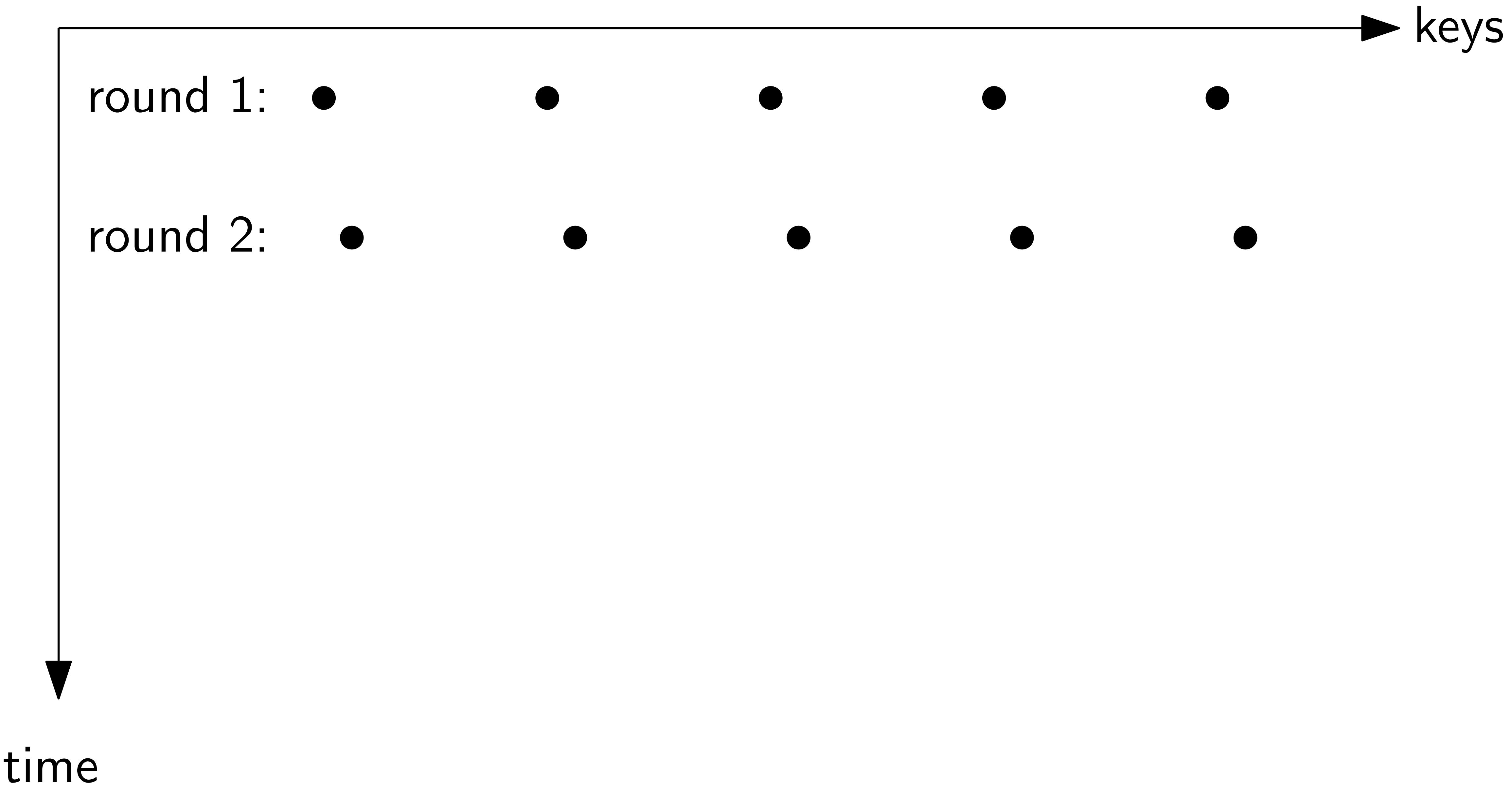

keys

round 1:

round 2:

time

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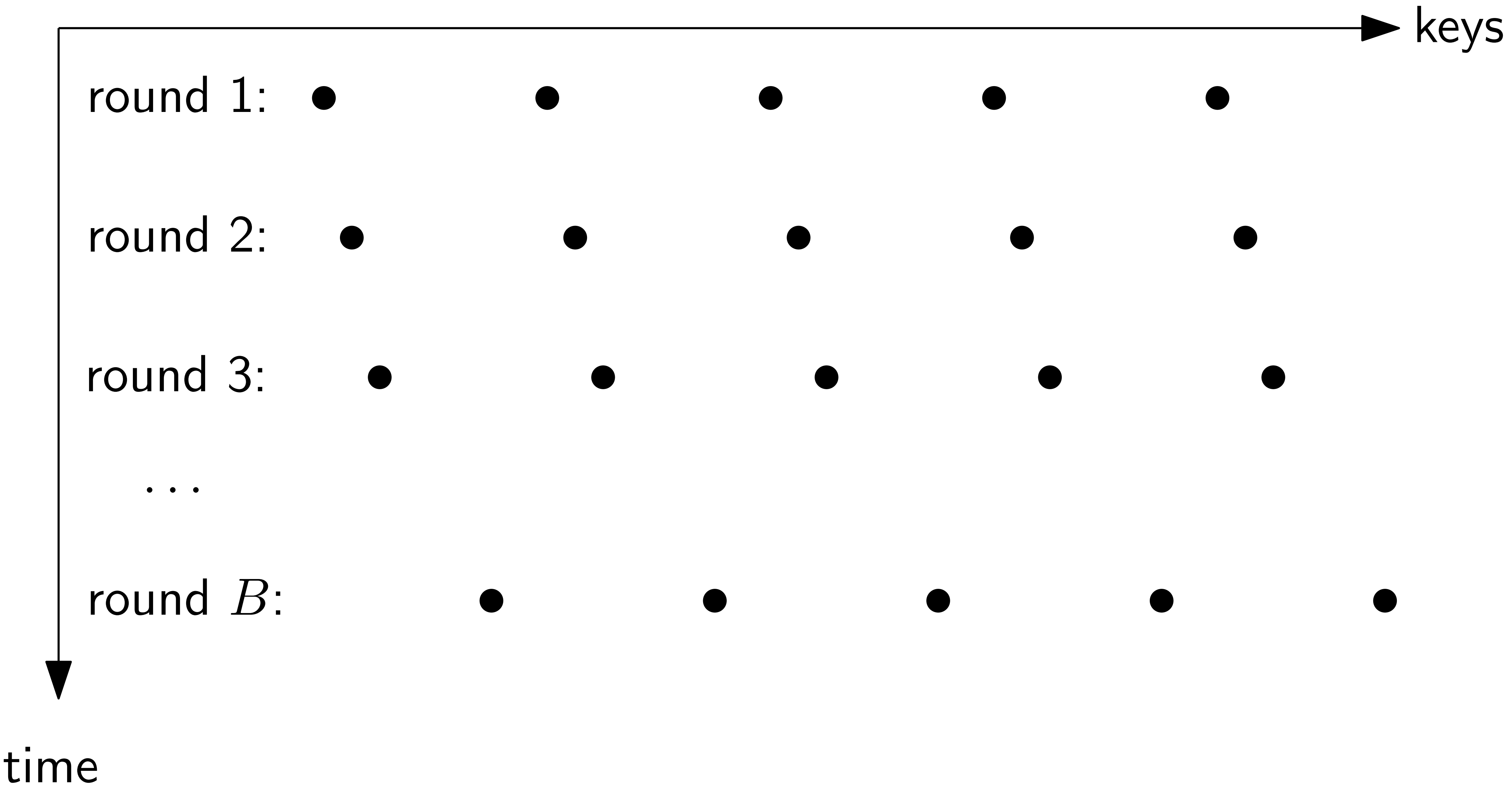

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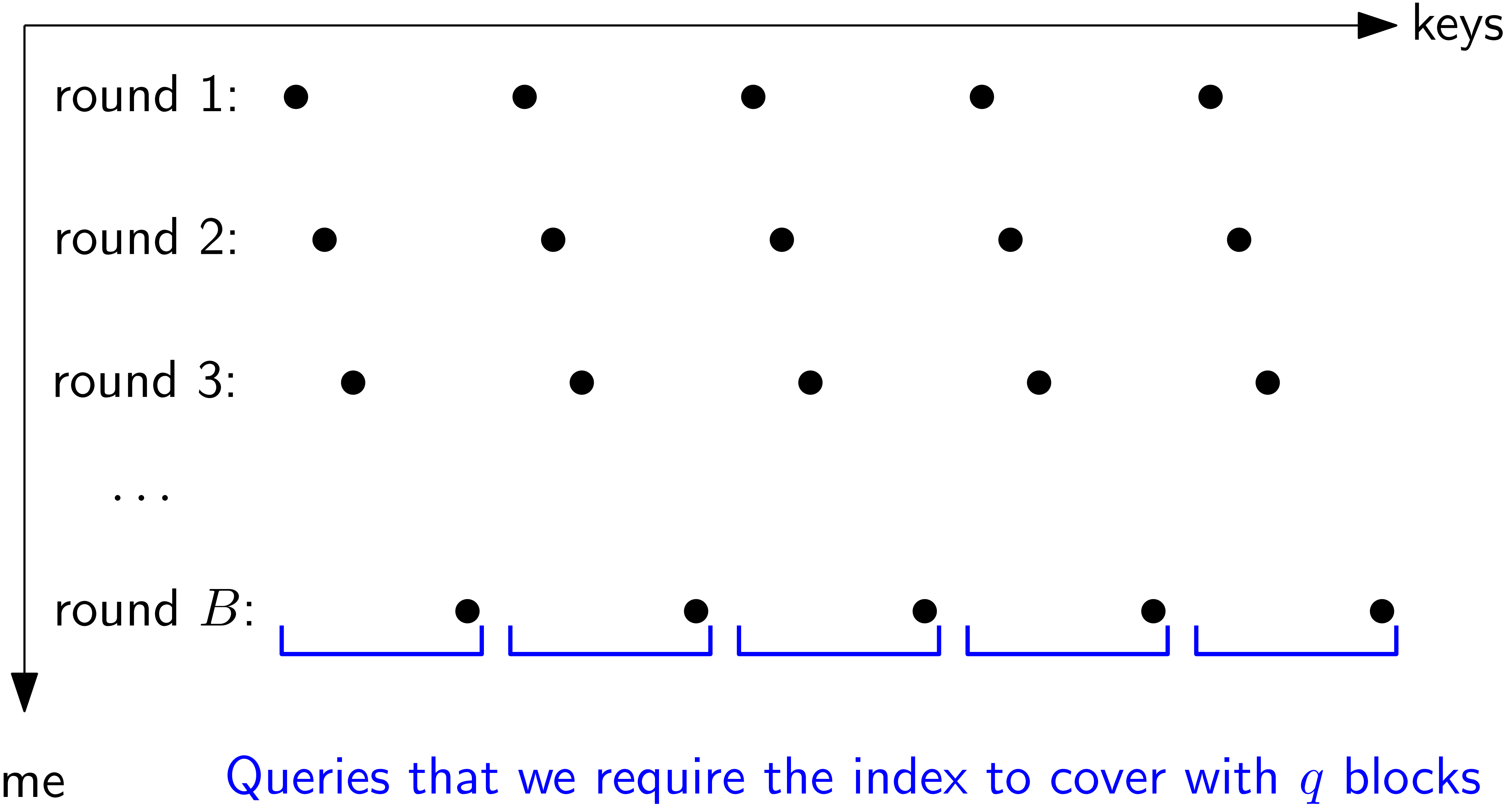

Queries that we require the index to cover with q blocks

\# queries $\geq 2MB$

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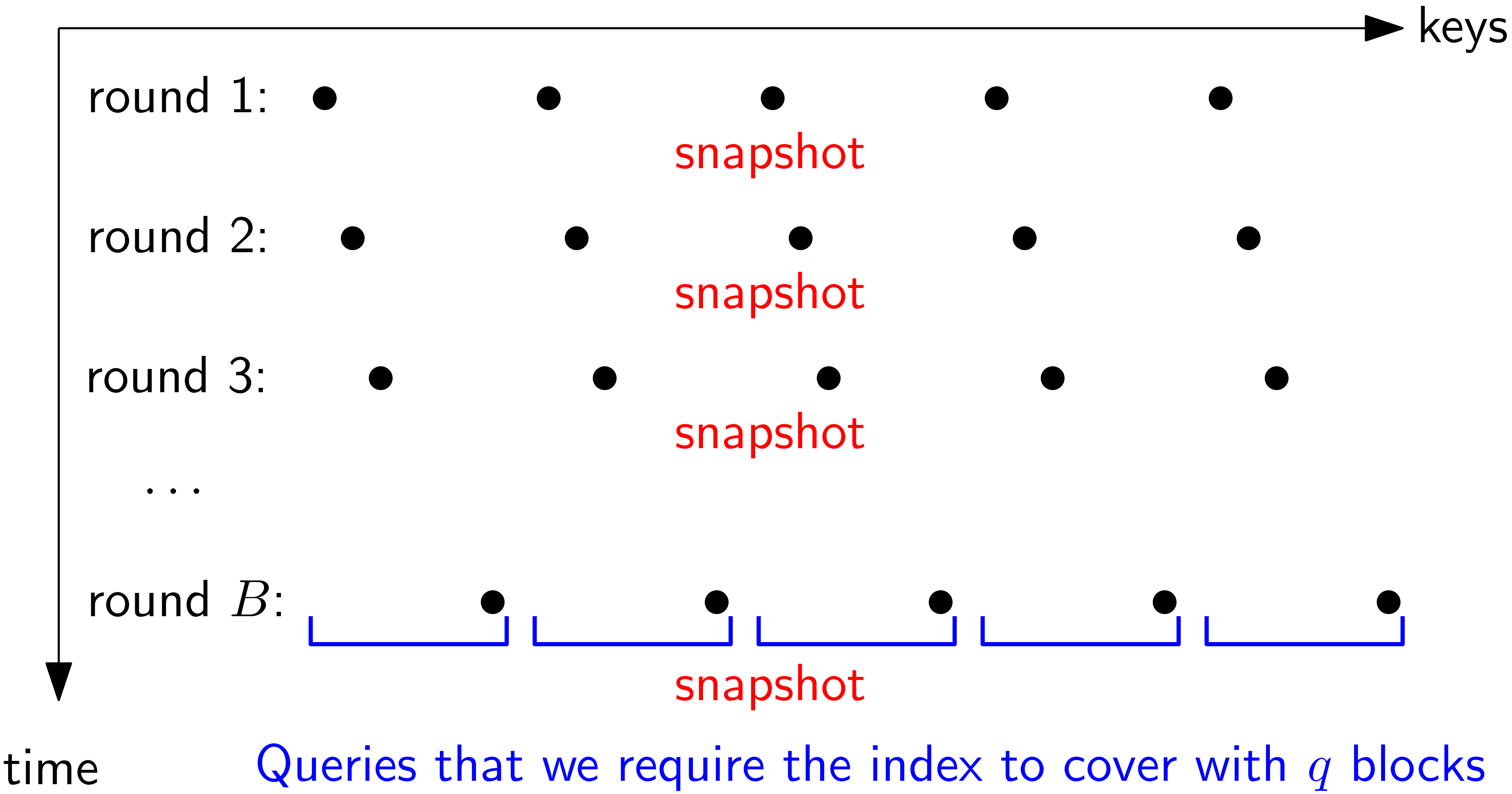

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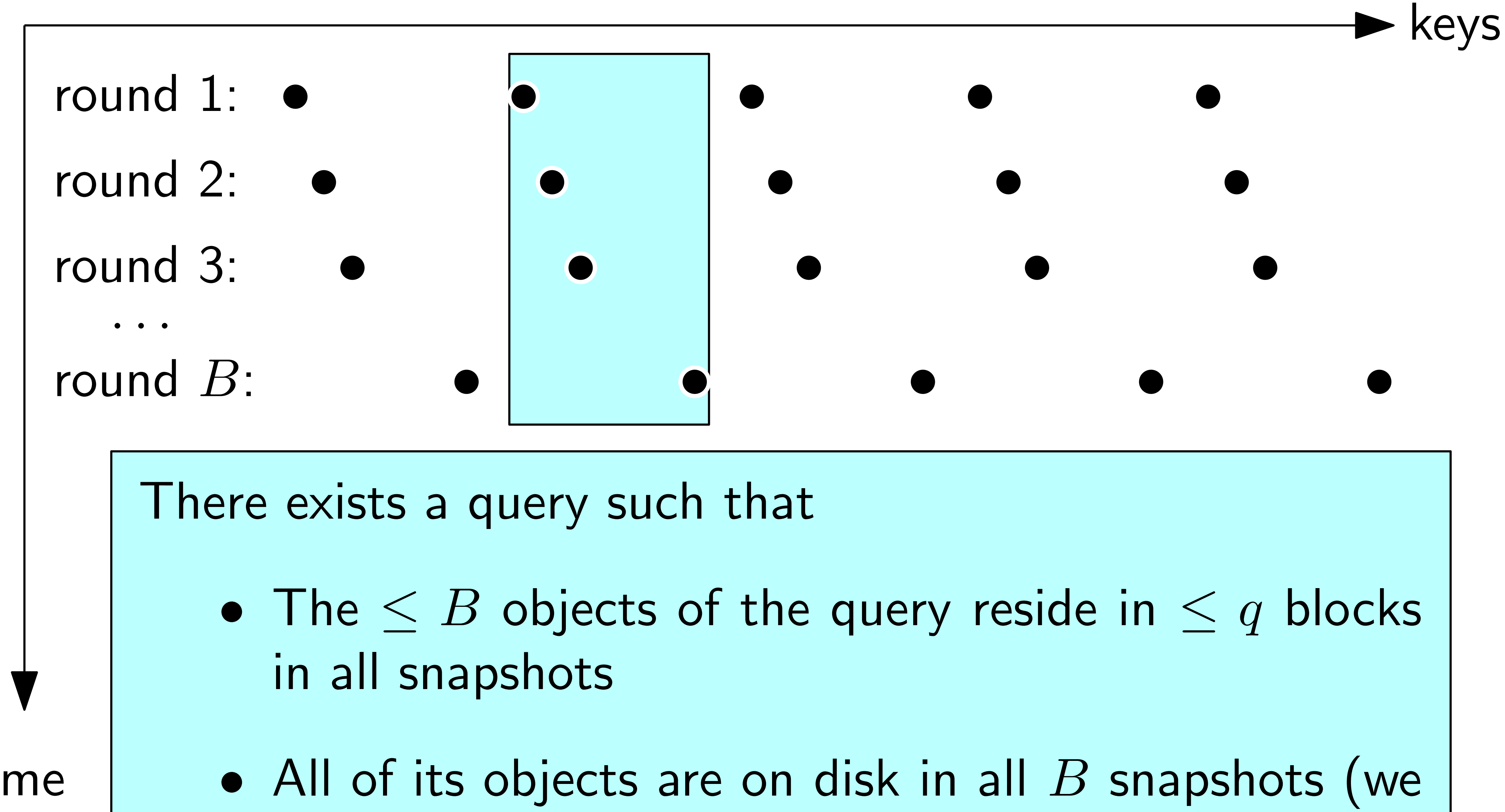
Snapshots of the dynamic index considered

The Workload Construction



There exists a query such that

- The $\leq B$ objects of the query reside in $\leq q$ blocks in all snapshots
- All of its objects are on disk in all B snapshots (we have $\geq MB$ queries)
- The index moves its objects uB^2 times in total

The Reduction

An index with update cost u and query A gives us a solution to the ball-shuffling game with cost uB^2 for B balls and q bins

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$$\begin{cases} \Omega(q \cdot B^{1+\Omega(1/q)}), & \text{for } q < \alpha \log B \text{ where } \alpha \text{ is any constant;} \\ \Omega(B \log_q B), & \text{for any } q. \end{cases}$$

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$$\Downarrow$$

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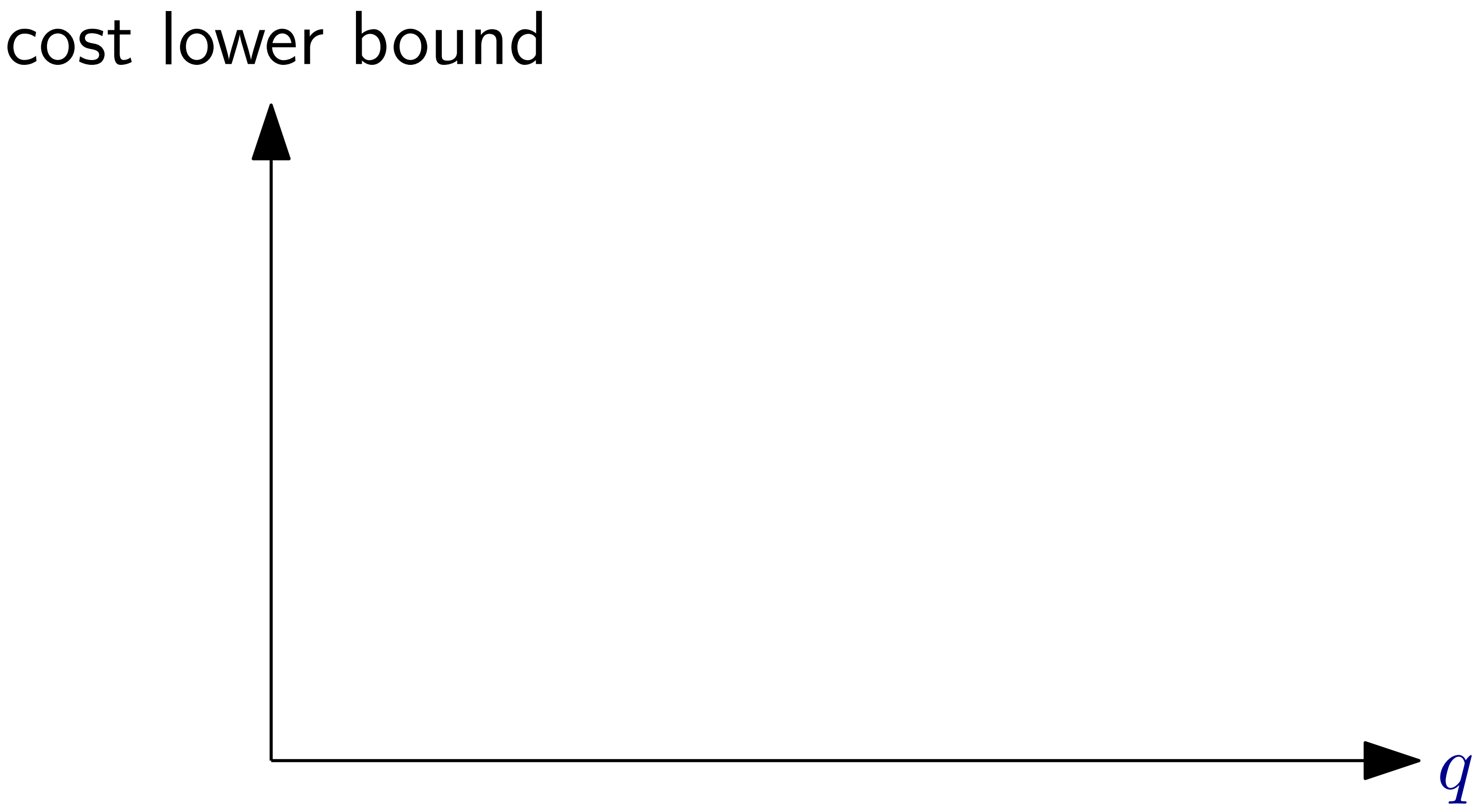

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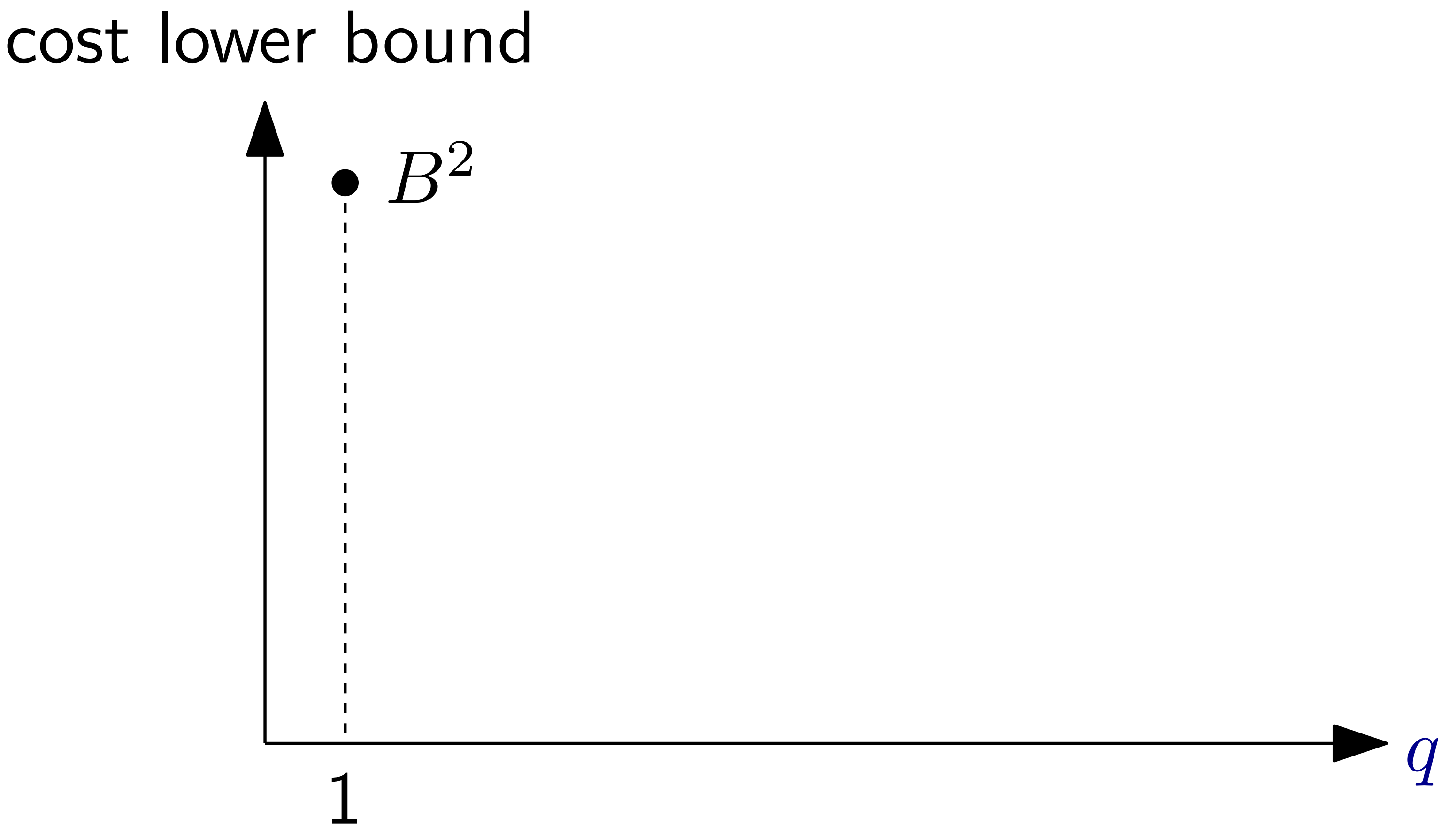

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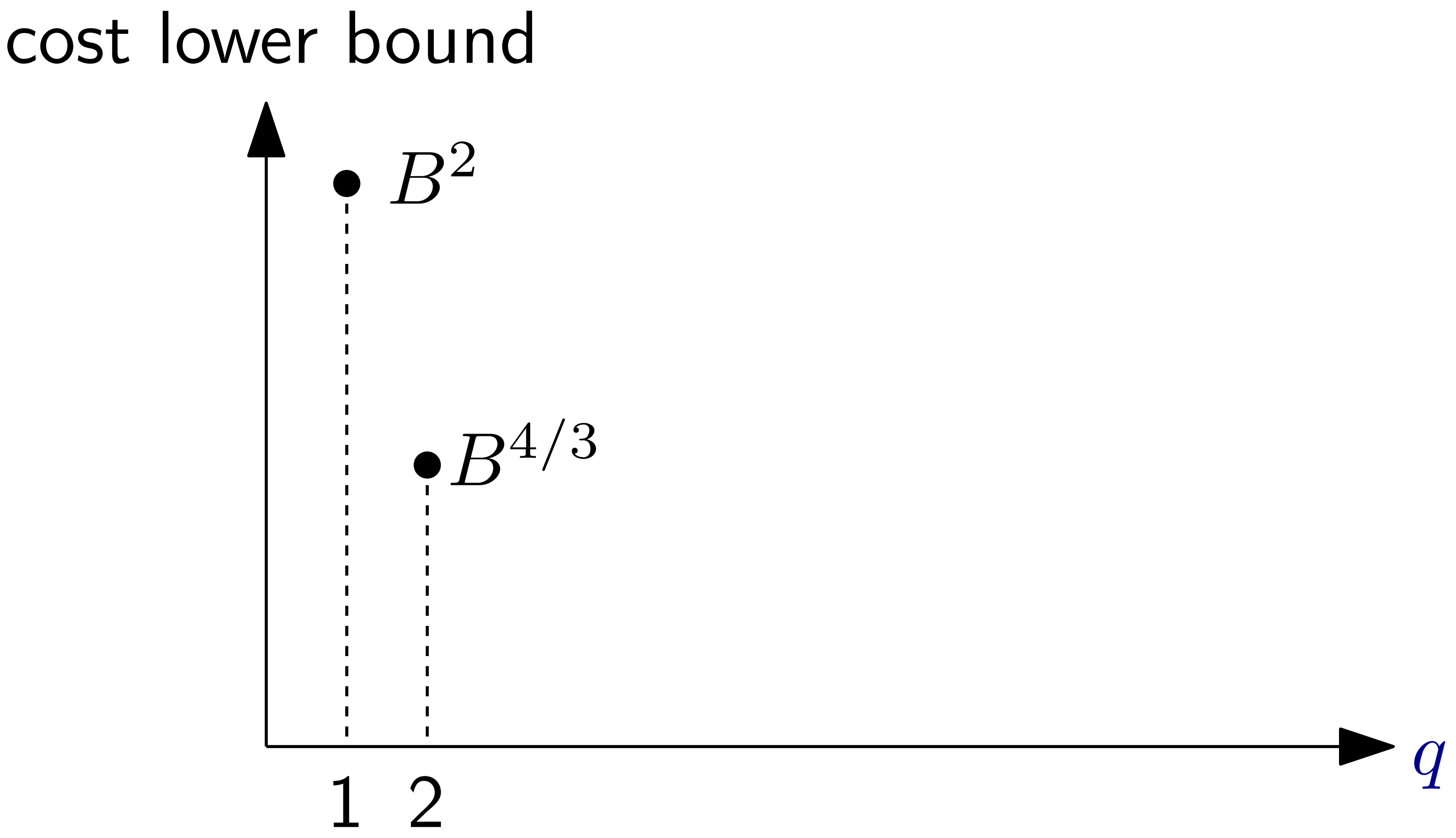

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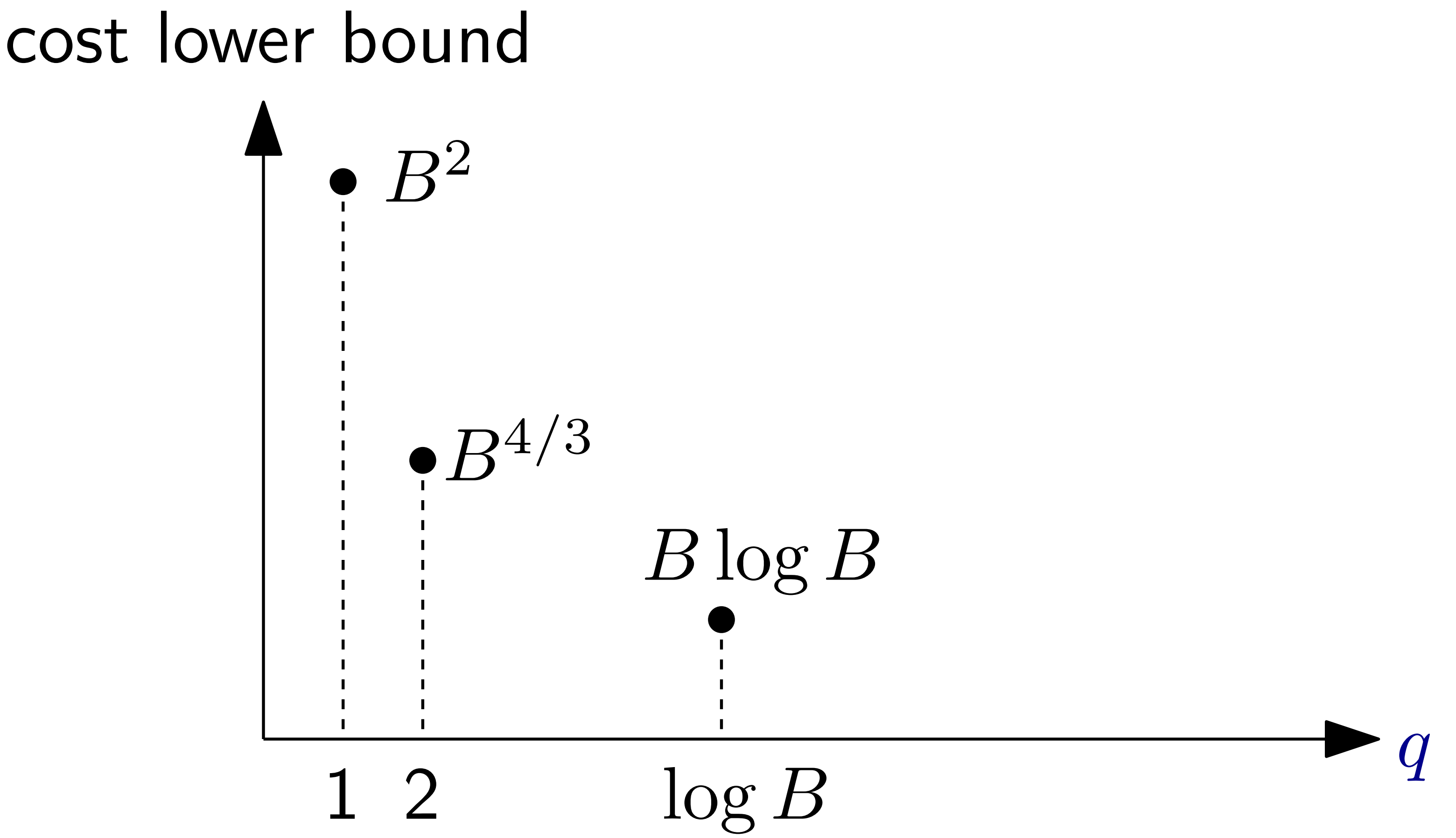

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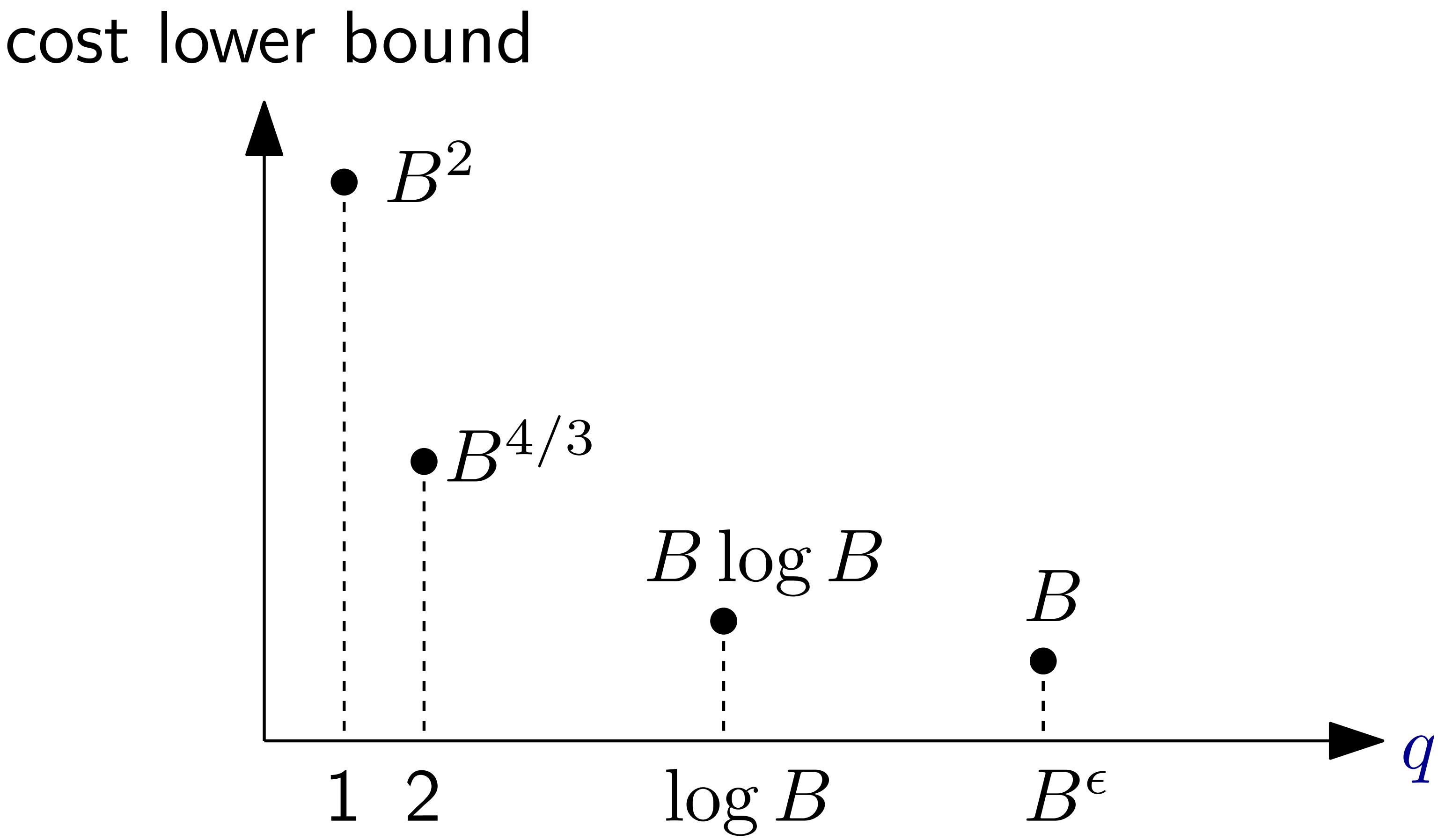

Dynamic B-trees

Dynamic Hash Tables

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Buffering is useless?

Dynamic Hash Tables (for successful queries)

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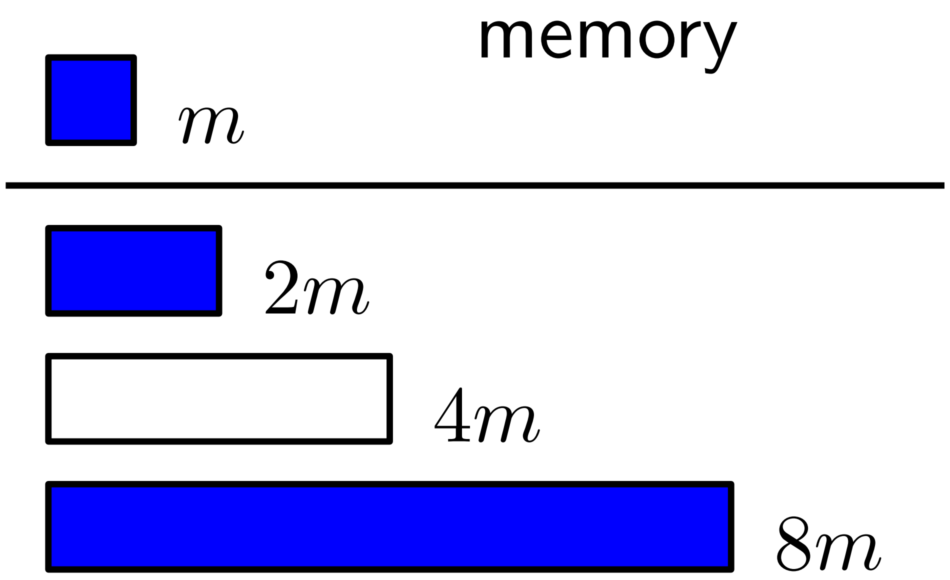

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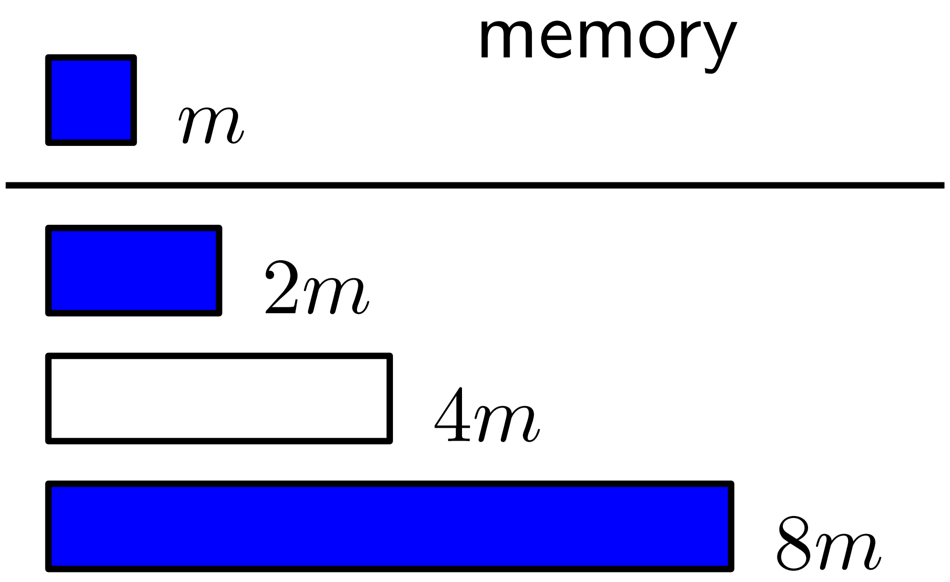

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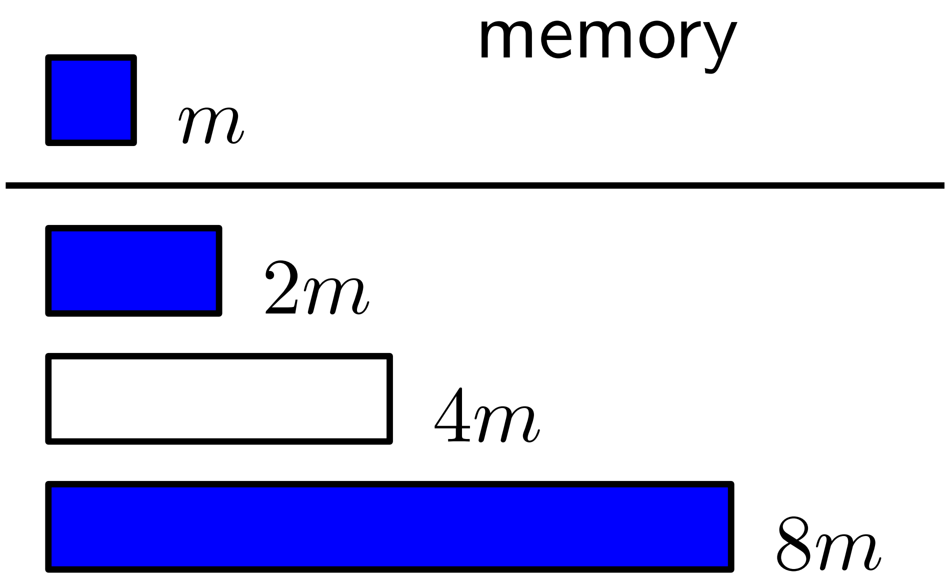

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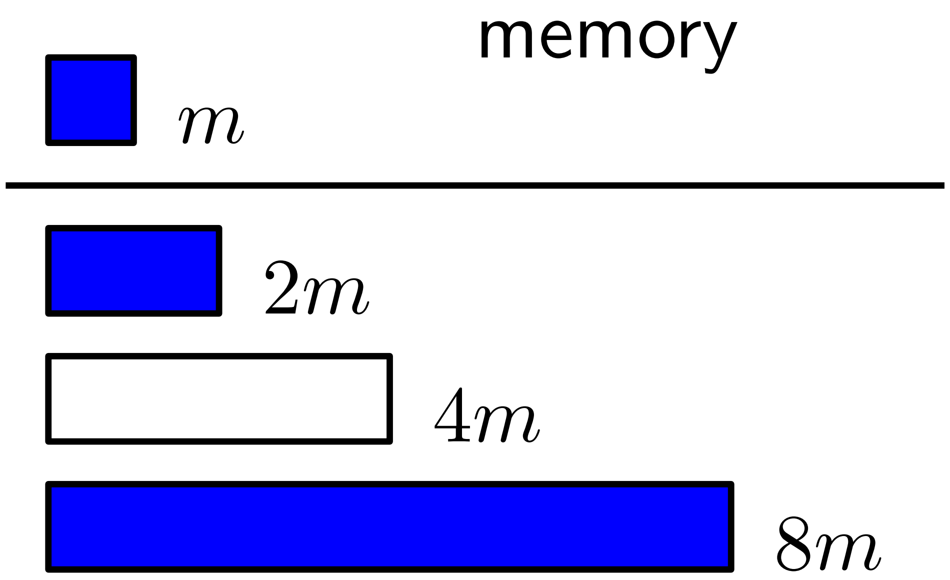

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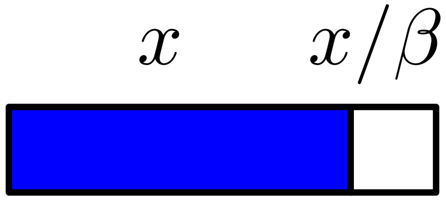

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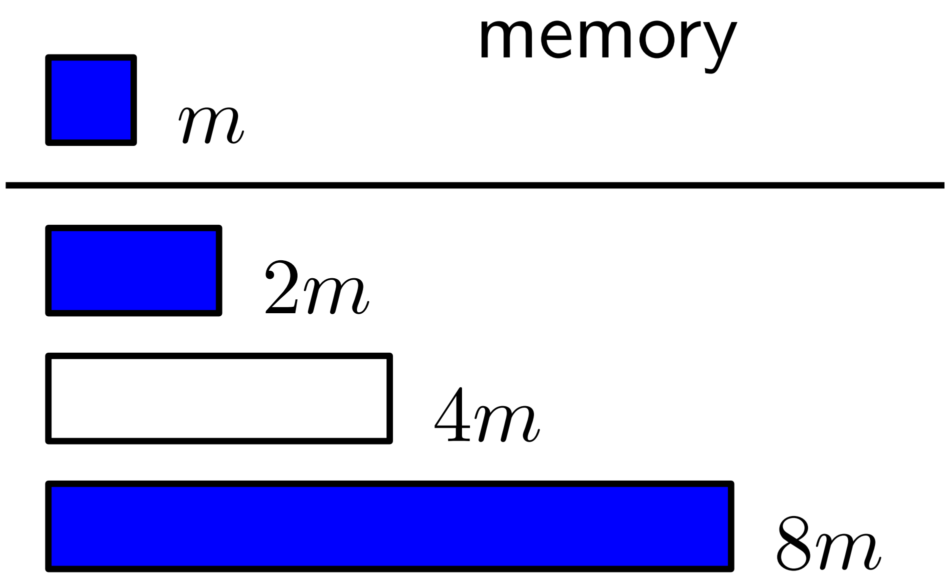

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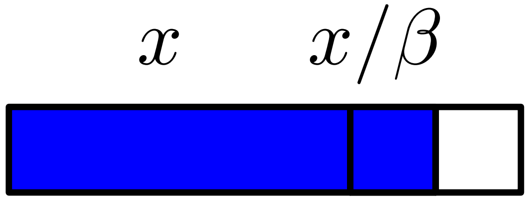

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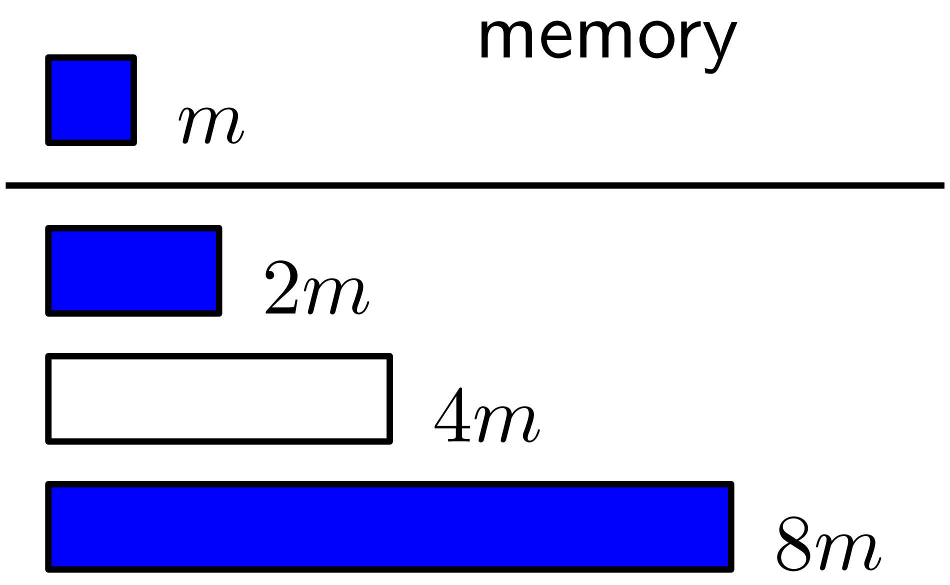

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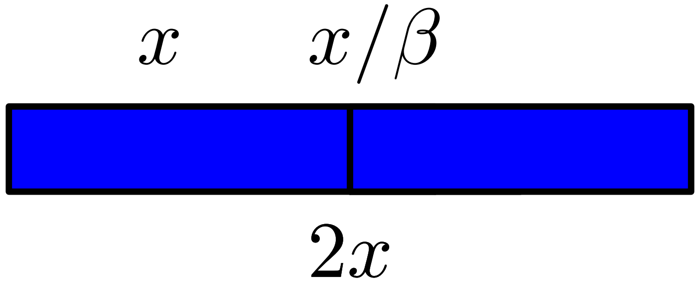

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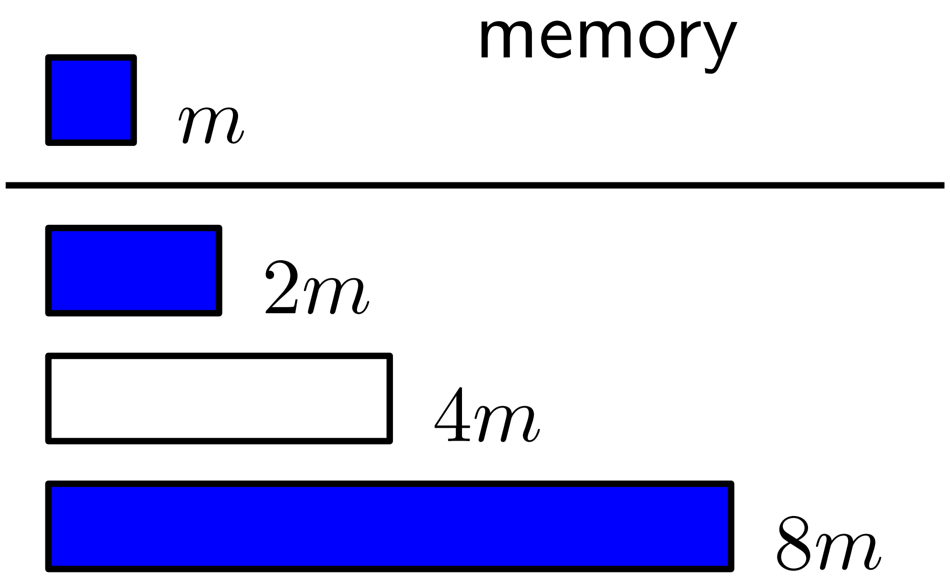

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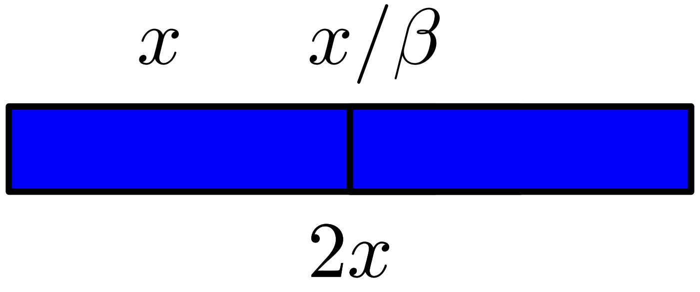

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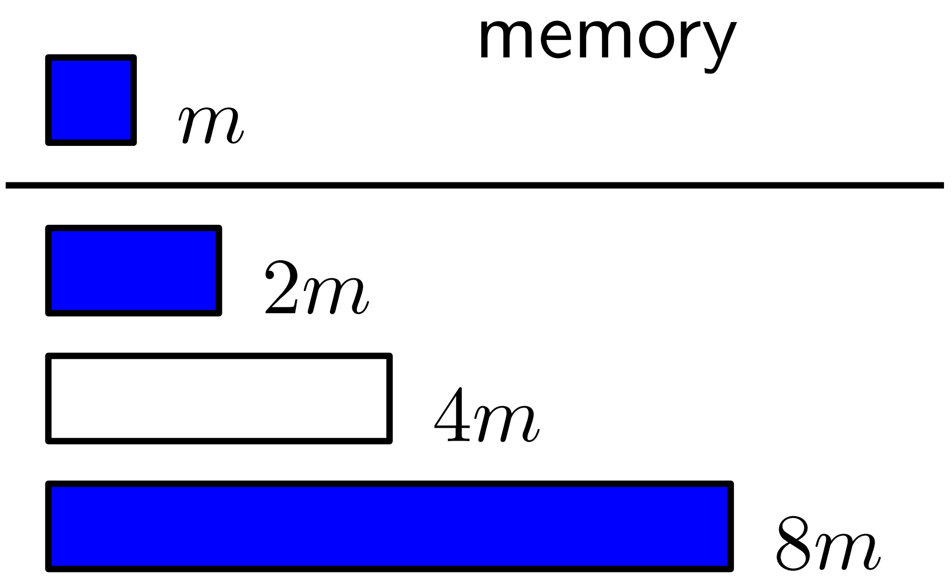

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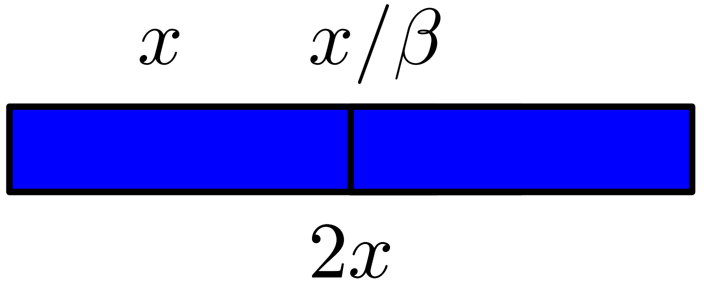

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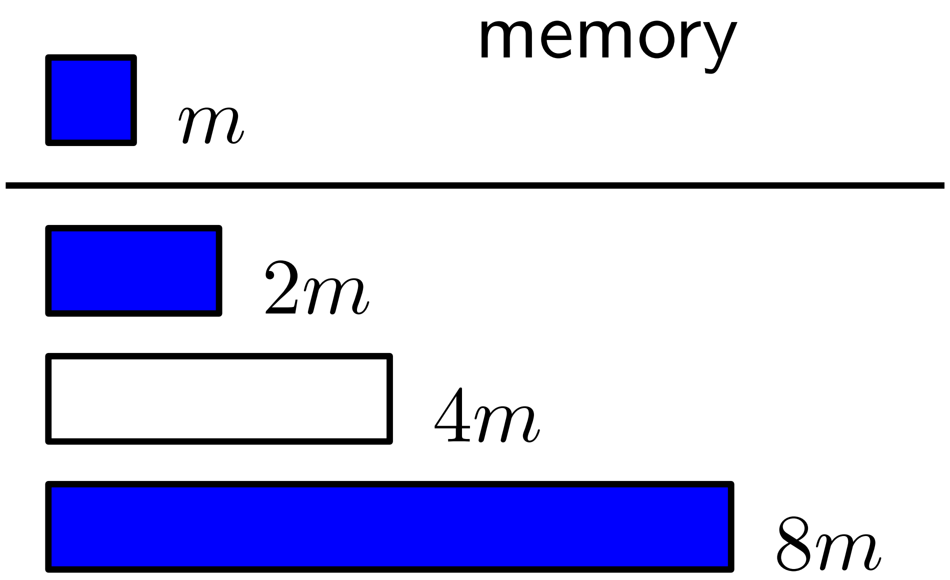

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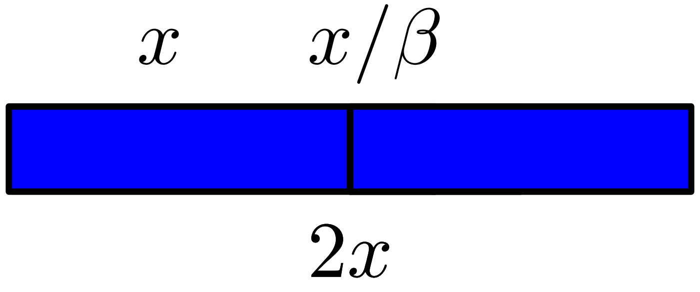
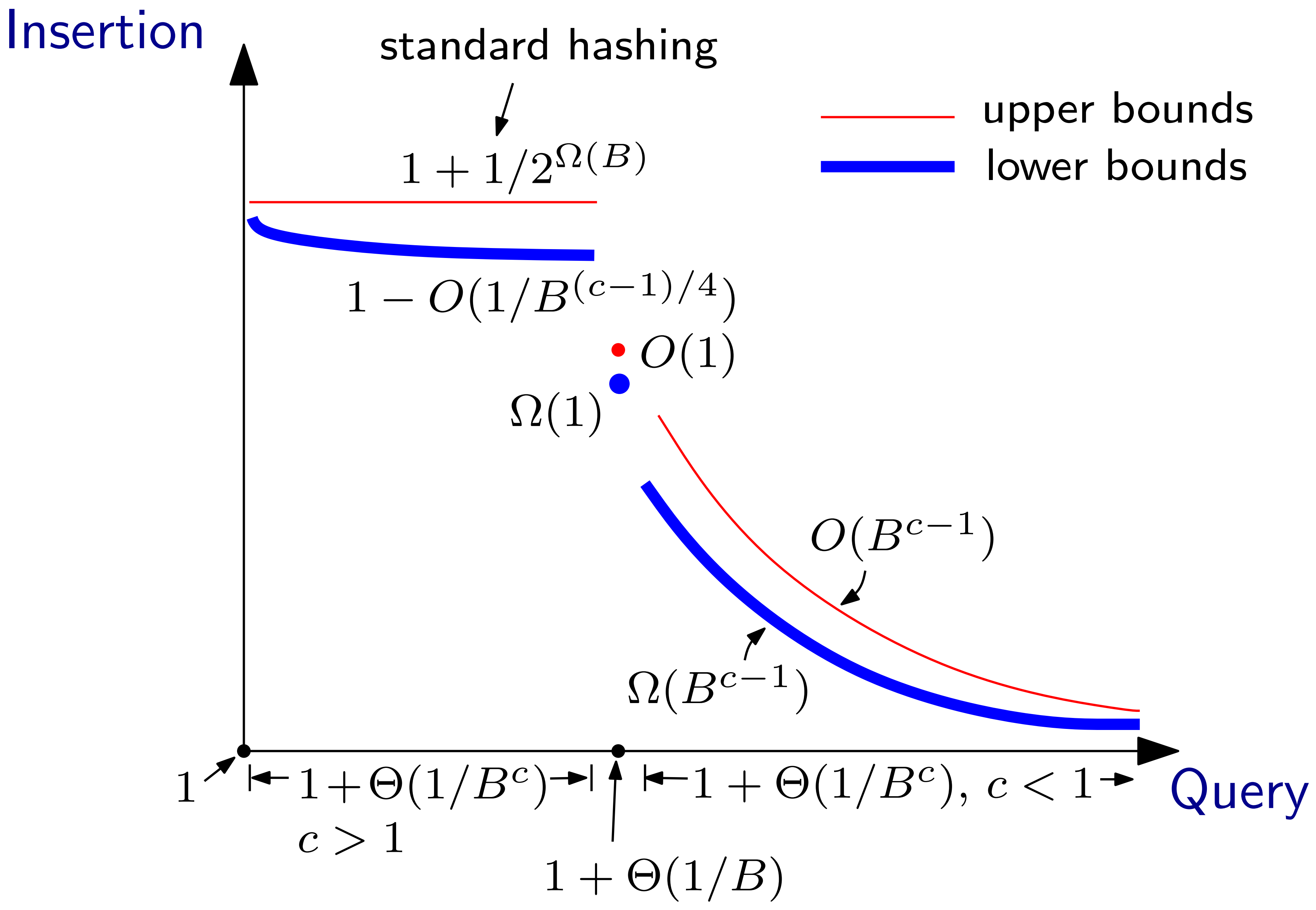


Query-Insertion Tradeoff for Successful queries

[Wei, Yi, Zhang, SPAA'09]



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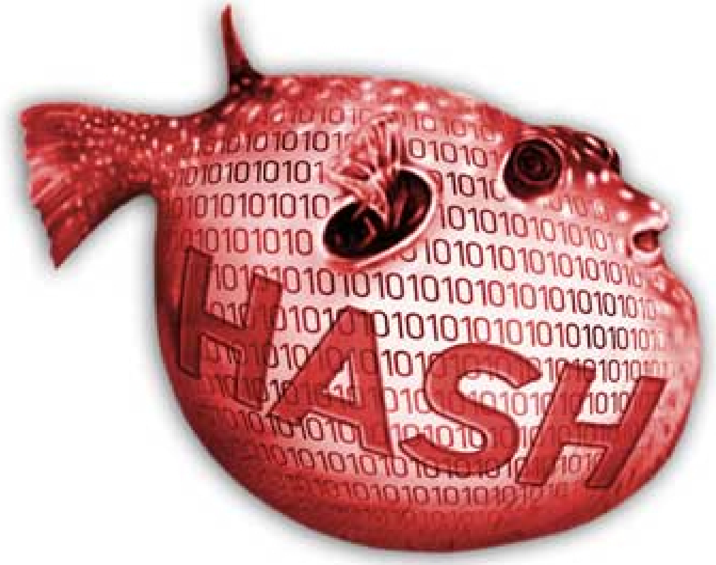

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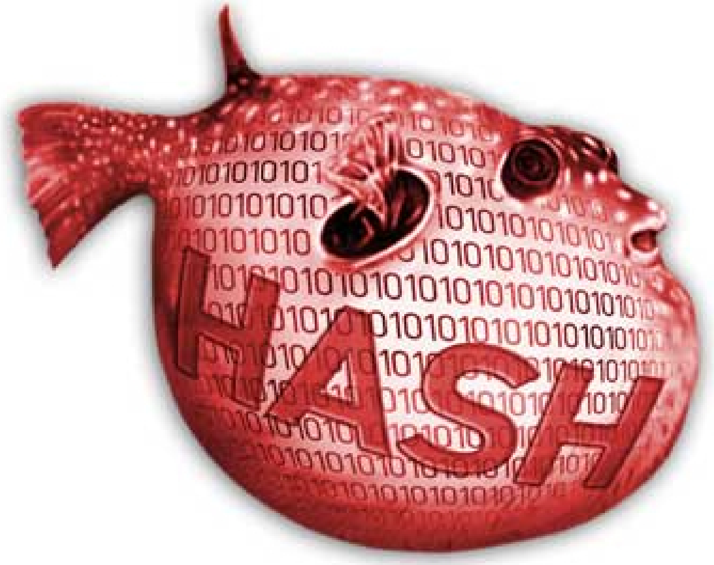


Too many possible mappings!

- Indexabilty $+$ information-theoretical argument

  If with only the information in memory, the hash table cannot locate the item, then querying it takes at least 2 I/Os.

The Abstraction

- Consider the layout of a hash table at any snapshot. Denote all the blocks on disk by $B_1, B_2, \ldots, B_d$. Let $f : U \rightarrow \{1, \ldots, d\}$ be any function computable within memory.

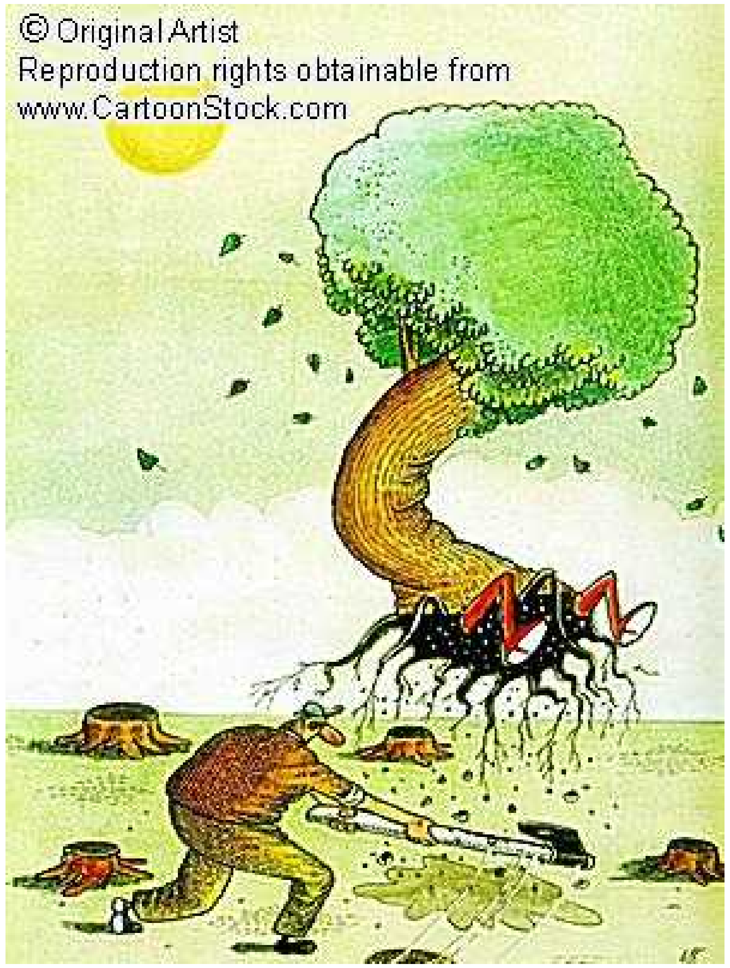
  We divide items inserted into 3 zones with respect to f.

  - Memory zone M: set of items stored in memory. $t_q = 0$.
  - Fast zone F: set of items x such that $x \in B_{f(x)}$. $t_q = 1$.
  - Slow zone S: The rest of items. $t_q = 2$.

The Key

The hash table can employ a family $\mathcal{F}$ of at most 2^M distinct f's.

Note that the current f adopted by the hash table is dependent upon the already inserted items, but the family $\mathcal{F}$ has to be fixed beforehand.

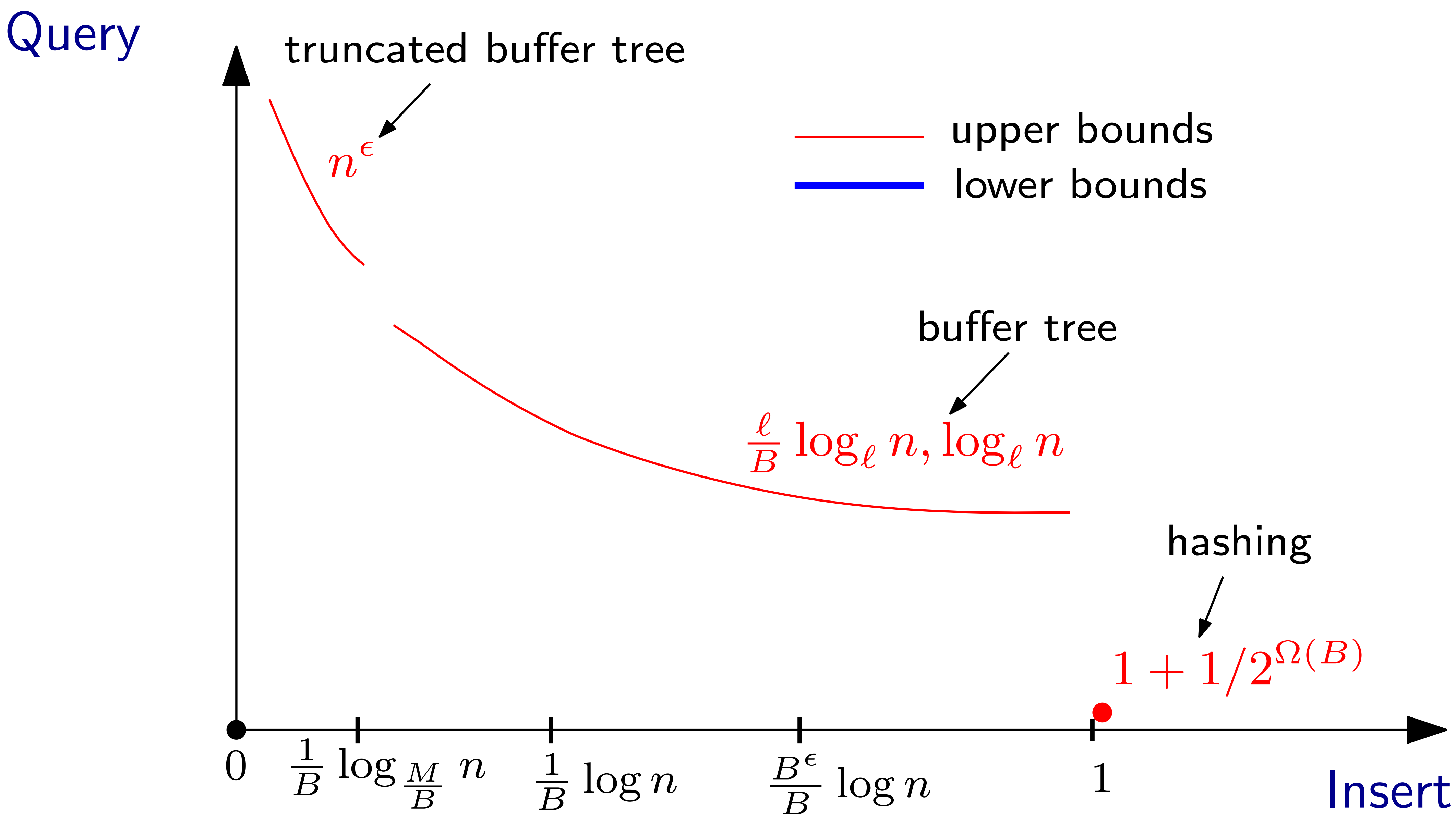


How about All Queries? (Latest results)

- We are essentially talking about the membership problem
    - Can't use indexability model
    - Have to use cell probe model

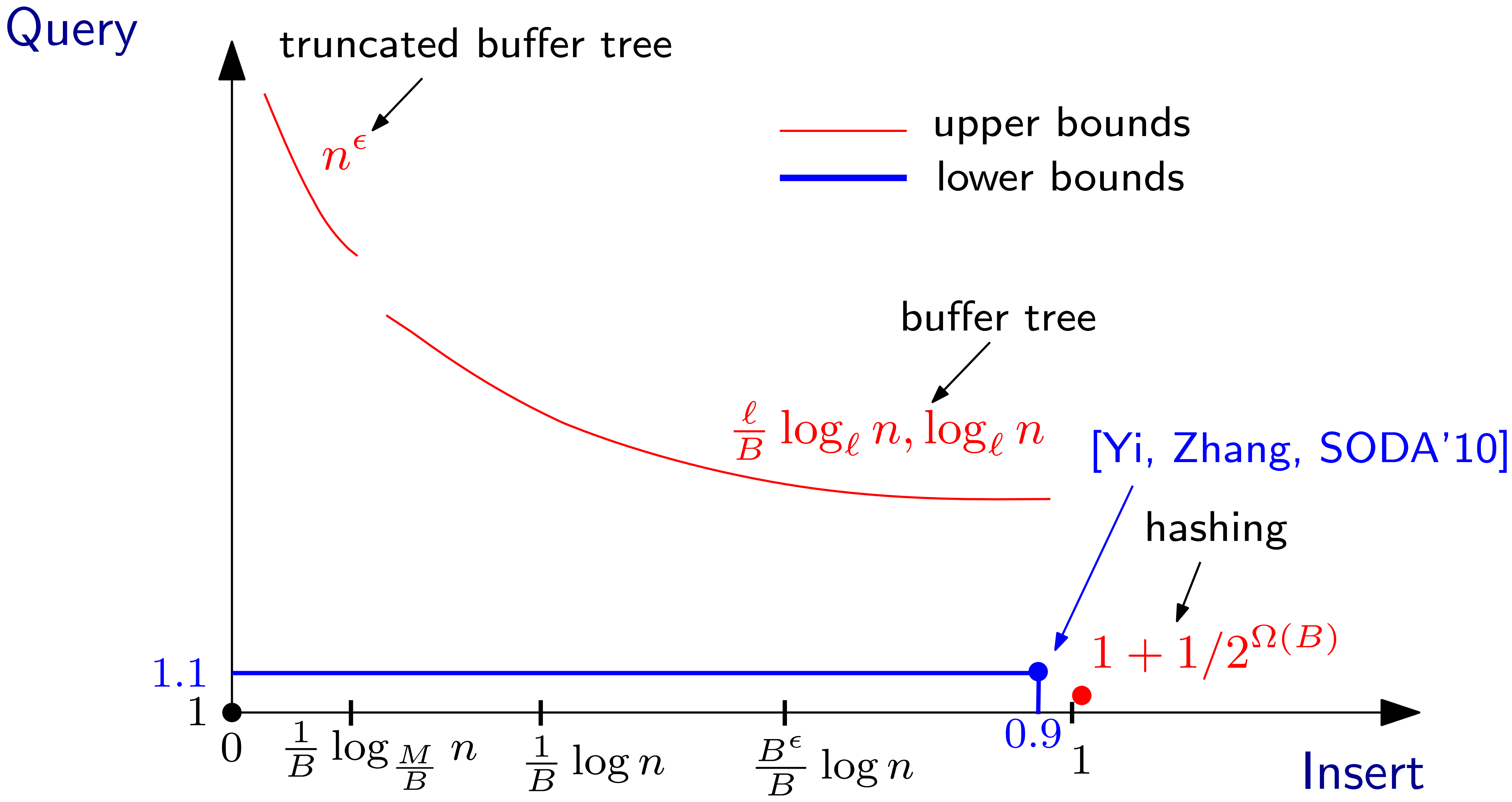
All queries (the membership problem)

(The cell probe model)



All queries (the membership problem)

(The cell probe model)



All queries (the membership problem)

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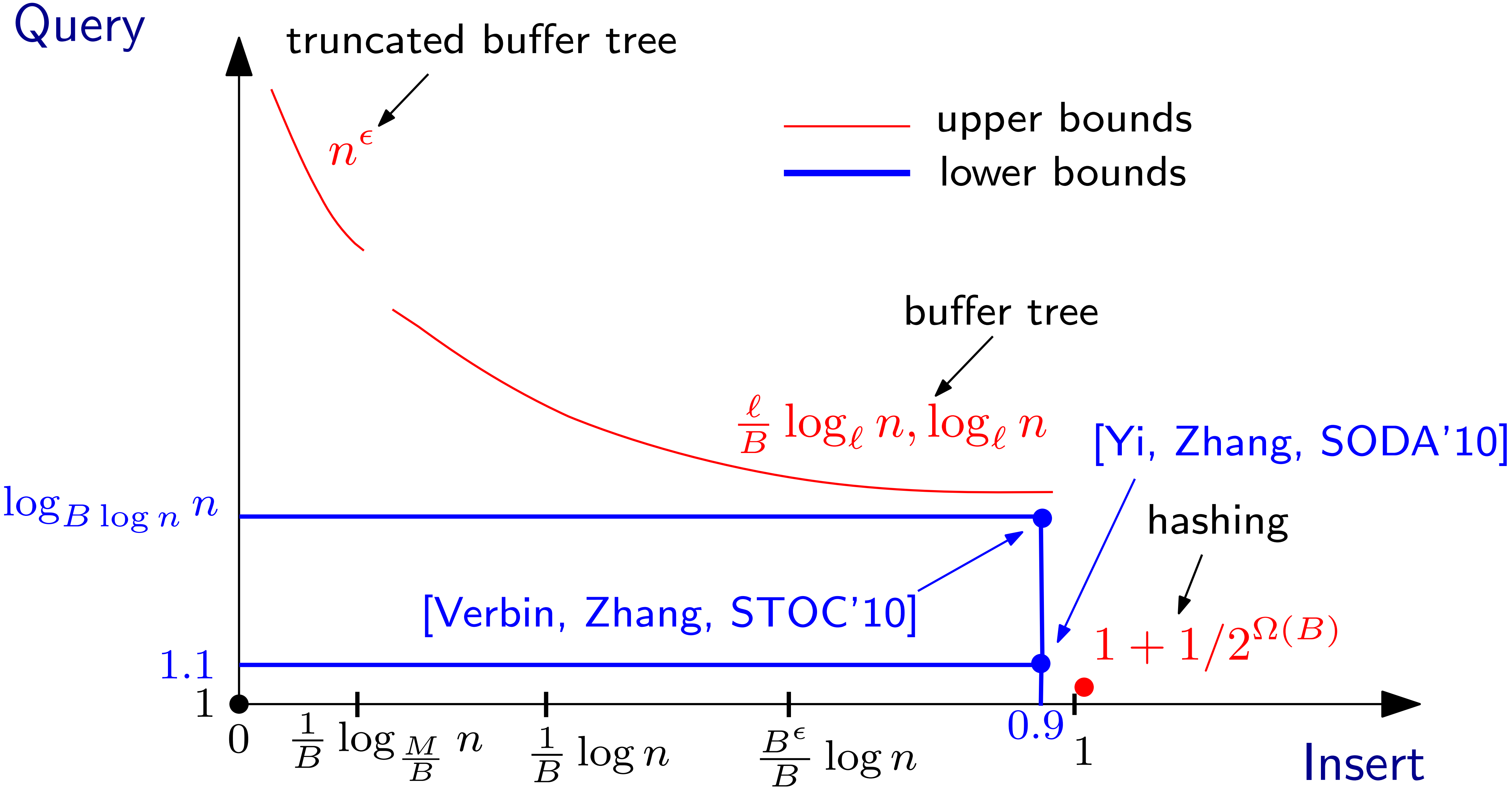

THE BIG BOLD CONJECTURE

All these fundamental data structure problems have the same query-update tradeoff in external memory when $u = o(1)$, for sufficiently large B.

Partial-sum: all B; Range reporting: $B > n^{\epsilon}$; Predecessor: unknown.

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Strong implication: The buffer tree (and many of the log method based structures) is simple, practical, versatile, and optimal!

The End

THANK YOU

Q and A