Beyond Simple Aggregates: Indexing for Summary Queries

Zhewei Wei and Ke Yi

Hong Kong University of Science and Technology

Reporting vs. Aggregation

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FROM Table T
WHERE 30 < age < 40
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$$\left.\begin{array}{l} \$32,000 \\ \$76,300 \\ \$54,400 \\ \ \ \ldots \\ \$68,000 \\ \$28,000 \end{array}\right\} 50,000 \text{ records}$$

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$\$52,312$

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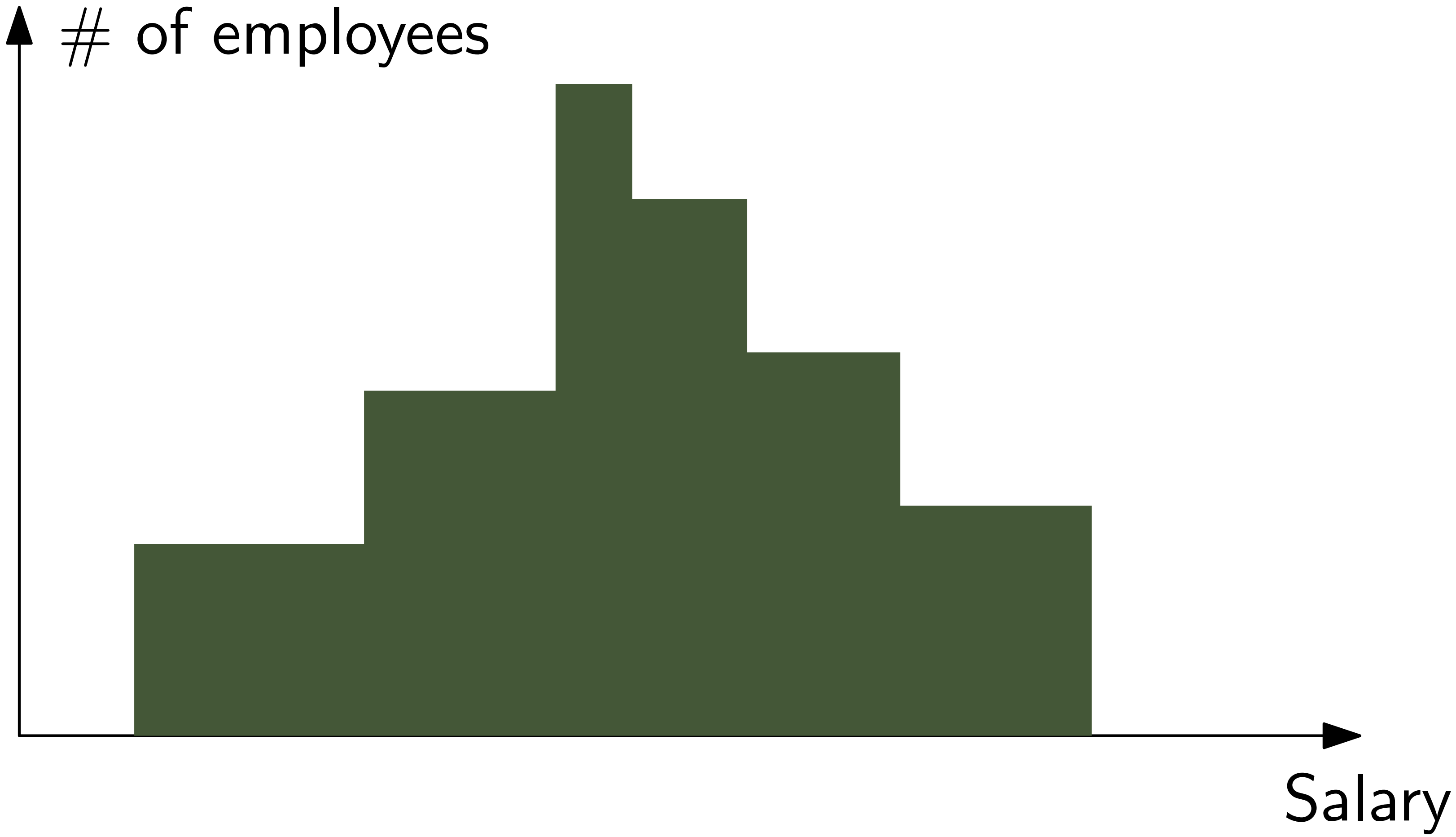

Reporting vs. Aggregation

Search Engine Log

Date	Keyword
2011.04.08	Masters 2011
2011.04.08	Libya
2011.04.07	Japan nuclear crisis
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2011.03.11	Japan earthquake
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$\Longrightarrow$

Keyword	Frequency
Libya	19.3%
Japan nuclear crisis	16.5%
Japan earthquake	10.2%
. . .	

Summary Queries

- Let $\mathcal{D}$ be a database containing N records. Each record $p \in \mathcal{D}$ is associated with query attribute $A_q(p)$ (age) and a summary attribute $A_s(p)$ (salary).

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- A summary query specifies a range constraint $[q_1, q_2]$ on A_q and the database returns a summary on the A_s attribute of all records whose A_q attribute is within the range.

Summary Queries

- Data summarization techniques
  - Heavy hitters (a.k.a. frequent items) [MG 82] [MAA 06] ...
  - Quantiles [MP 80] [GK 01] ...
  - Histograms [PHIJ 96] [JKMPSS 98] [GGIKMS 02] ...
  - Wavelets [MVW 98] [VM 99] [GKMS 01] ...
  - Various sketches ([AMS 99], Count-Min [CM 05], ...)
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  . . .

- Past research focuses on computing summaries on the whole data set: offline or streaming

Algorithm Problem vs. Data Structure Problem

	The algorithm problem	The data structure problem
Space		
Time		

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Space	offline: $O(N)$ streaming: sublinear	$O(N)$: data must be stored
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Algorithm Problem vs. Data Structure Problem

	The algorithm problem	The data structure problem
Space	offline: $O(N)$ streaming: sublinear	$O(N)$: data must be stored
Time	$\tilde{O}(N)$ sublinear when sampling works	preprocessing time: less important
		query time: $O(\log N + s_\varepsilon)$ internal mem $O(\log_B N + s_\varepsilon / B)$ external mem s_ε: summary size B: block size

Quantile Summaries

- ϕ-quantile: the value ranked at $\phi|D|$ in D.
- ε-approximate ϕ-quantile: any value whose rank is between $[(\phi - \varepsilon)|D|, (\phi + \varepsilon)|D|]$.
- Quantile summary: for any $0 < \phi < 1$, an ε-approximate ϕ-quantile can be extracted.

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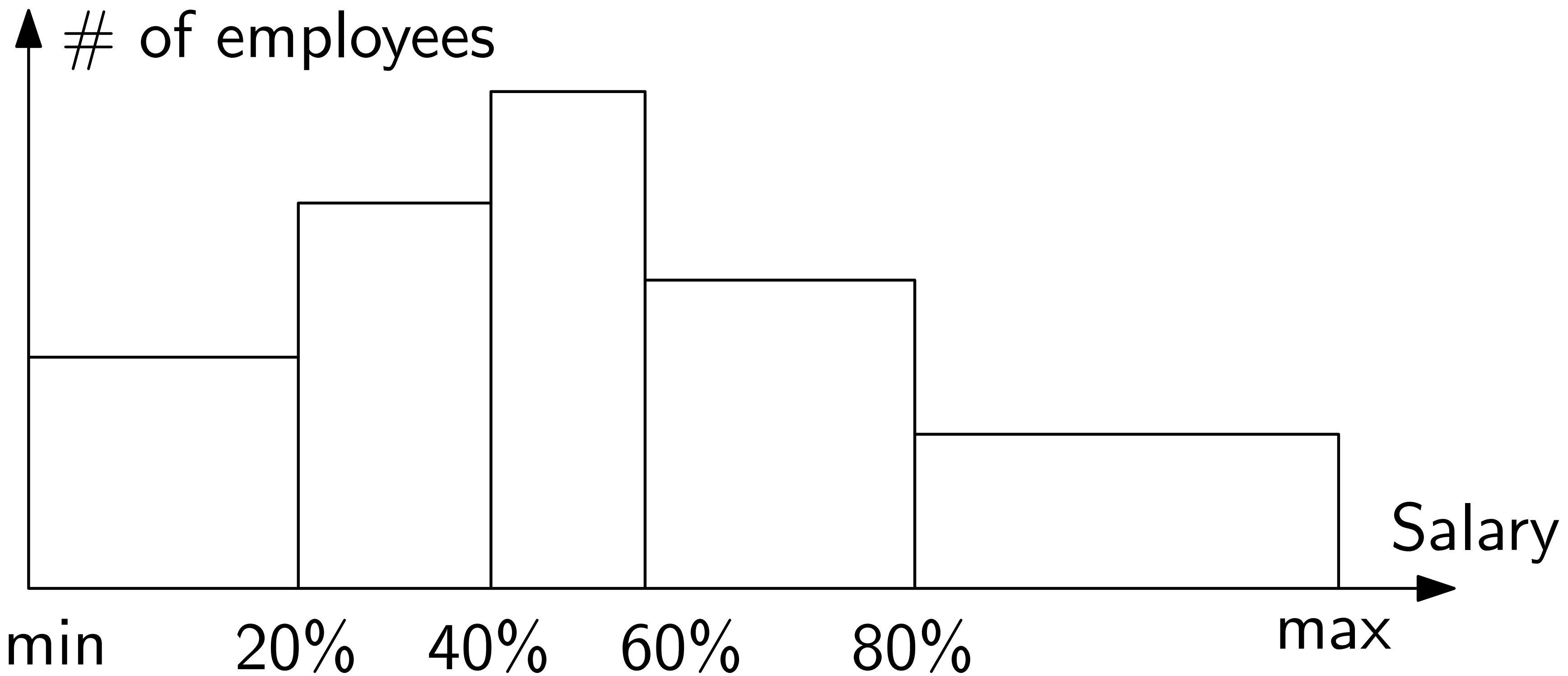
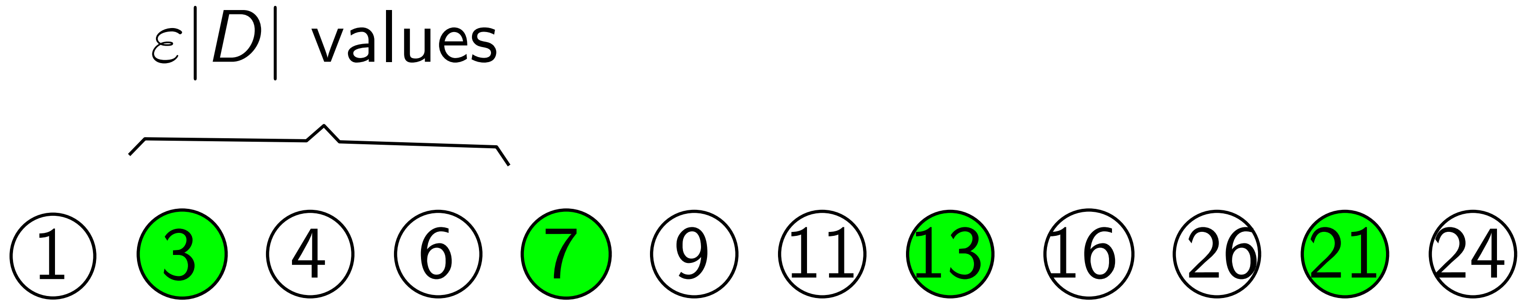


Quantile Summaries



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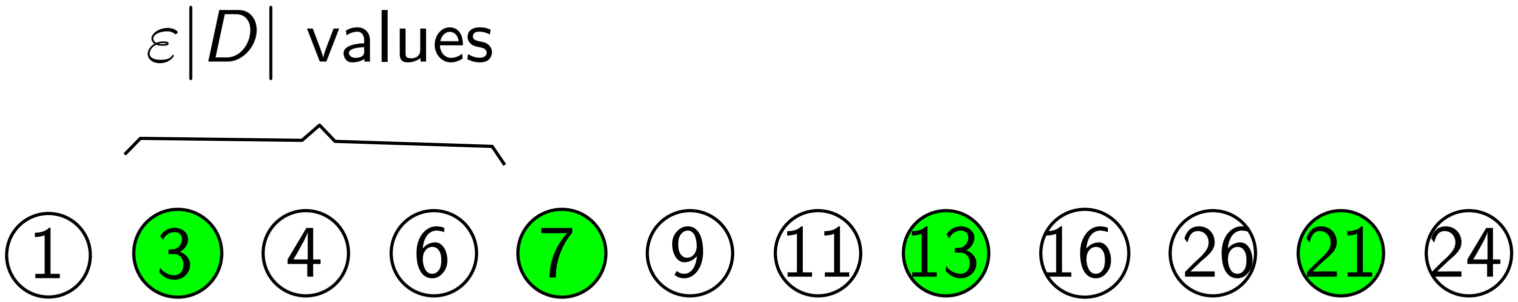

Size: $s_\varepsilon = \Theta(1/\varepsilon)$; Error: $\varepsilon|D|$

A Baseline Solution

- Decomposable summaries

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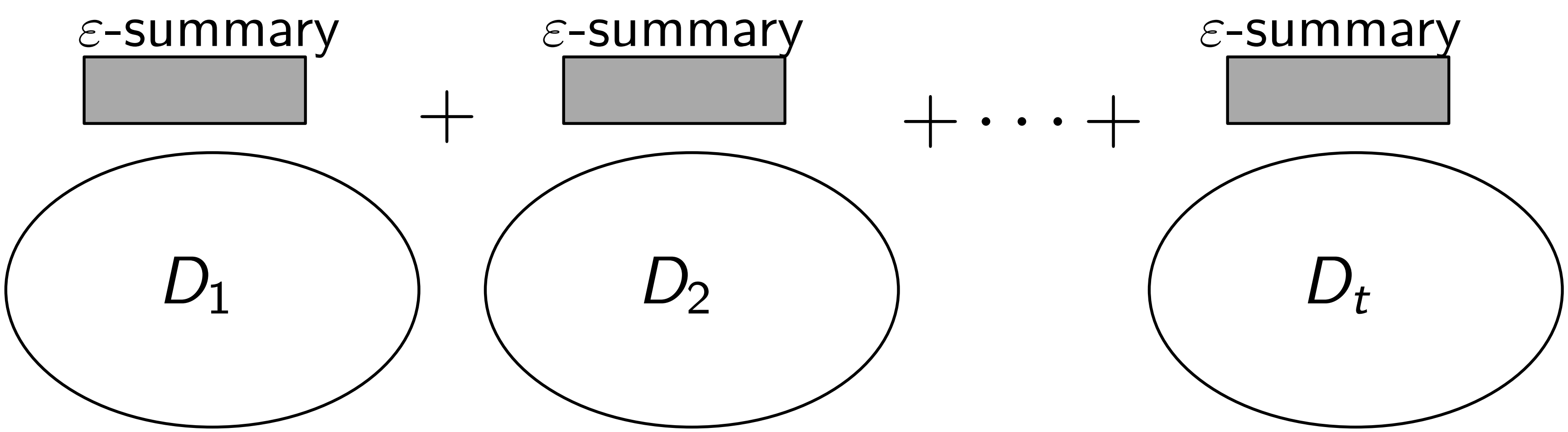

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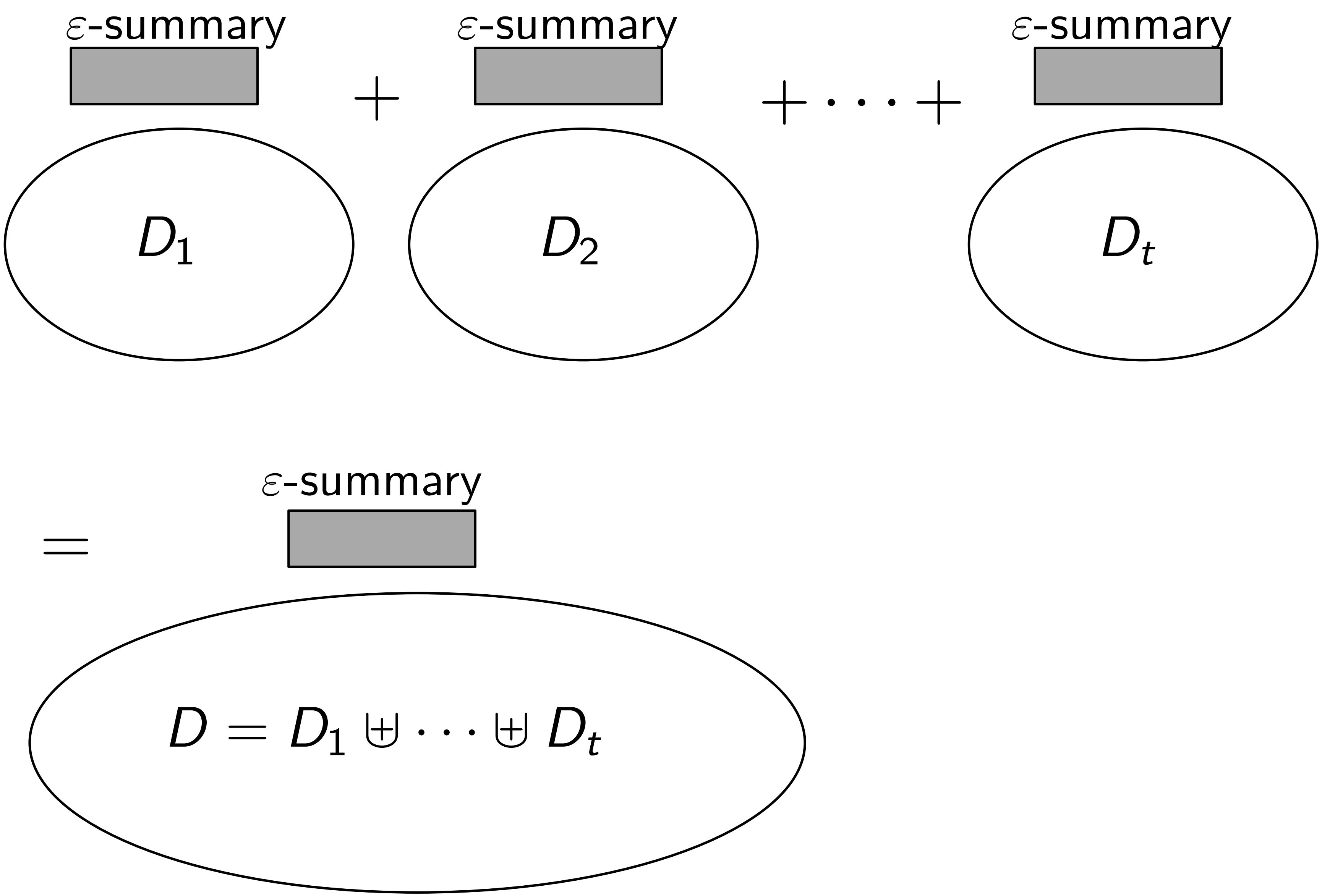

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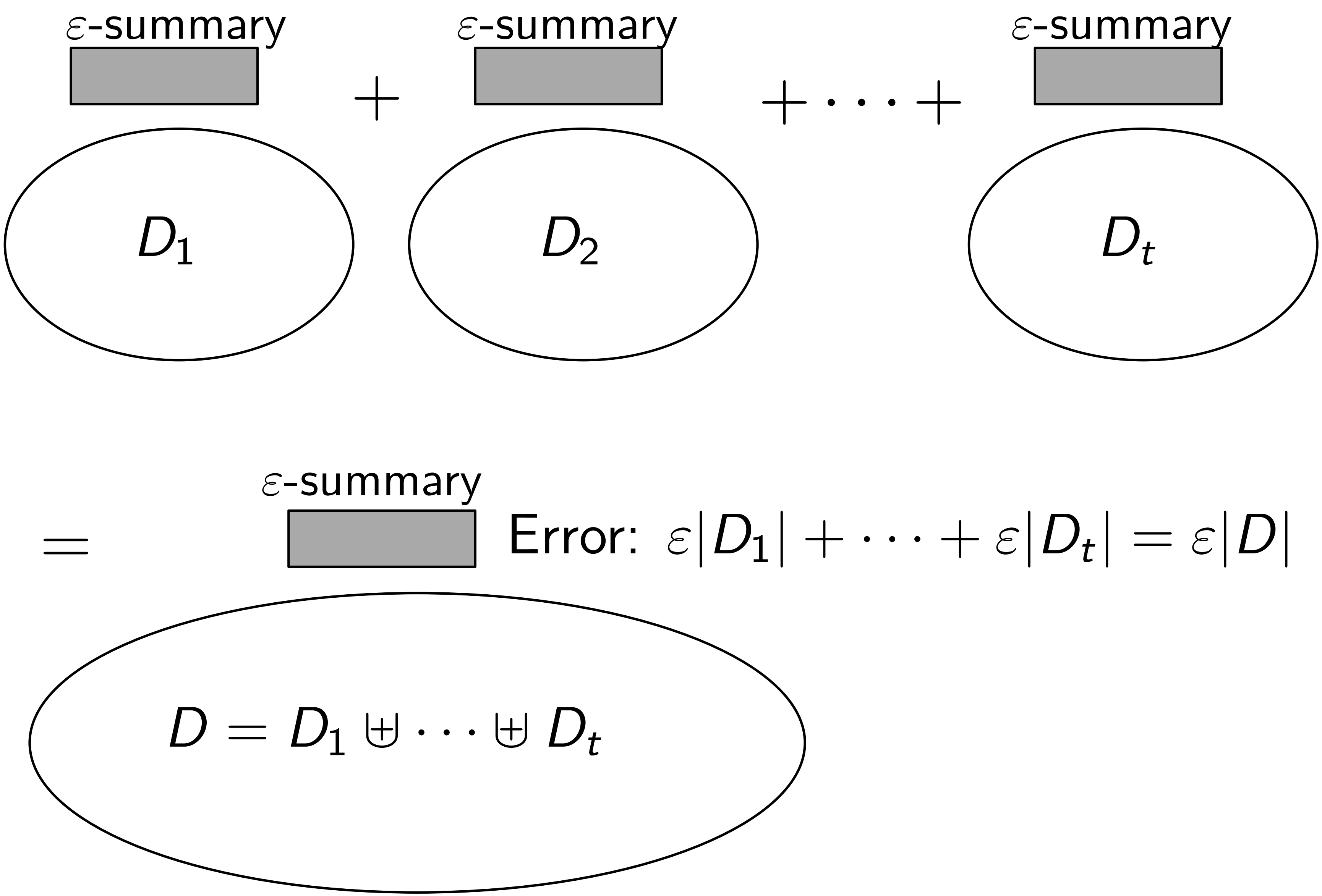

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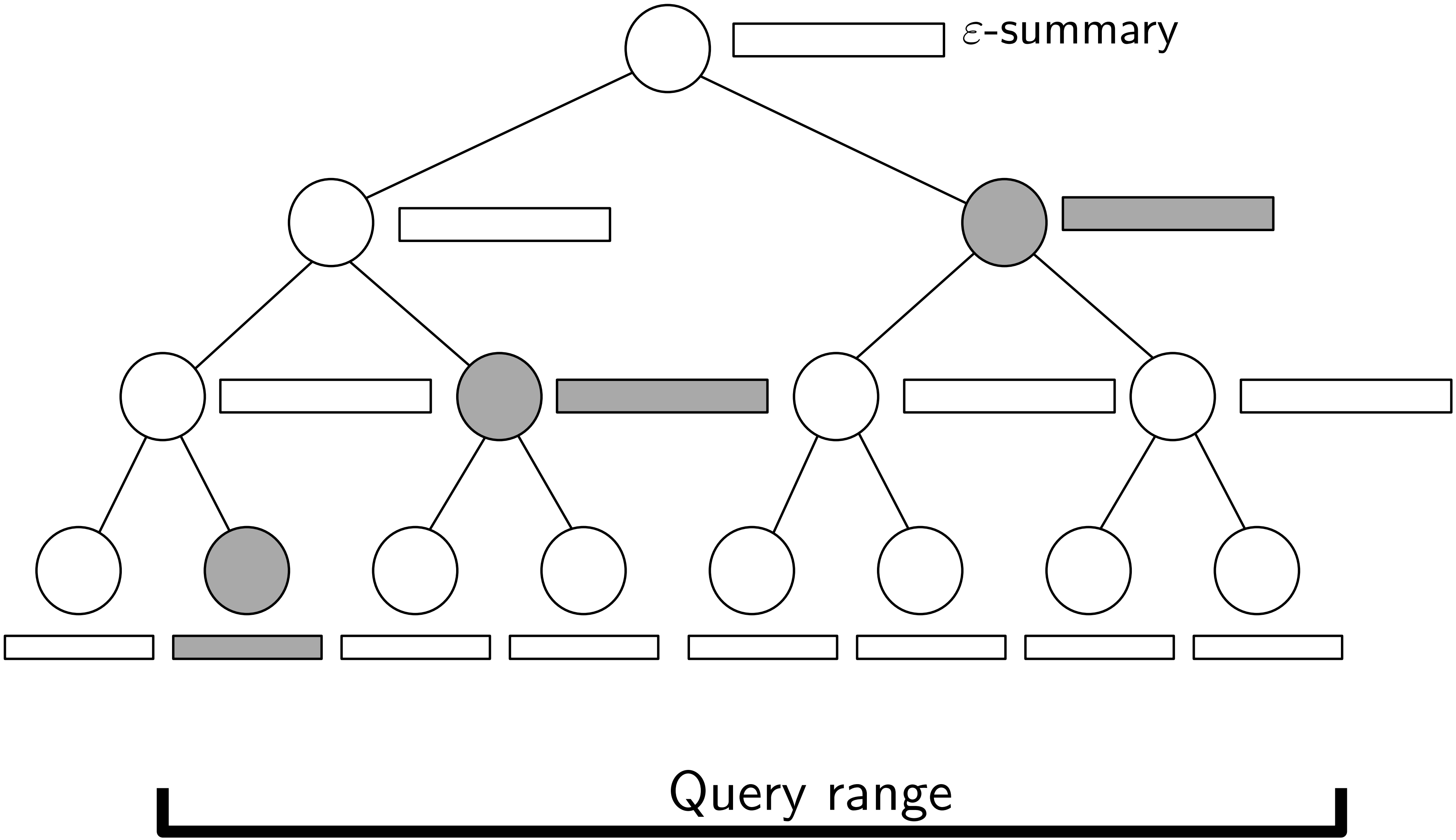
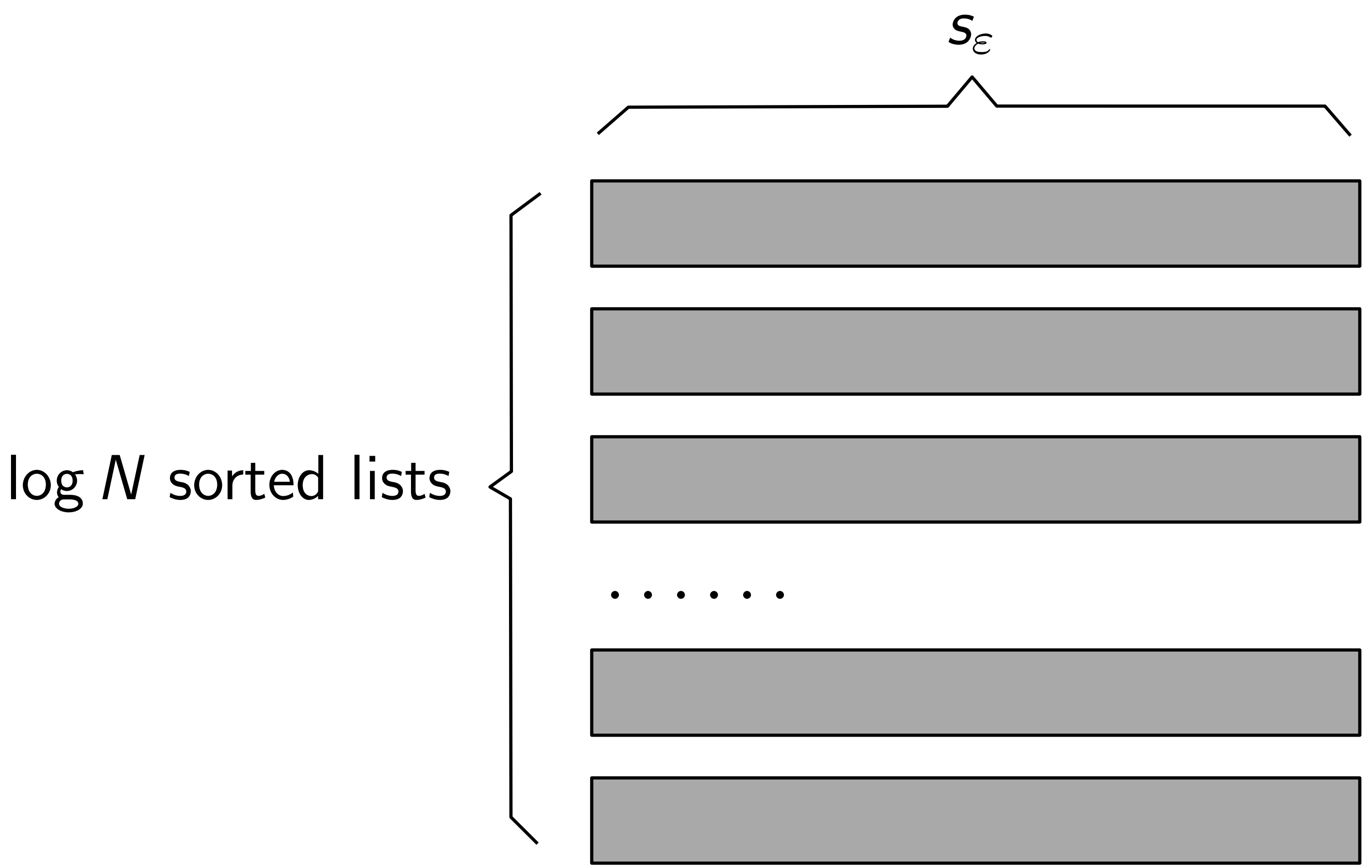


Query Cost



log N-way merging: $O(s_\varepsilon \log N \log \log N)$

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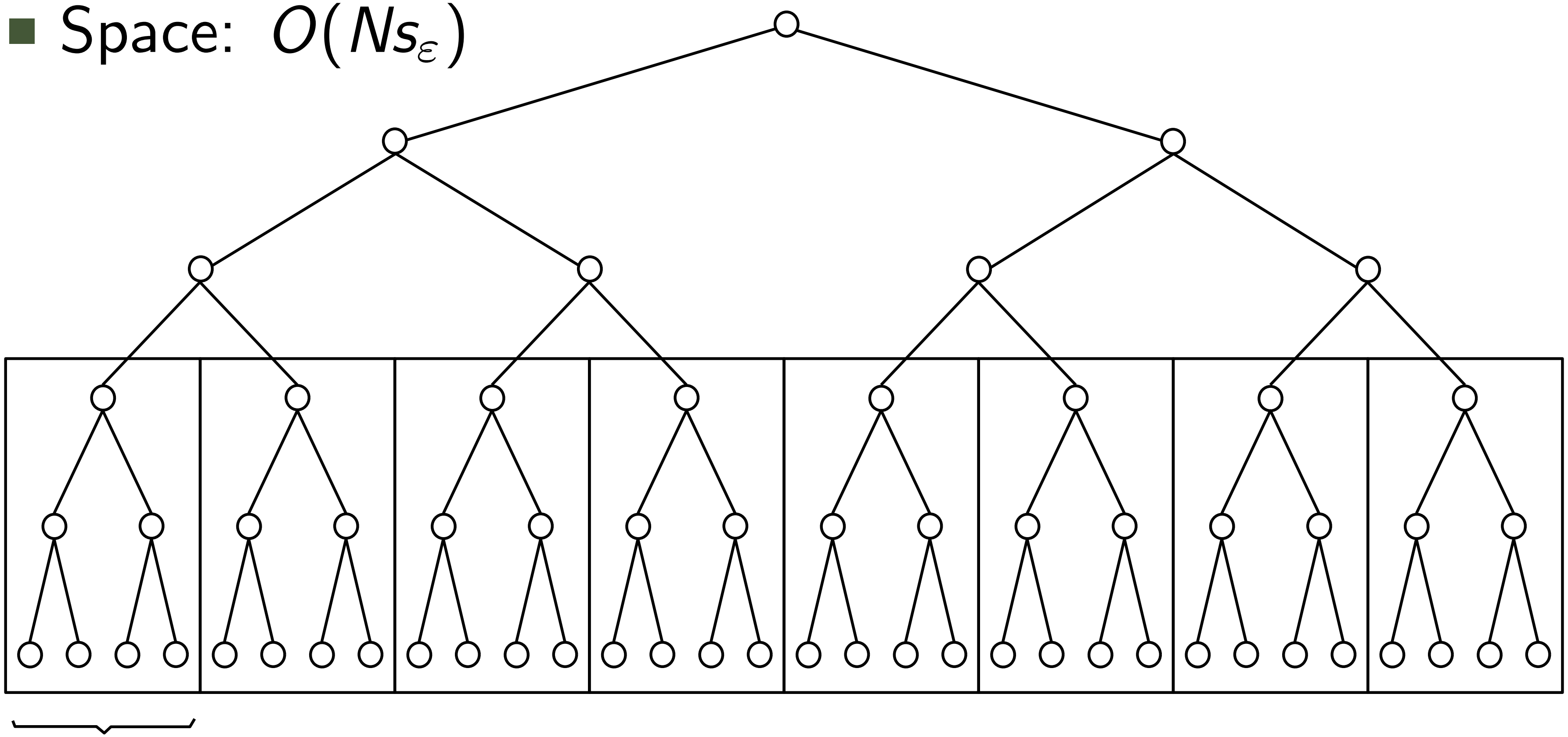

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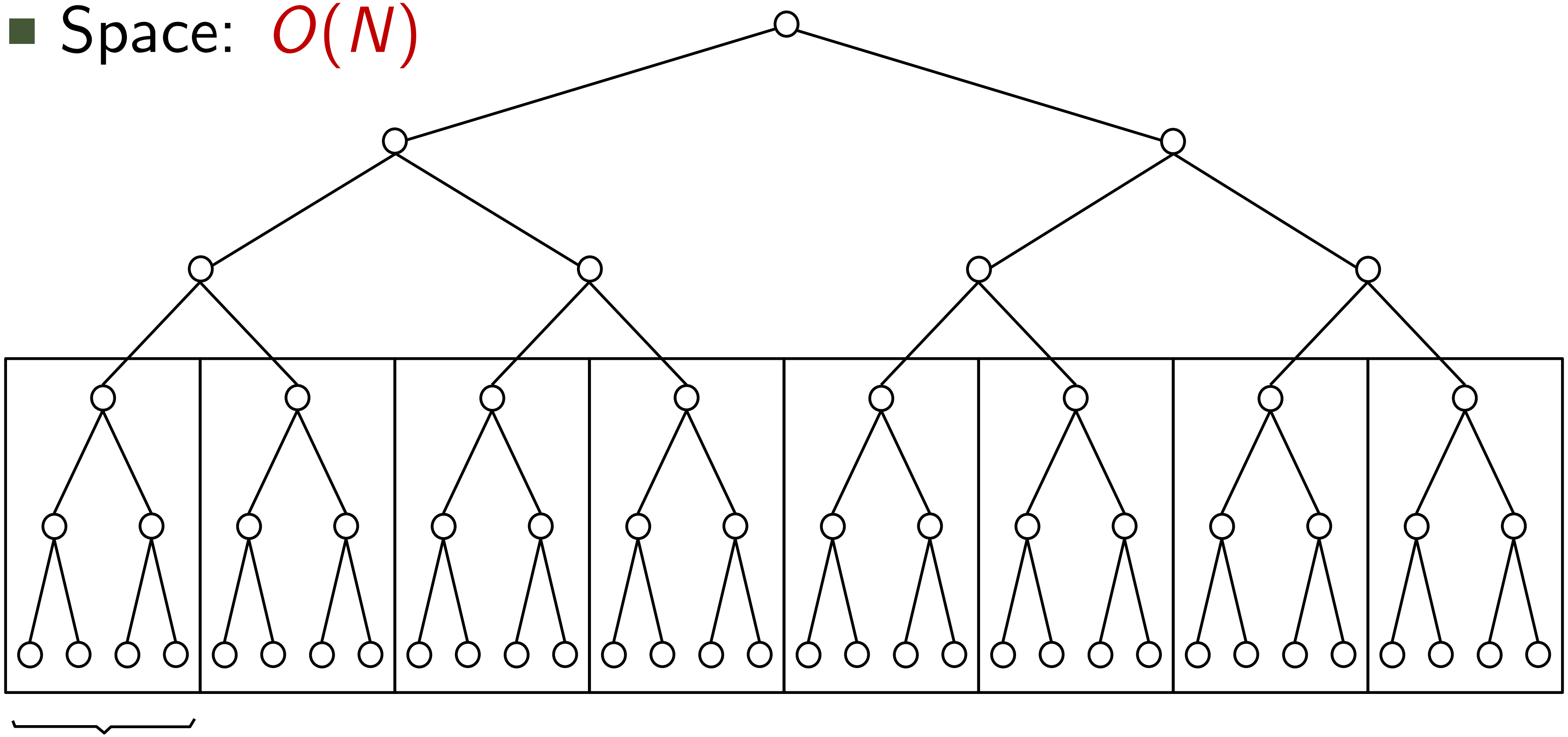
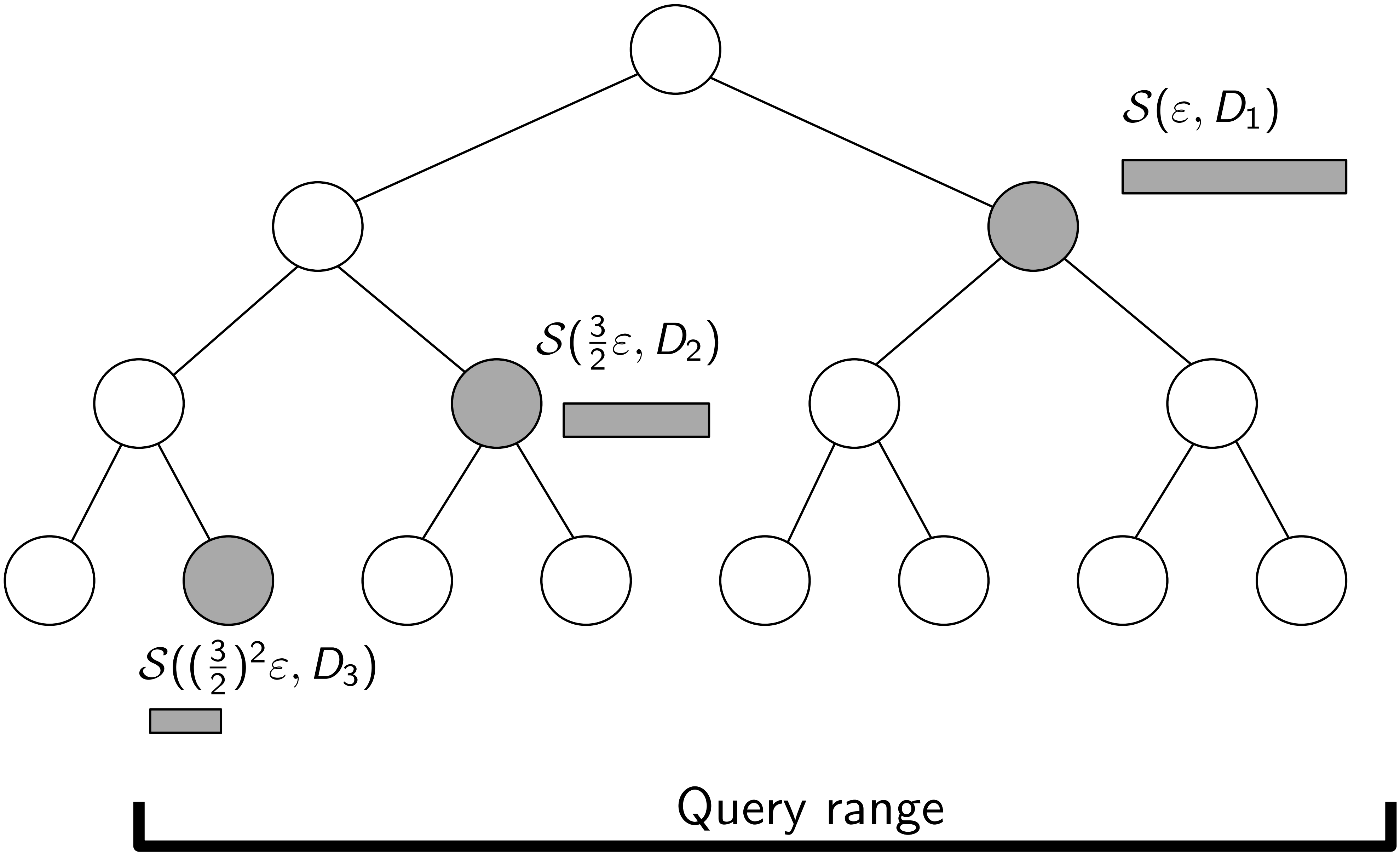


Fat leaf: s_ε

Optimal Data Structure



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- Quantile summary
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Data set	Data size	Error param.	Summary size	Absolute error
D_1	k	ε	$\frac{1}{\varepsilon}$	εk
D_2	$\frac{k}{2}$	$\frac{3}{2}\varepsilon$	$\frac{2}{3}\frac{1}{\varepsilon}$	$\frac{3}{4}\varepsilon k$
D_3	$\frac{k}{4}$	$\left(\frac{3}{2}\right)^2 \varepsilon$	$\left(\frac{2}{3}\right)^2 \frac{1}{\varepsilon}$	$\left(\frac{3}{4}\right)^2 \varepsilon k$
$\ldots$				
D_t	$\frac{k}{2^{t-1}}$	$\left(\frac{3}{2}\right)^{t-1} \varepsilon$	$\left(\frac{2}{3}\right)^{t-1} \frac{1}{\varepsilon}$	$\left(\frac{3}{4}\right)^{t-1} \varepsilon k$
D	$\Theta(k)$		$O(\frac{1}{\varepsilon})$	$O(\varepsilon k)$

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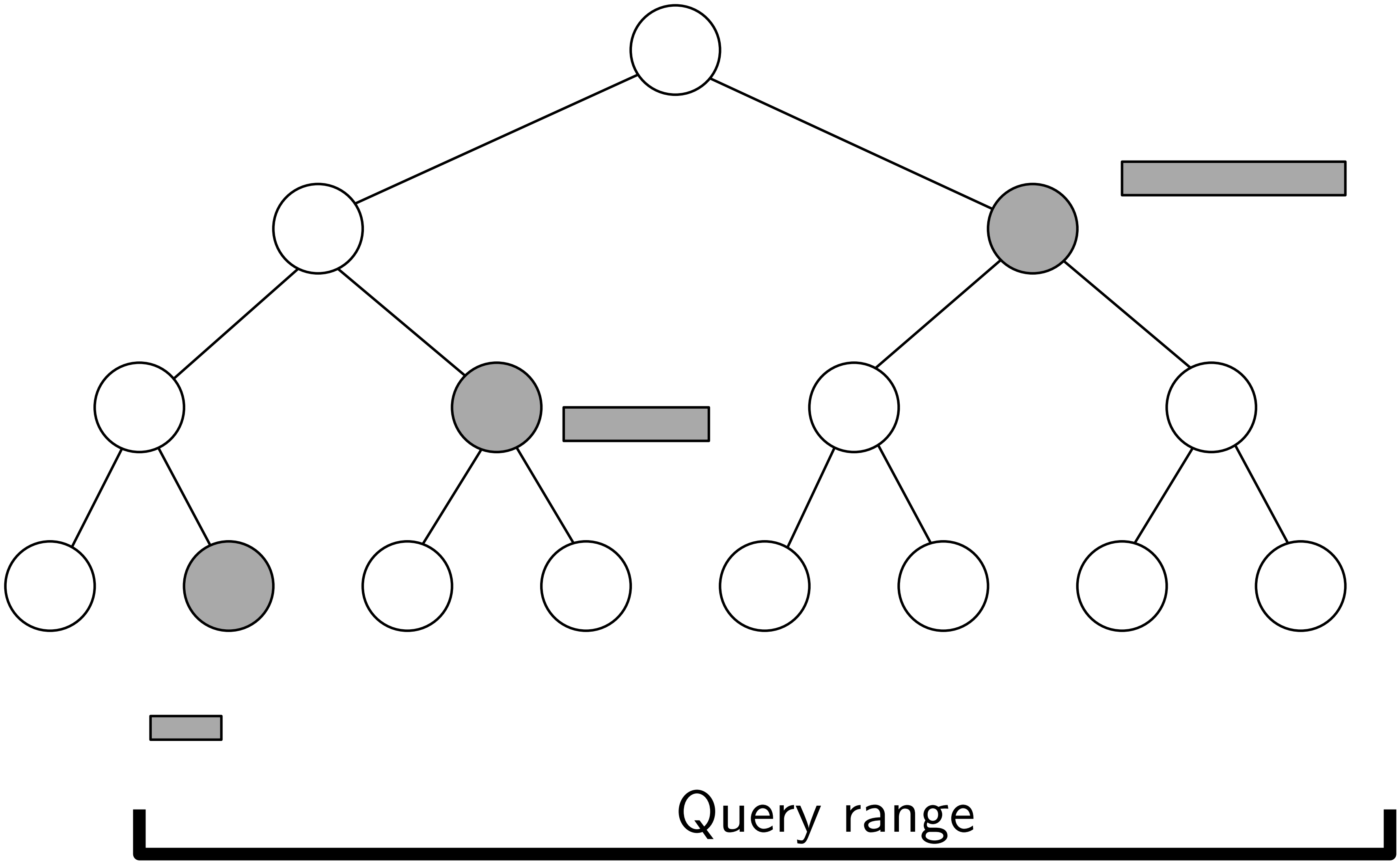

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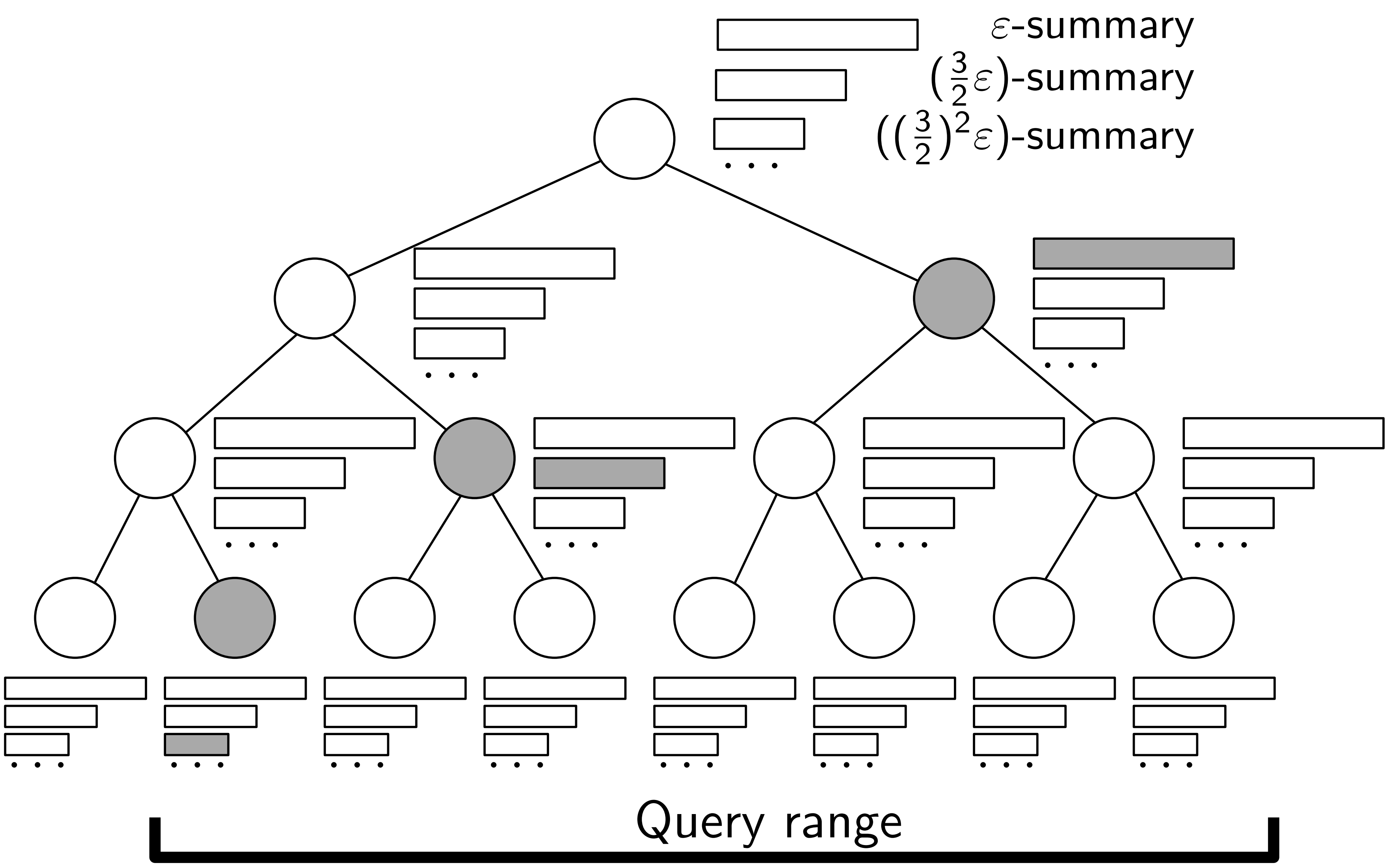
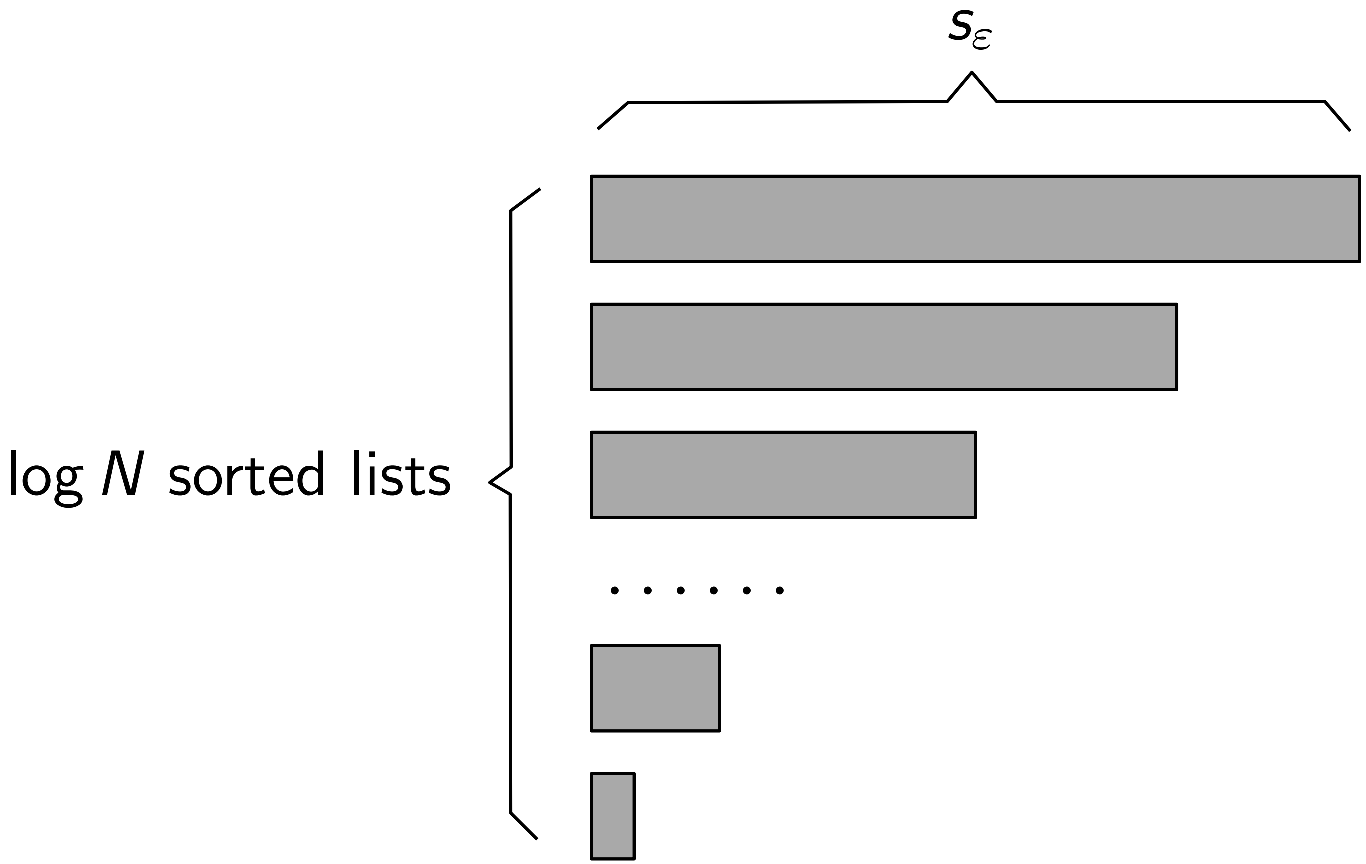


Query Cost



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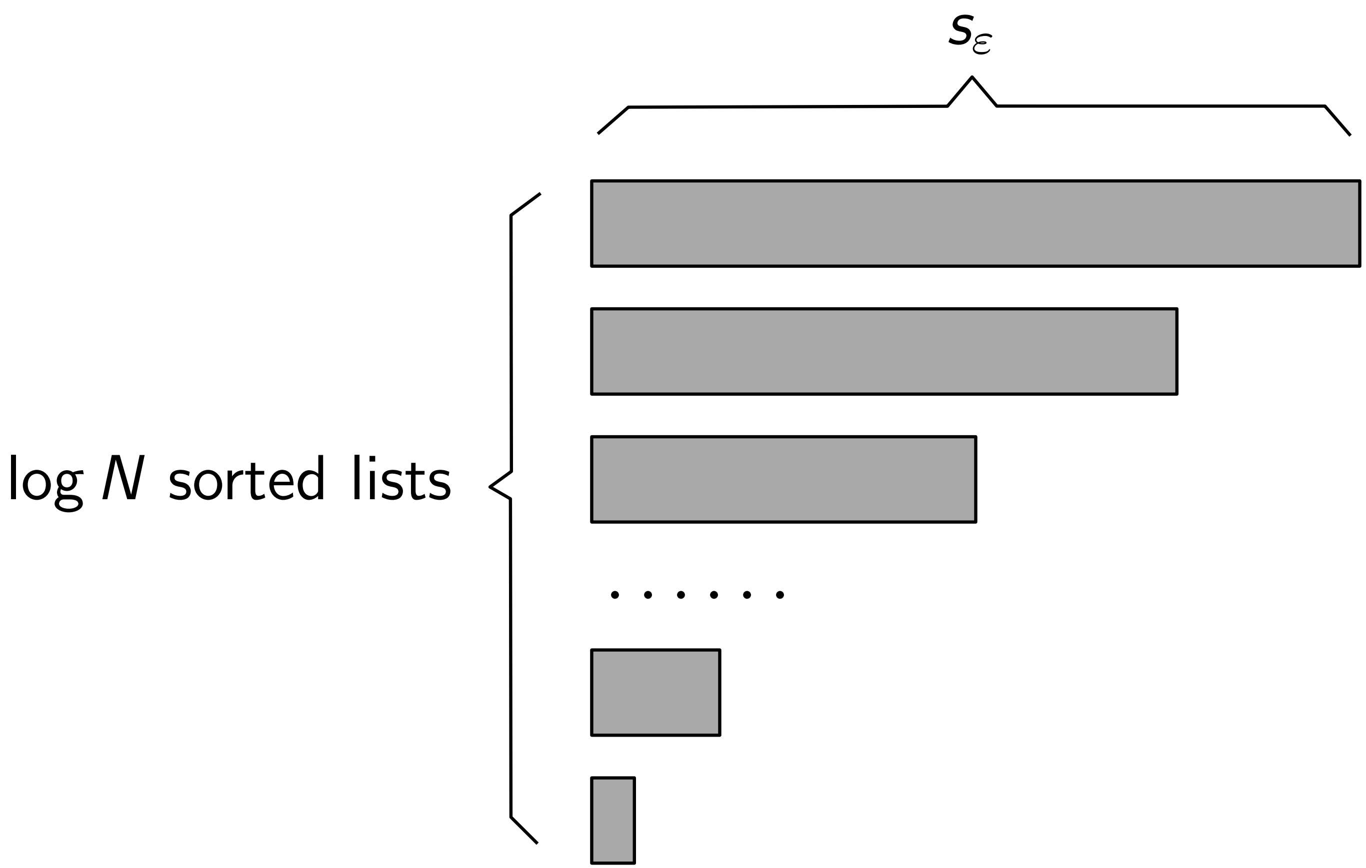

log N-way merging: $\Theta(s_\varepsilon \log\log N)$

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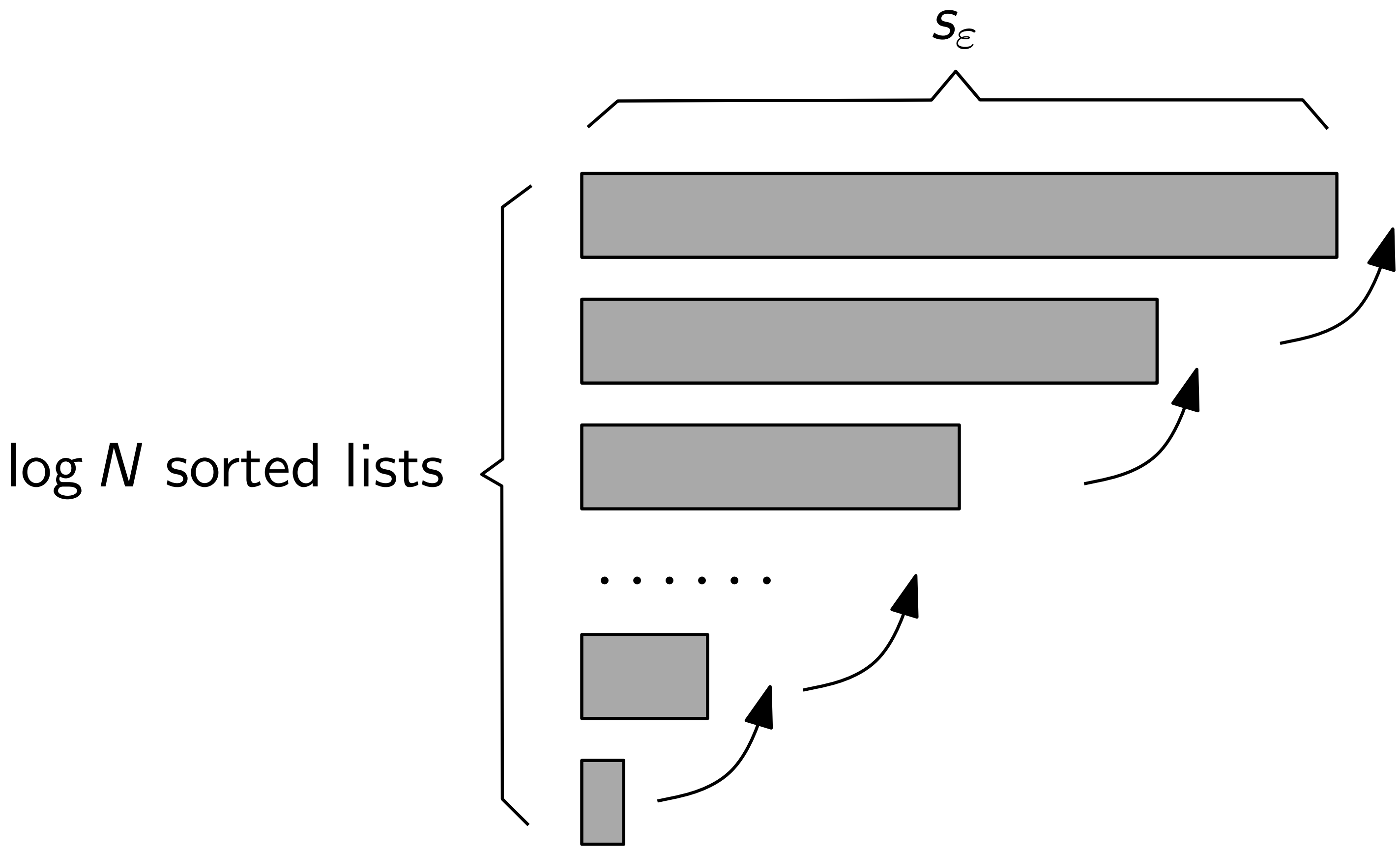

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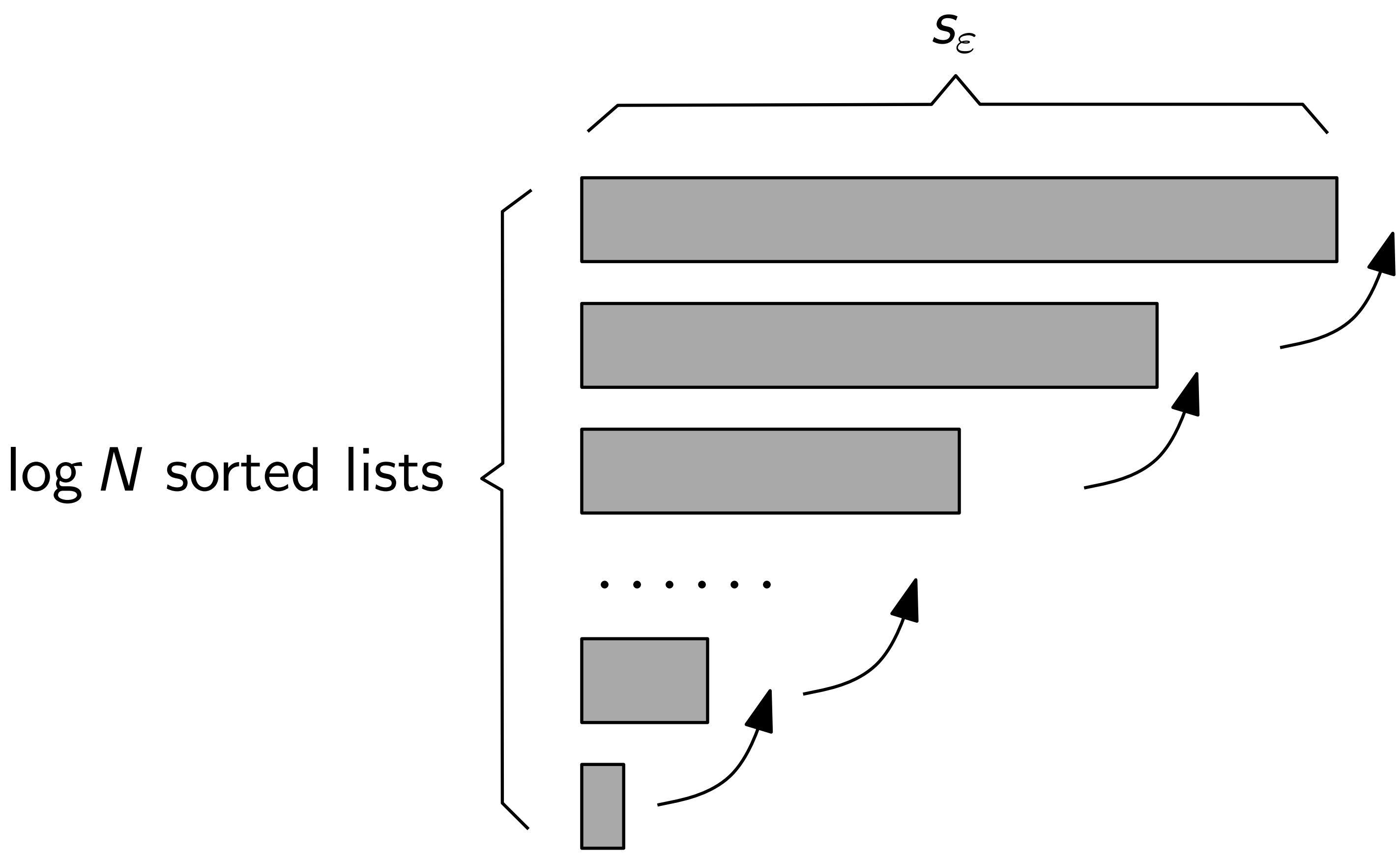

Bottom-up two-way merging: $O(s_\varepsilon)$

α-Exponentially Decomposable

- Multisets $D_1, \ldots, D_t$ with $F_1(D_i) \leq \alpha^{i-1} F_1(D_1)$, $\exists$ constant c, s.t. given $\mathcal{S}(\varepsilon, D_1), \mathcal{S}(c\varepsilon, D_2) \ldots, \mathcal{S}(c^{t-1}\varepsilon, D_t)$:
  - We can construct an $O(\varepsilon)$-summary for $D_1 \uplus \cdots \uplus D_t$.
  - The total size of $\mathcal{S}(\varepsilon, D_1), \ldots, \mathcal{S}(c^{t-1}\varepsilon, D_t)$ is $O(s_\varepsilon)$ and they can be combined in $O(s_\varepsilon)$ time.
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Theorem

For any $(1/2)$-exponentially decomposable summary, a database $\mathfrak{D}$ of N records can be stored in an internal memory structure of linear size so that a summary query can be answered in $O(\log N + s_\varepsilon)$ time.

Optimal Data Structure - External Memory

- Standard B-tree blocking with fat leaves

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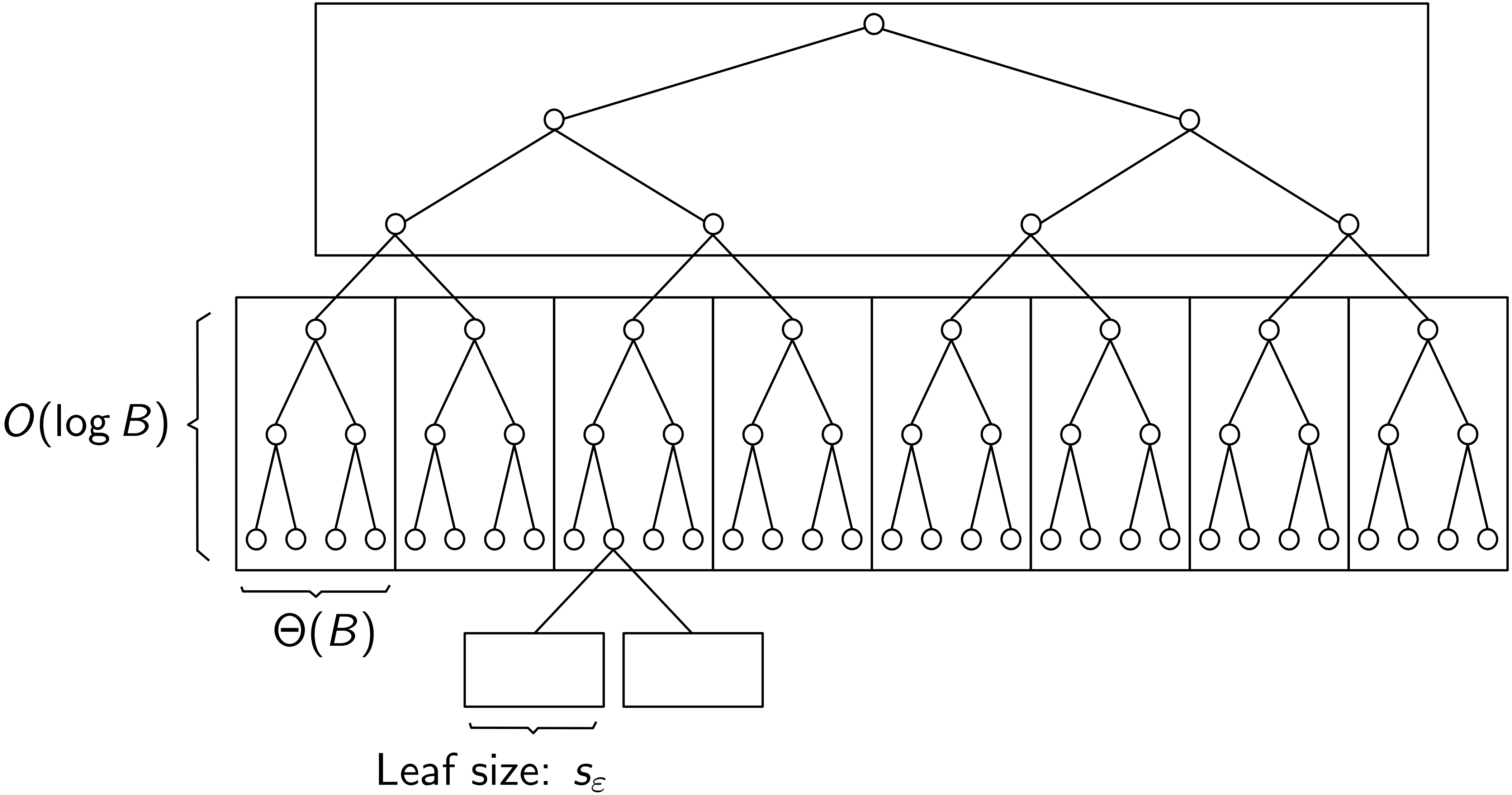
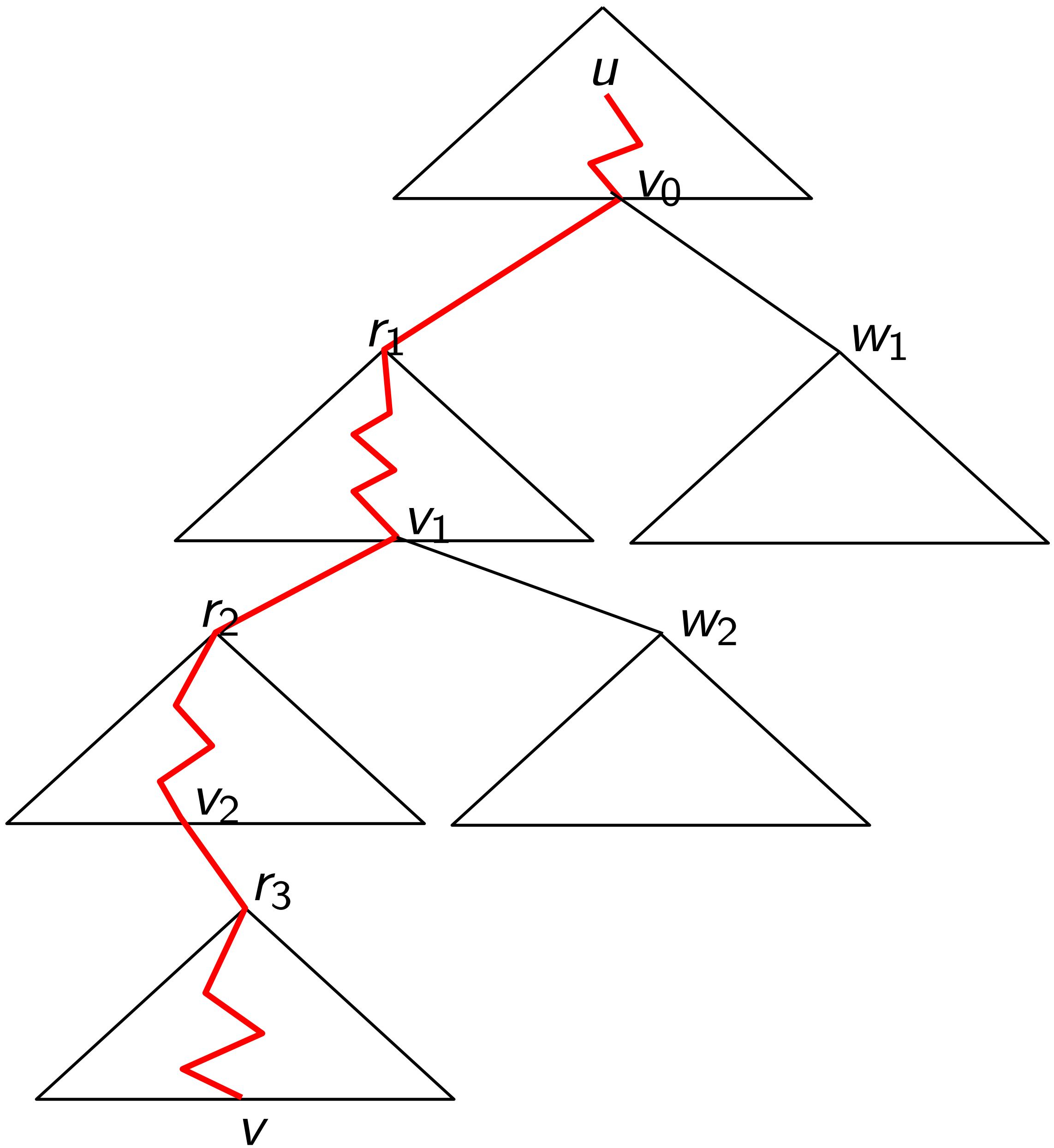
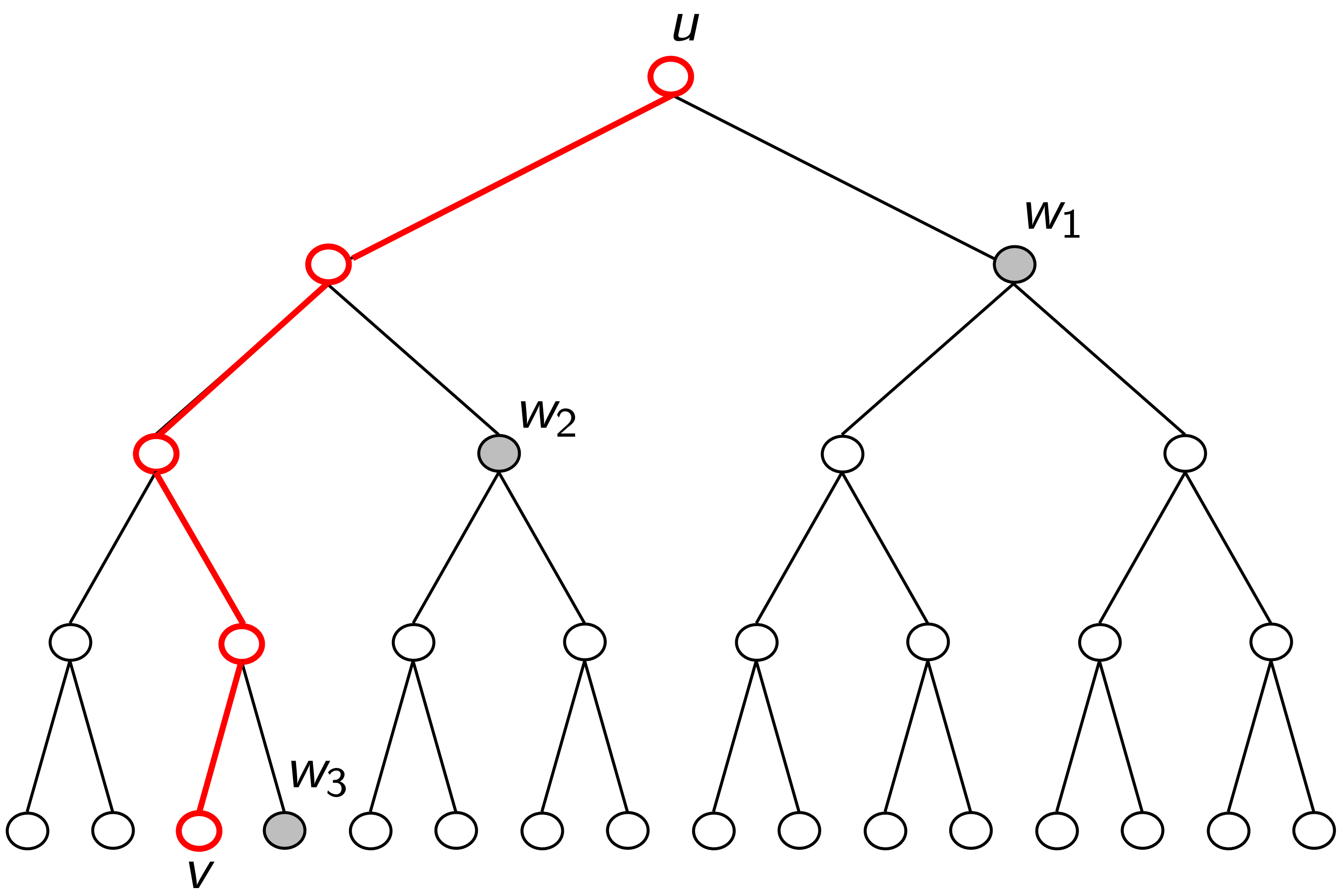


Query Path

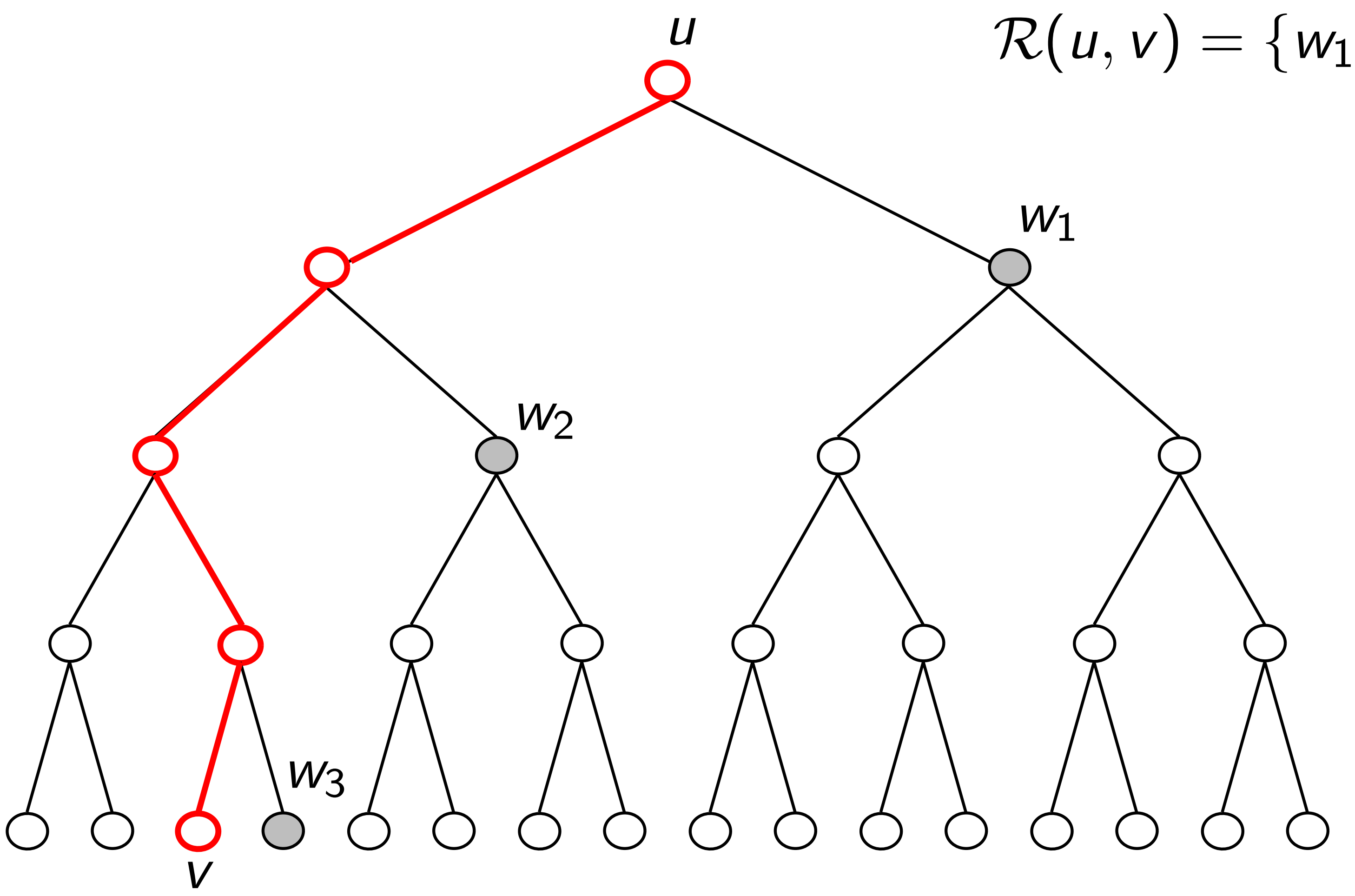


Summary Set



Summary Set

$$\mathcal{R}(u, v) = \{w_1, w_2, w_3\}$$



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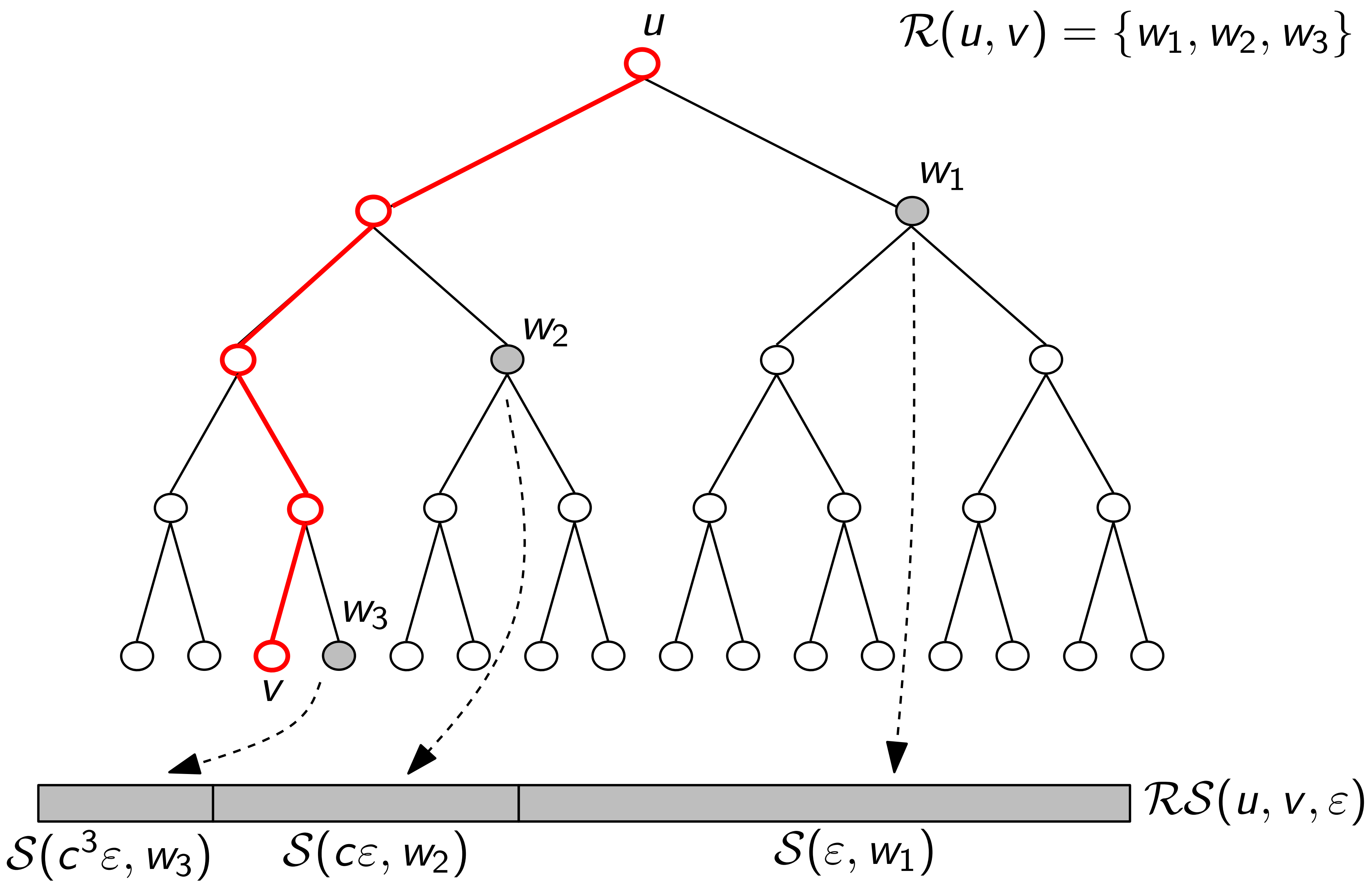
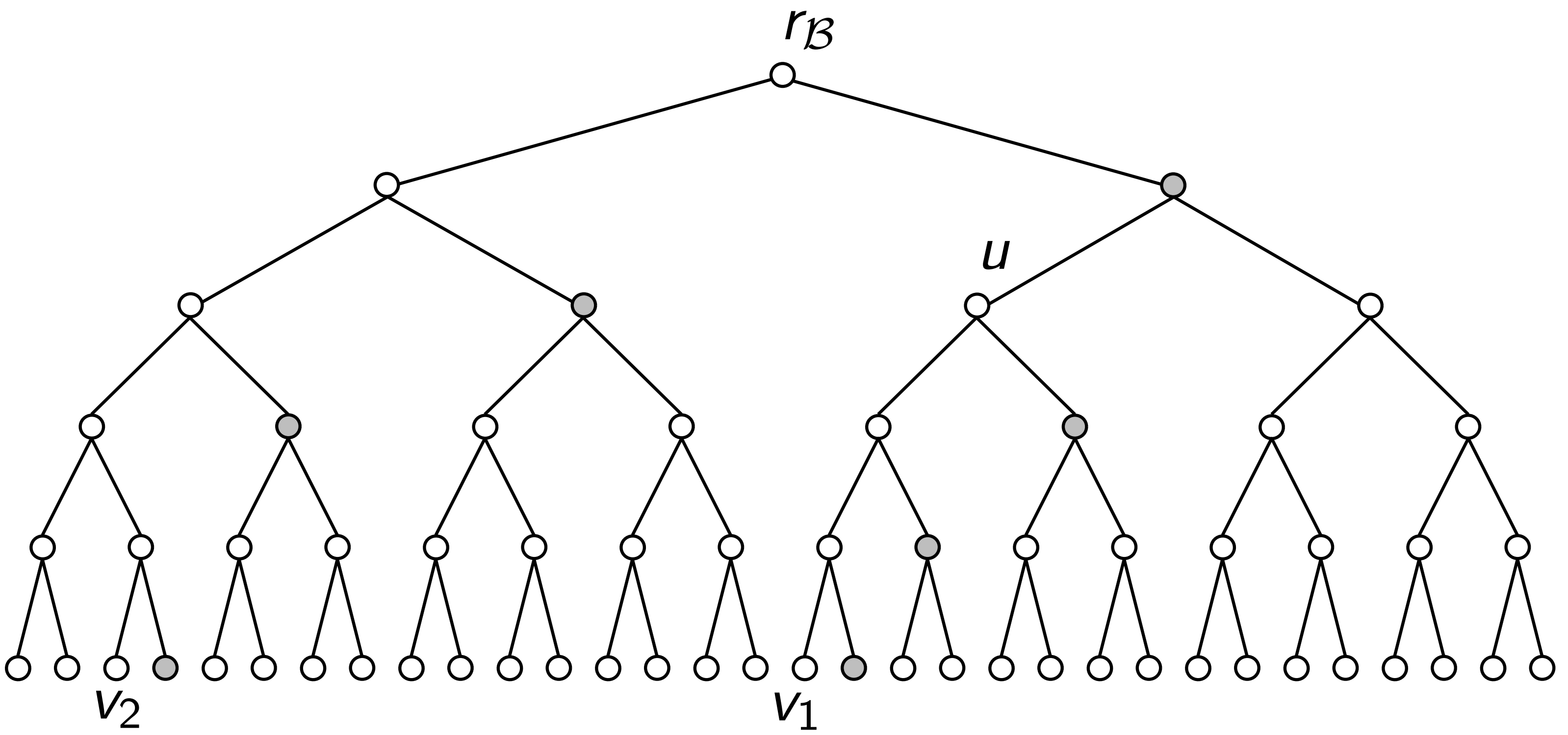


u
$\mathcal{R}(u, v) = \{w_1, w_2, w_3\}$
w_1
w_2
w_3
v
$\mathcal{RS}(u, v, \varepsilon)$
$\mathcal{S}(c^3\varepsilon, w_3)$
$\mathcal{S}(c\varepsilon, w_2)$
$\mathcal{S}(\varepsilon, w_1)$

Focus on a Block



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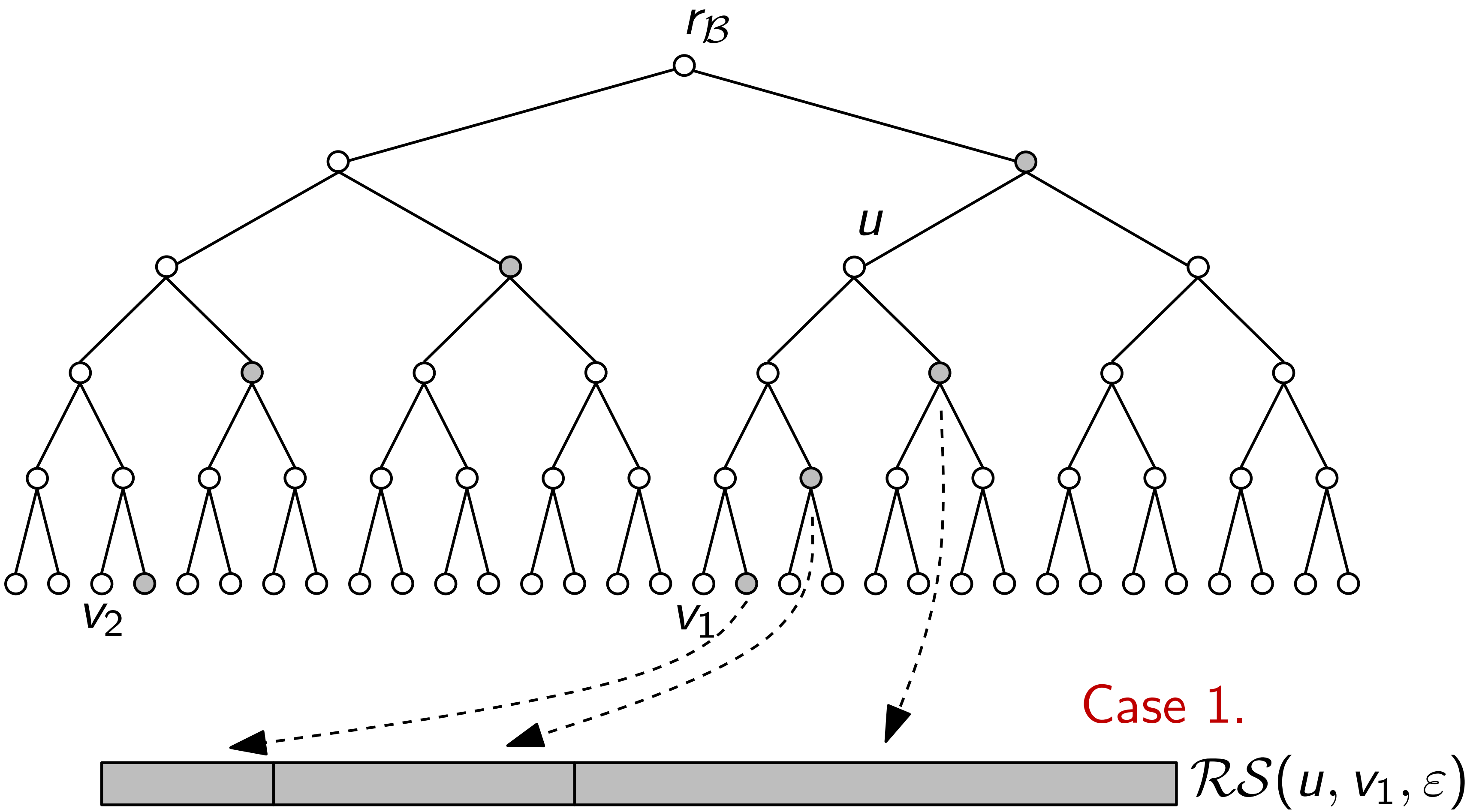

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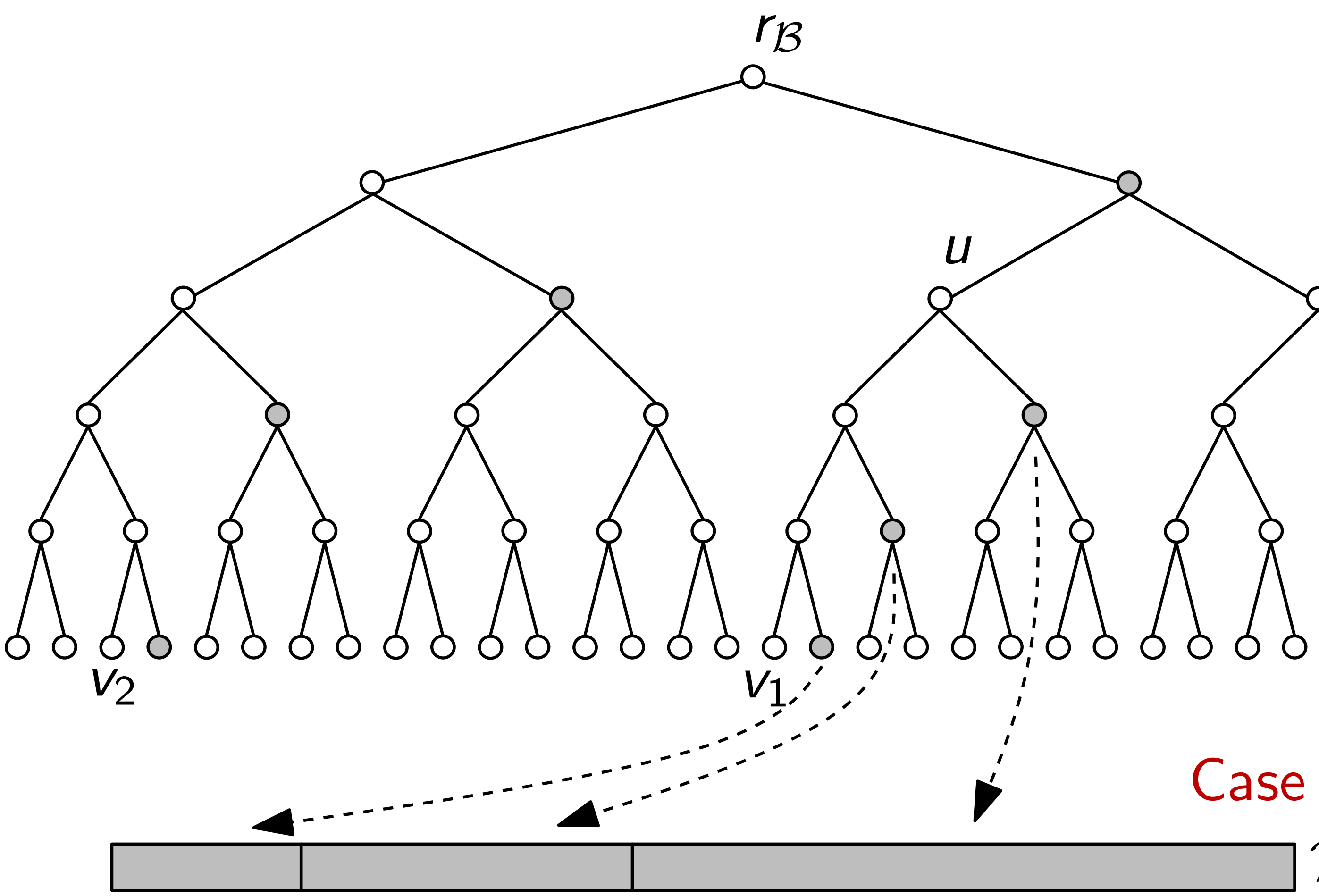

Case 1. Size: $s_\varepsilon B \log B$

$\mathcal{RS}(u, v_1, \varepsilon)$

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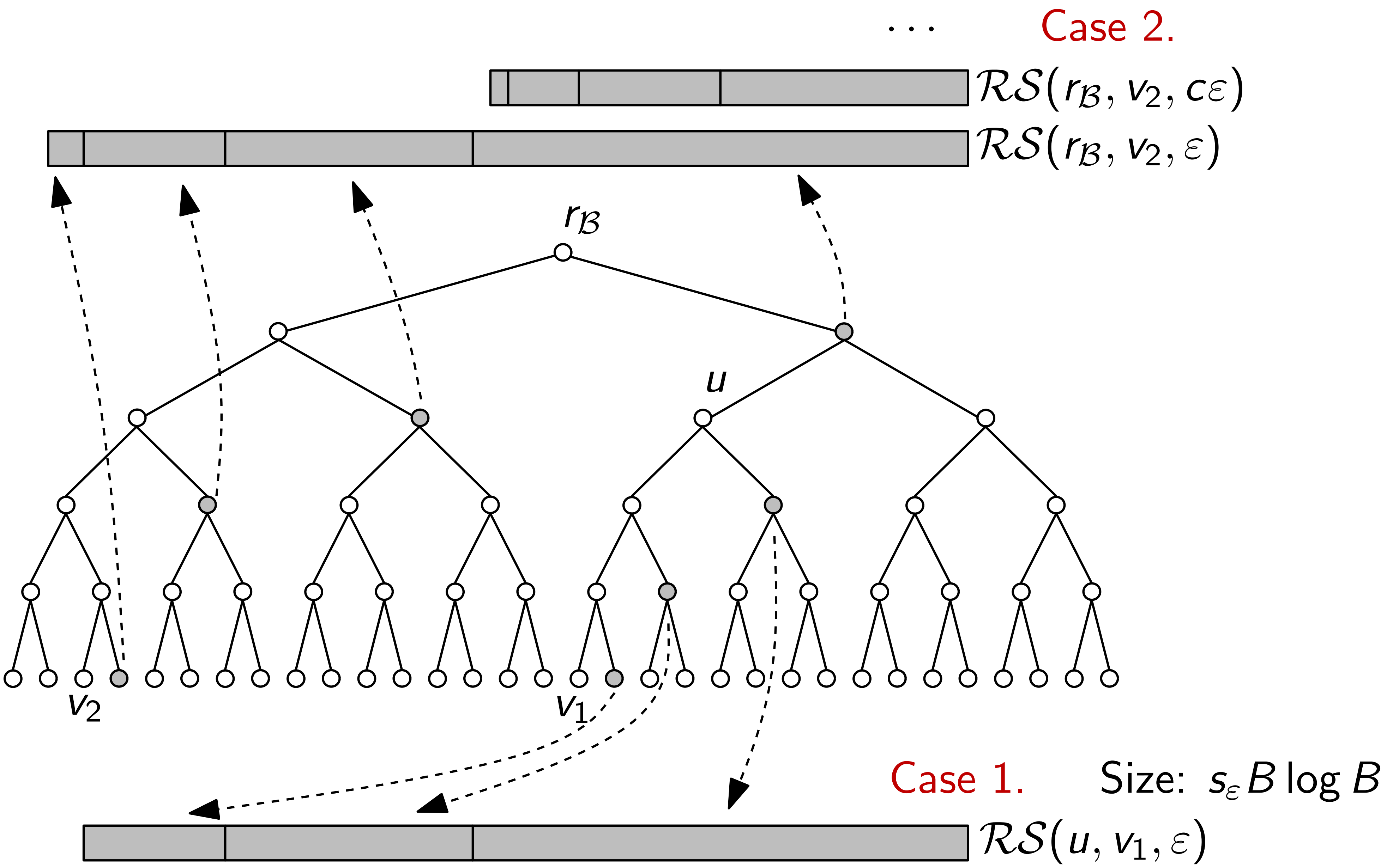

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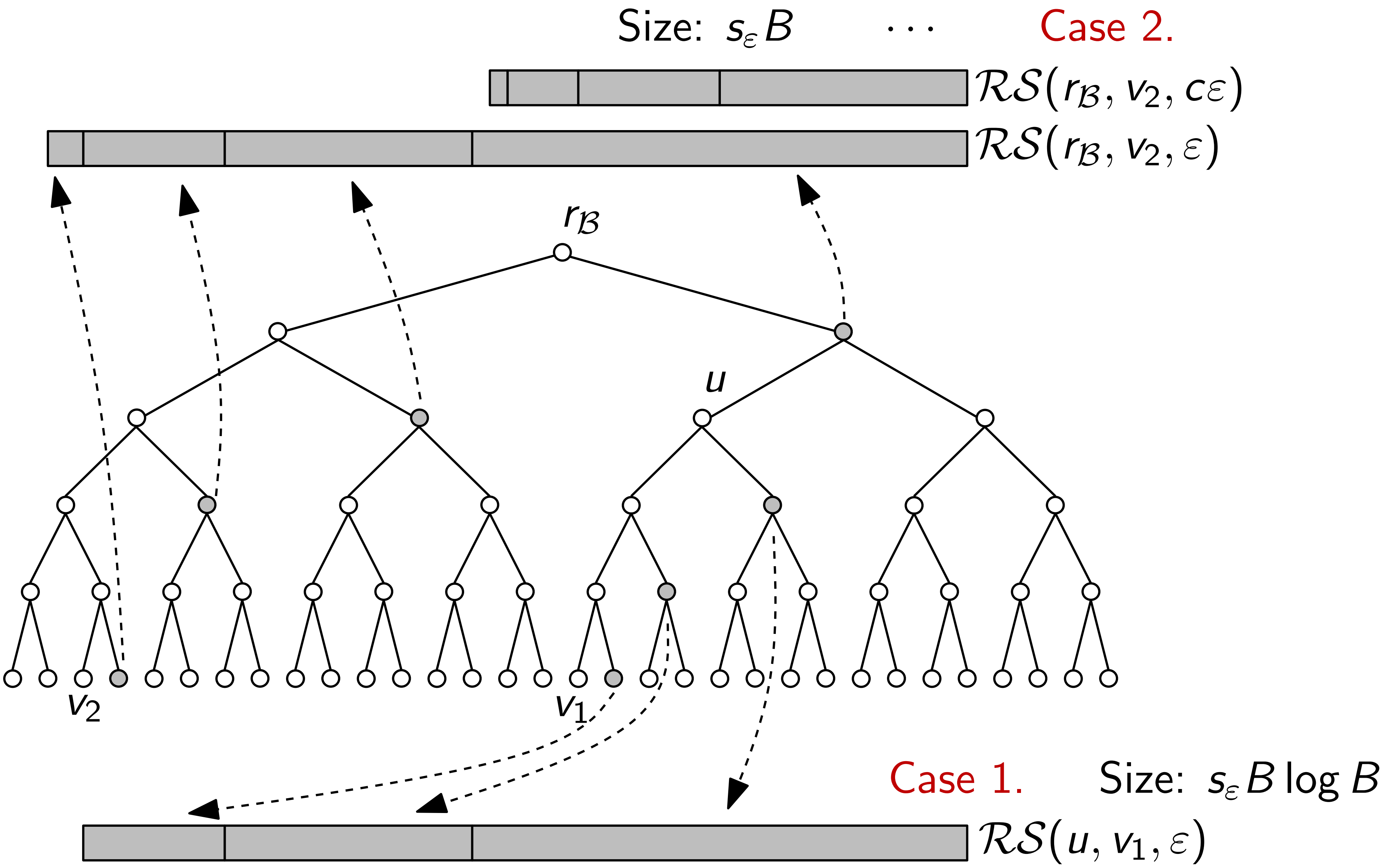

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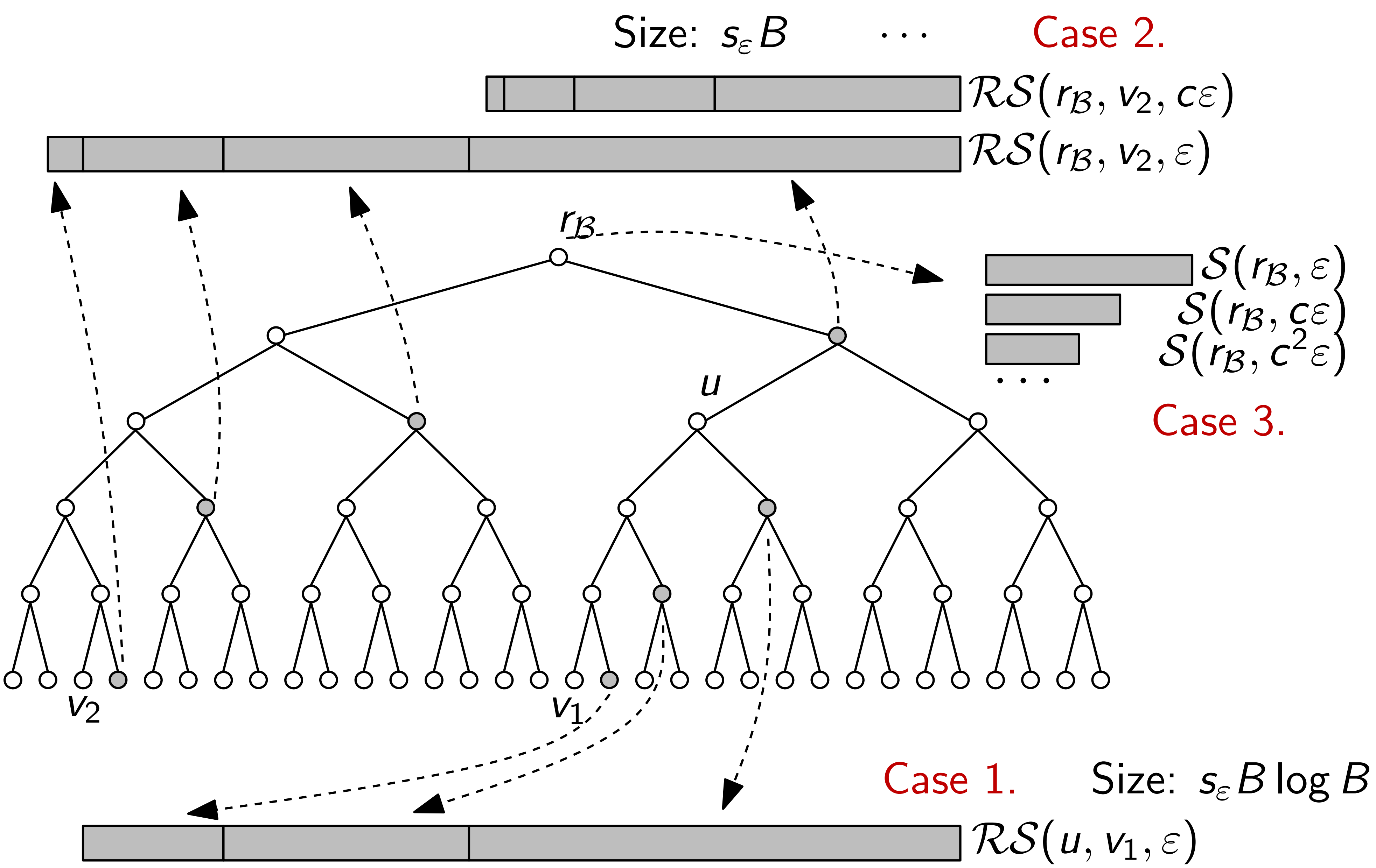

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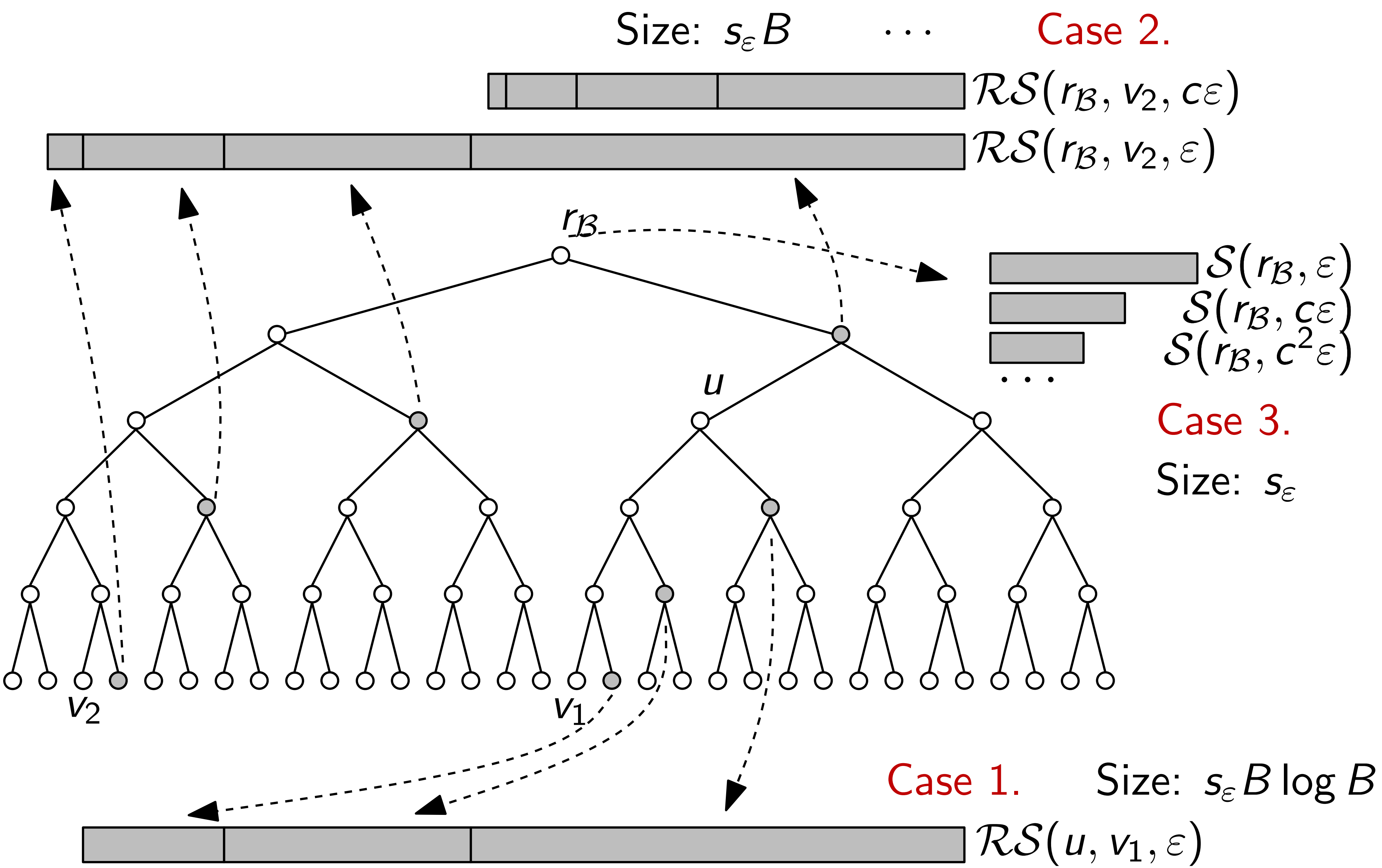
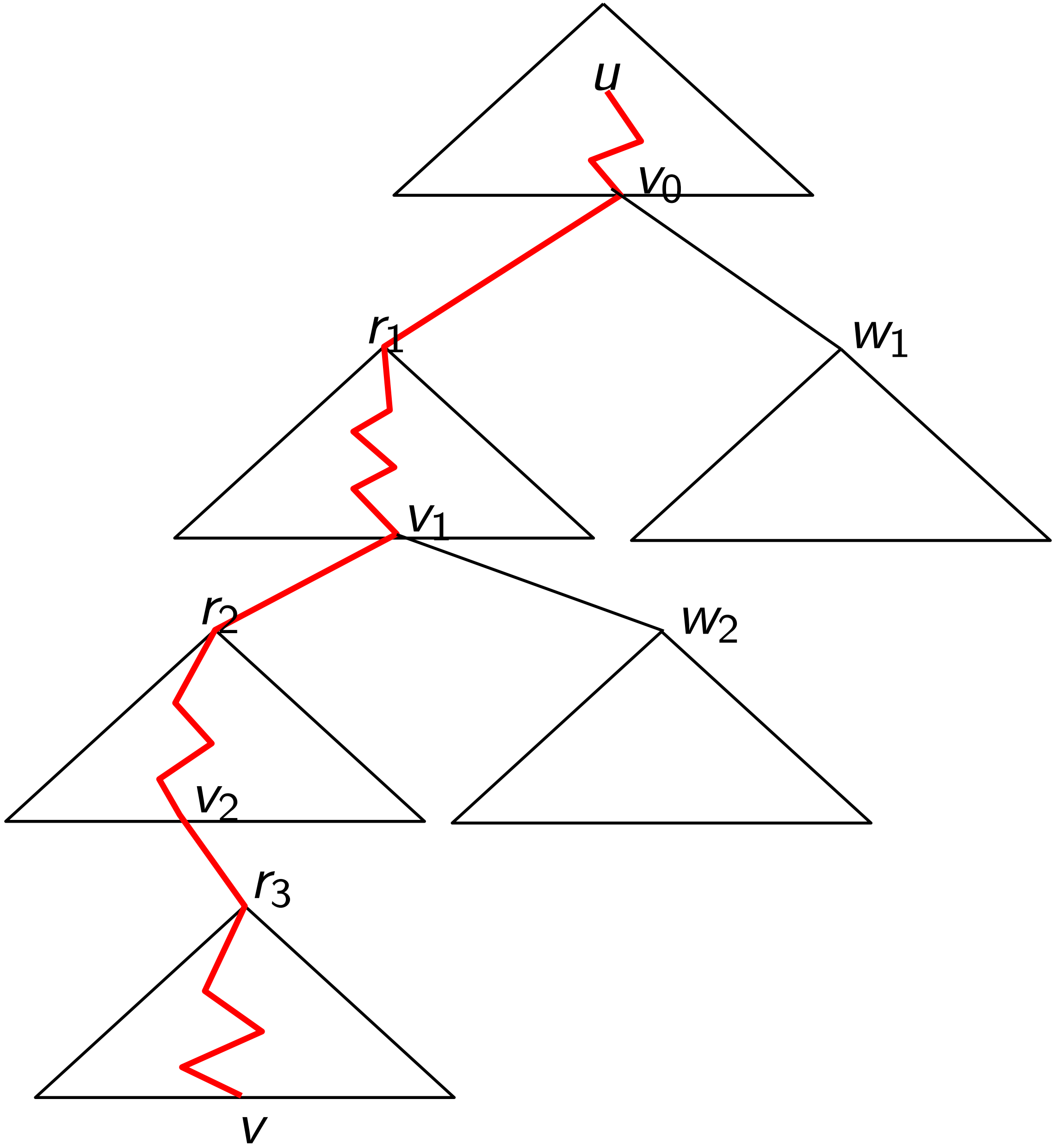


Query Process



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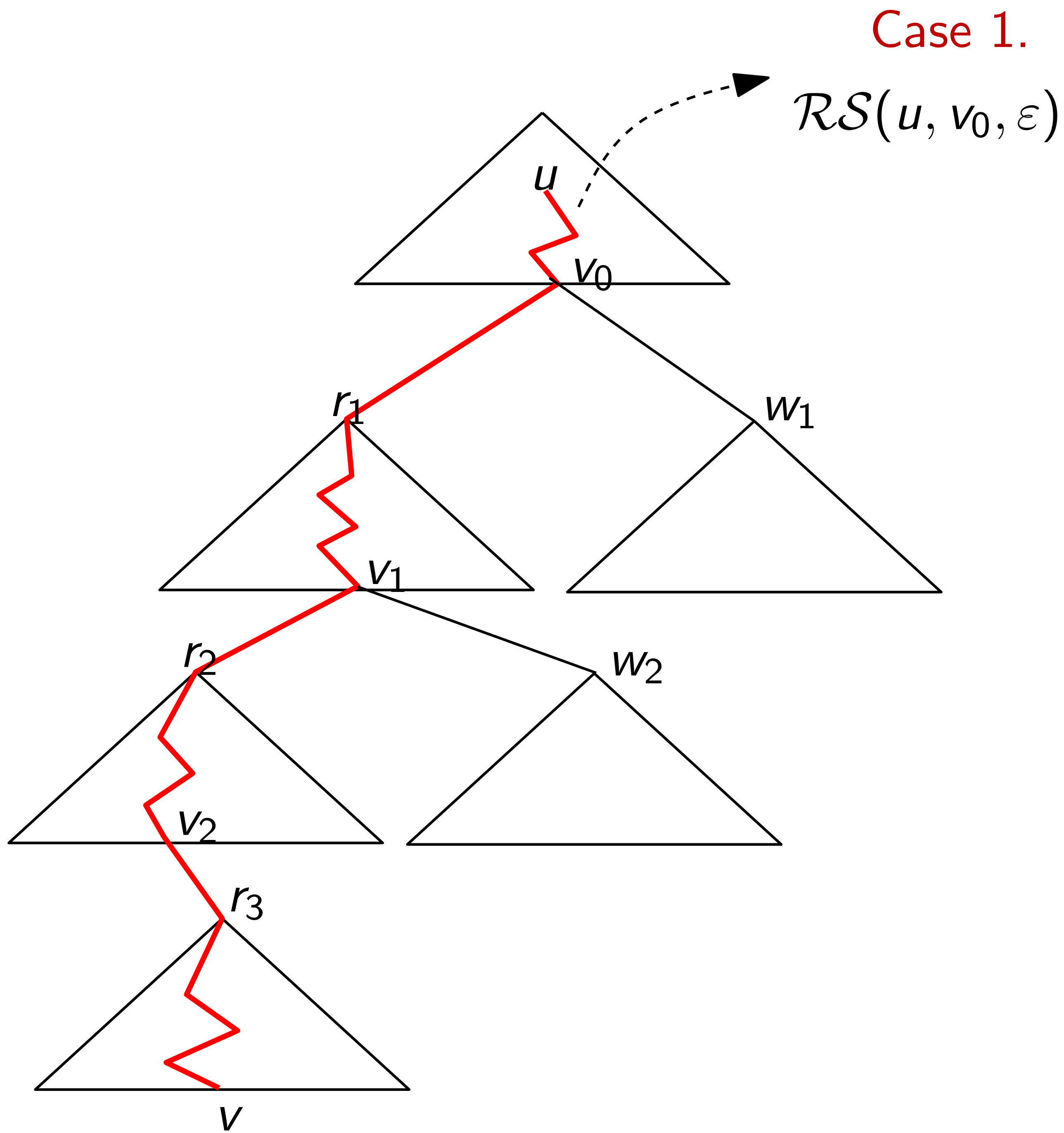

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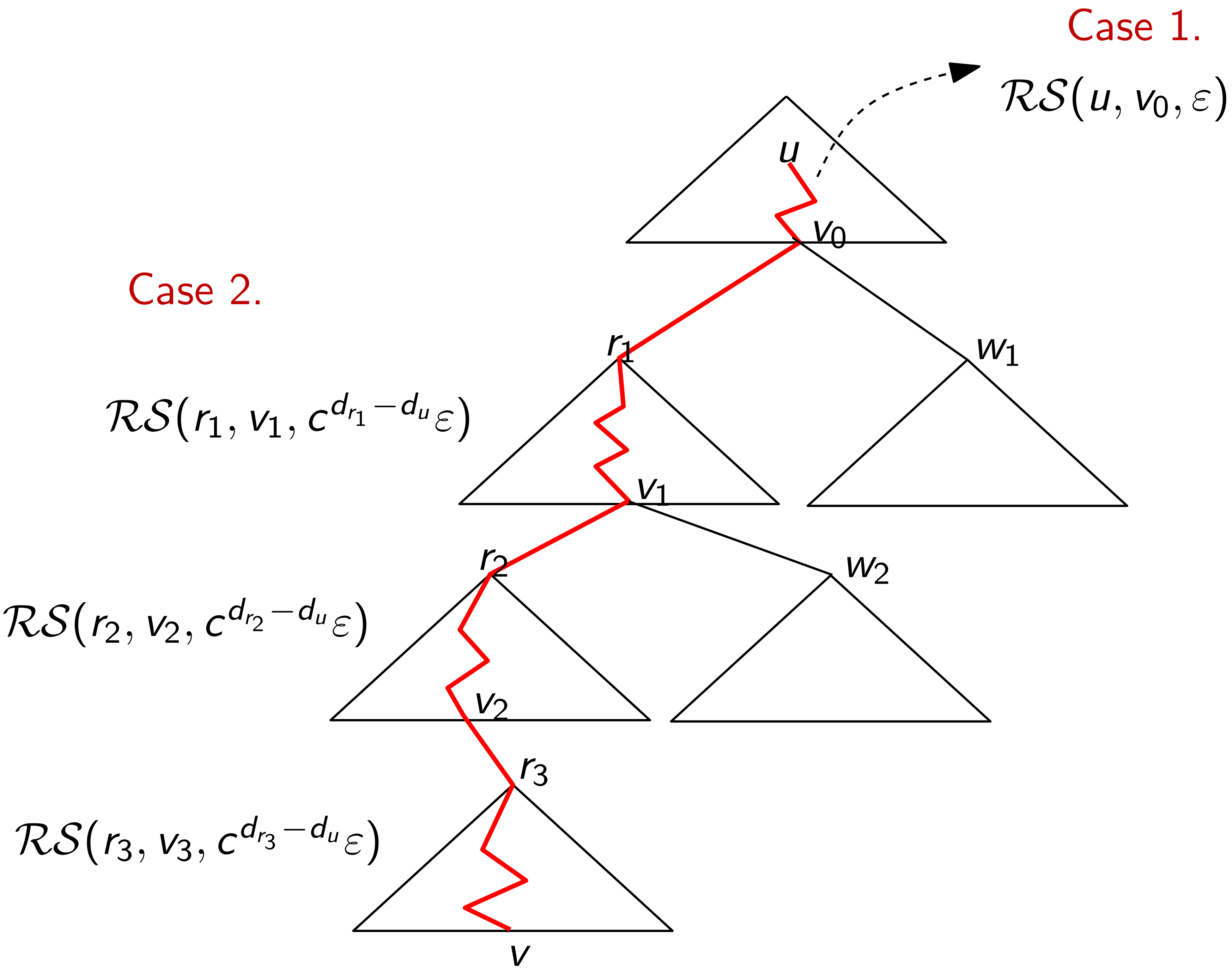

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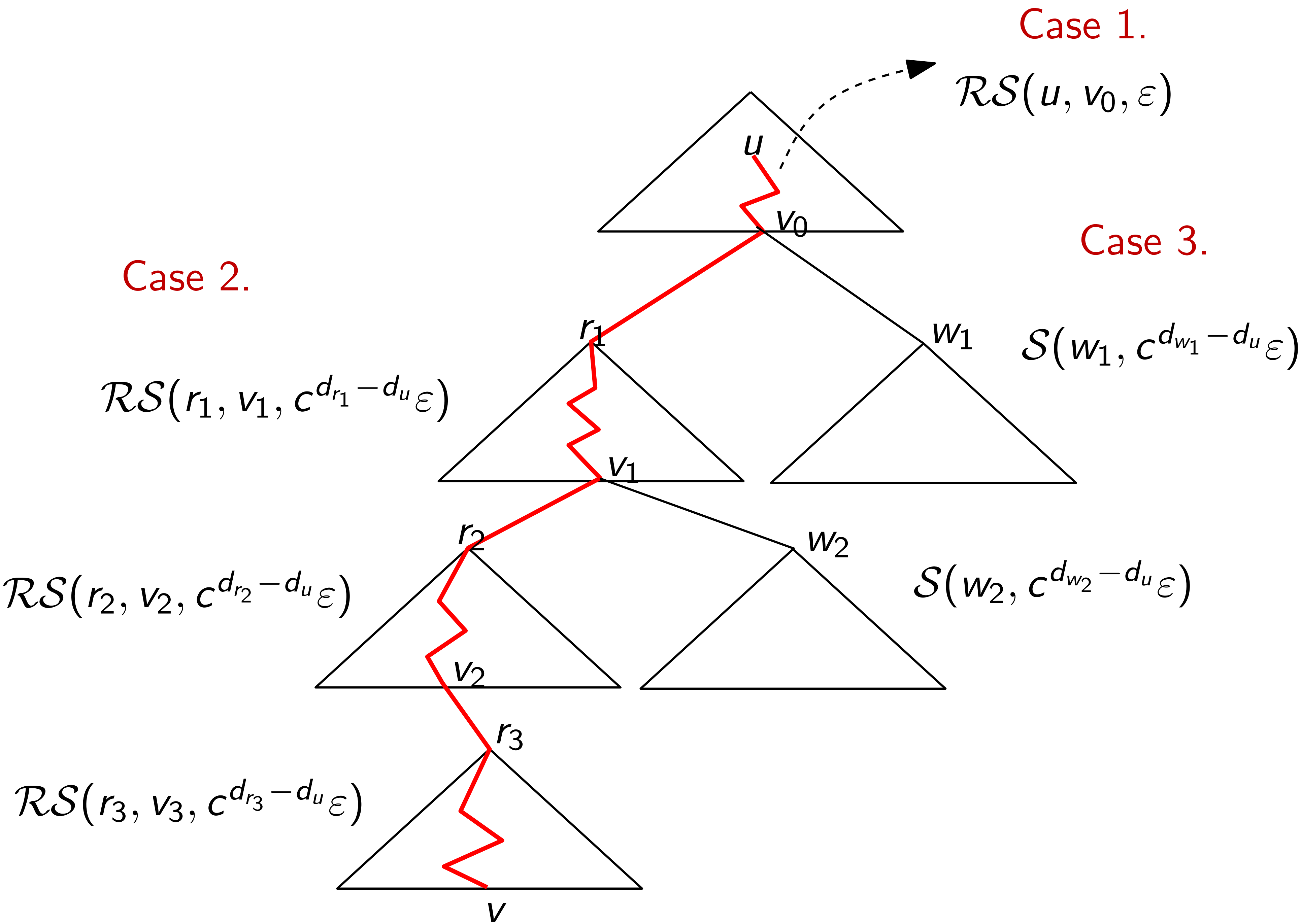
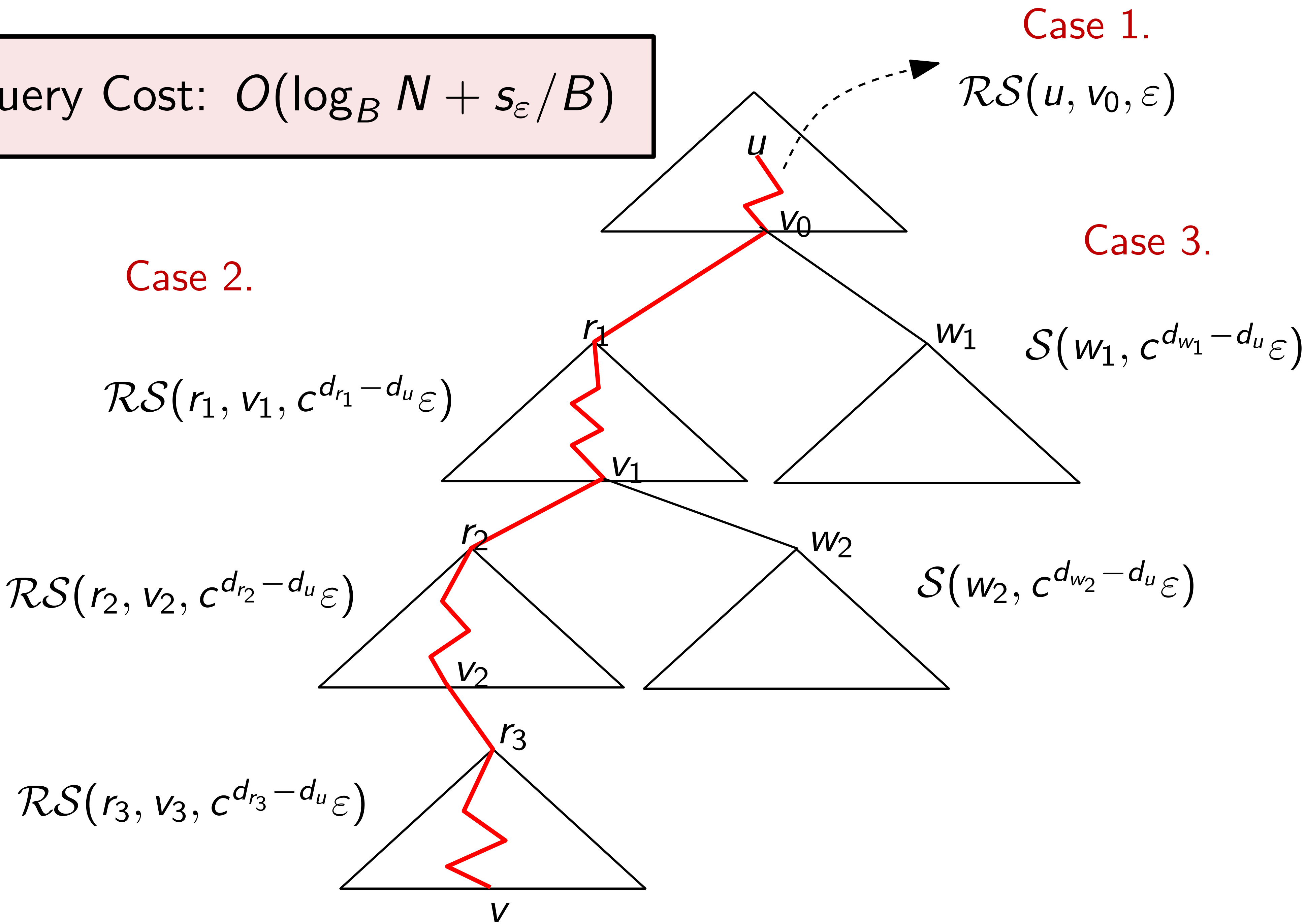


Query Process

Query Cost: $O(\log_B N + s_\varepsilon/B)$



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Query Cost: $O(\log_B N + s_\varepsilon / B)$
Space Usage: $O(N \log B)$

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$\Downarrow$

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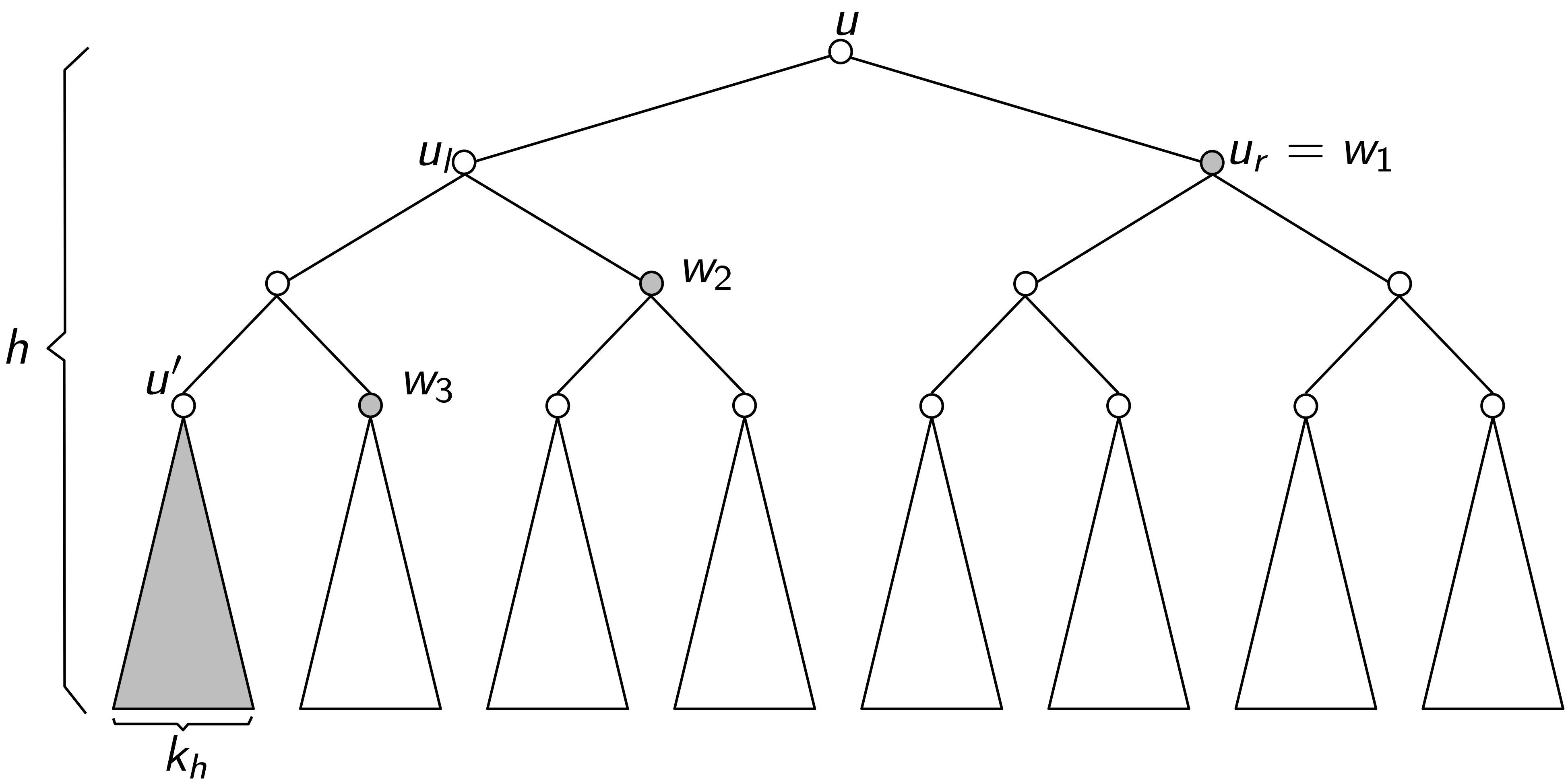
Idea: pack some leaves of u to reduce space usage

Packed Structure



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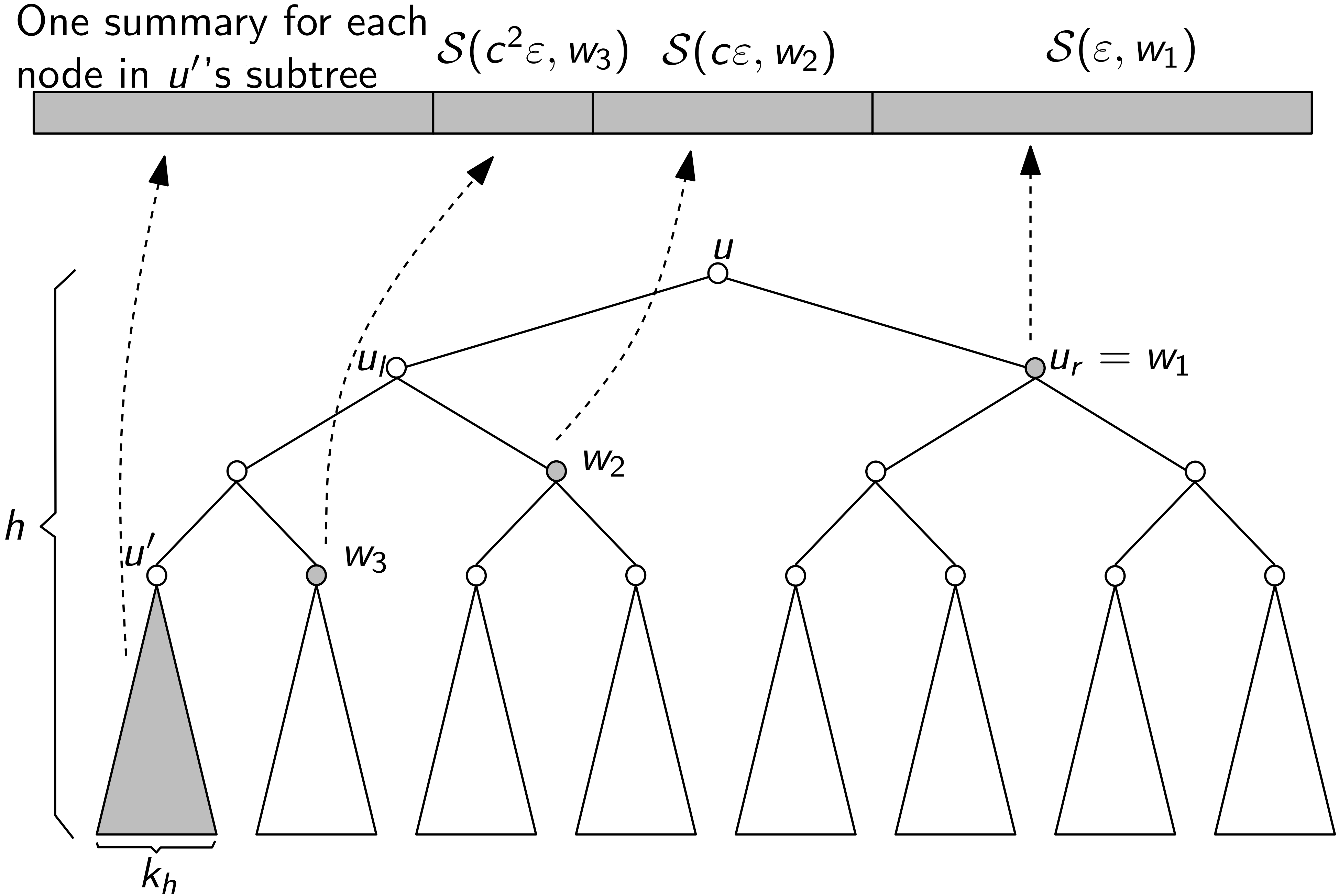
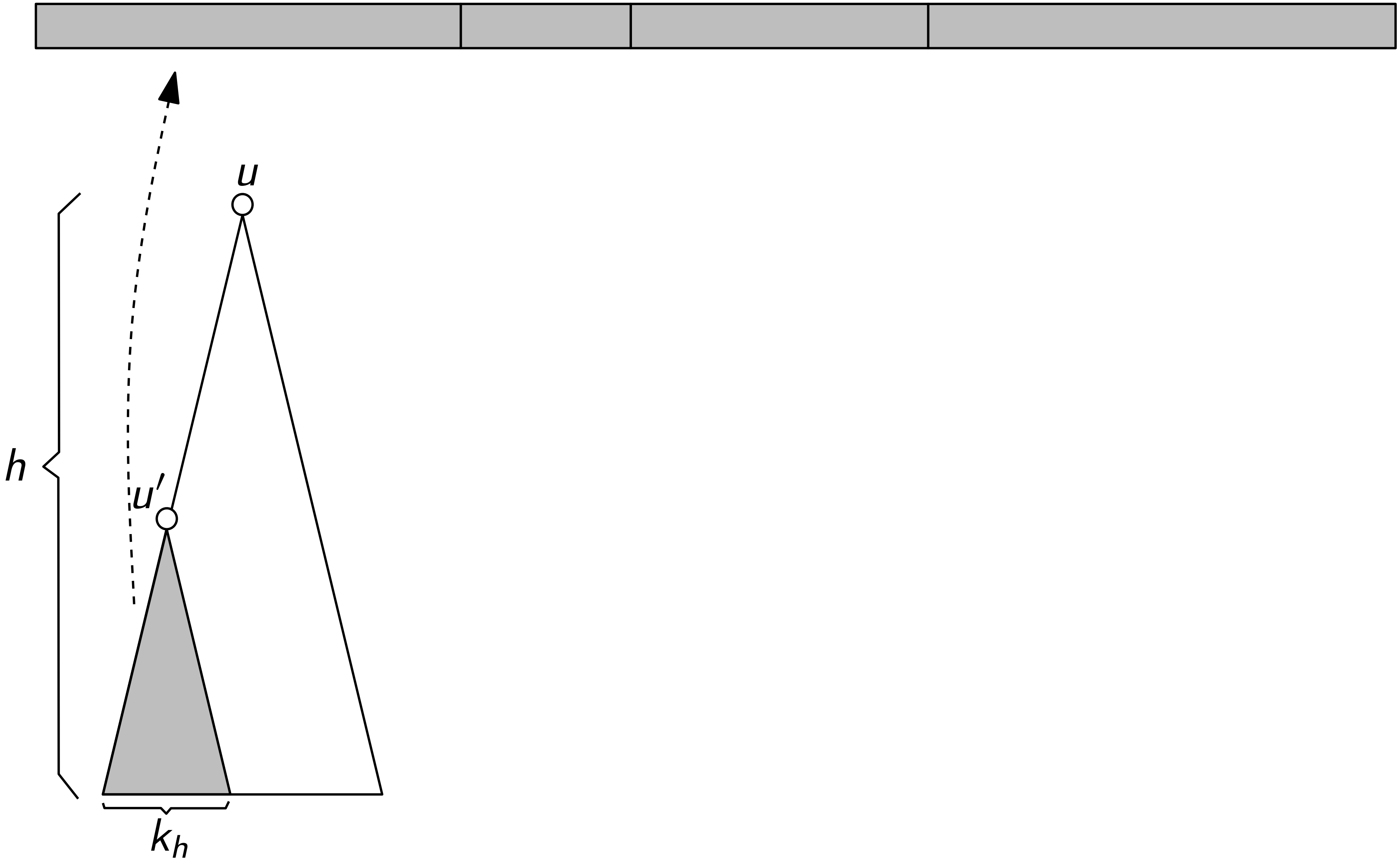


Packed Structure

One summary for each node in u''s subtree

$\mathcal{S}(c^2\varepsilon, w_3)$ $\mathcal{S}(c\varepsilon, w_2)$ $\mathcal{S}(\varepsilon, w_1)$



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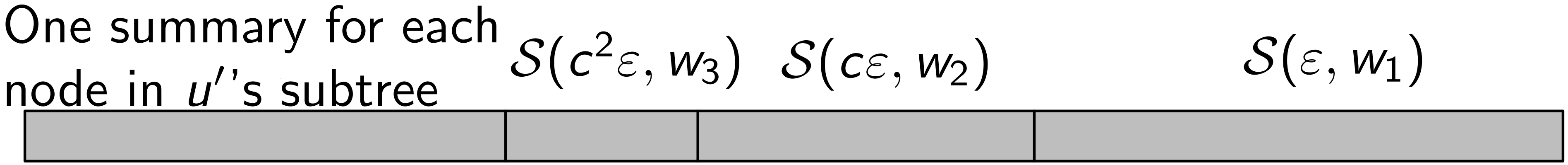
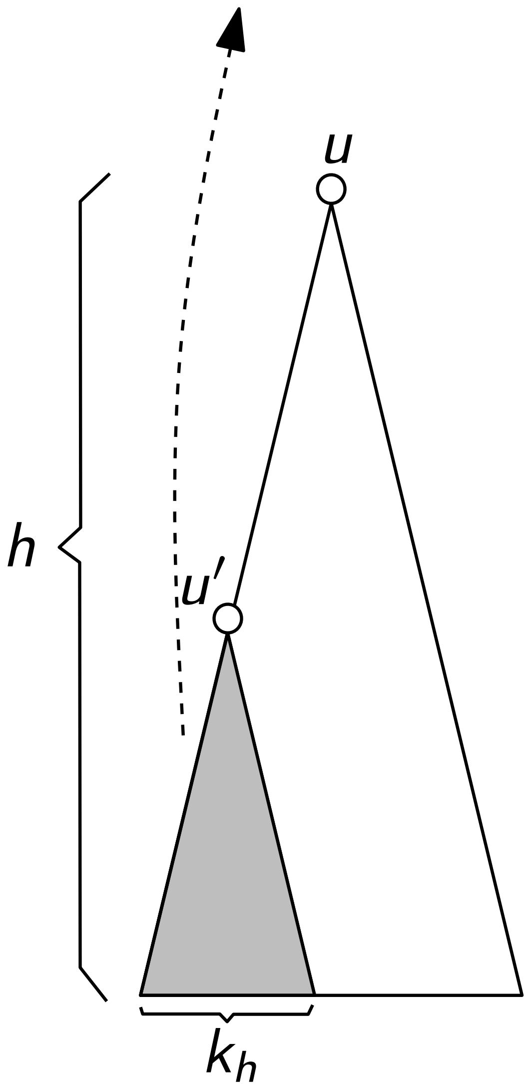


The total size of all summaries below u':

$$\sum_{i=0}^{\log k_h} \frac{k_h}{2^i} s_{c^{h-i-1}\varepsilon}. \tag{1}$$



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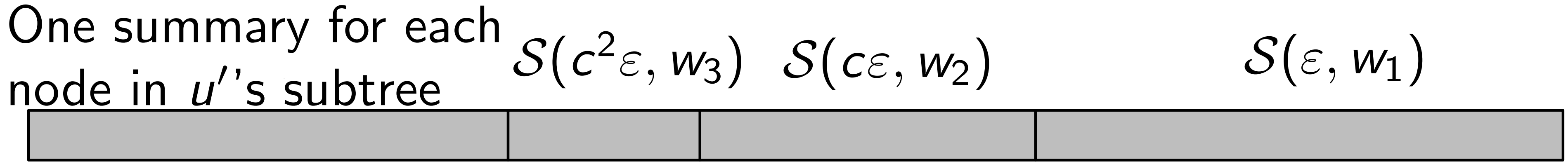

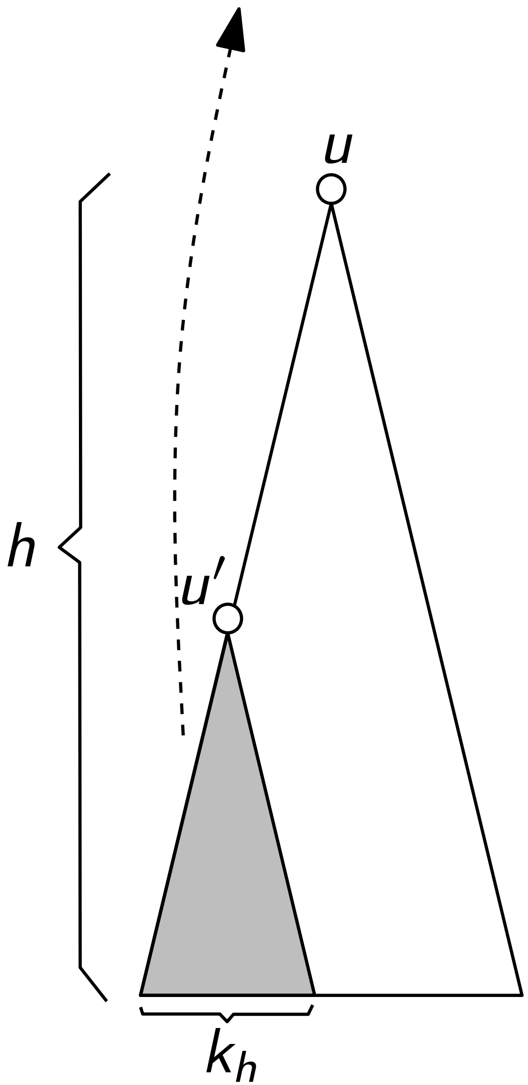

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Choose k_h such that (1) is $\Theta(s_\varepsilon)$.

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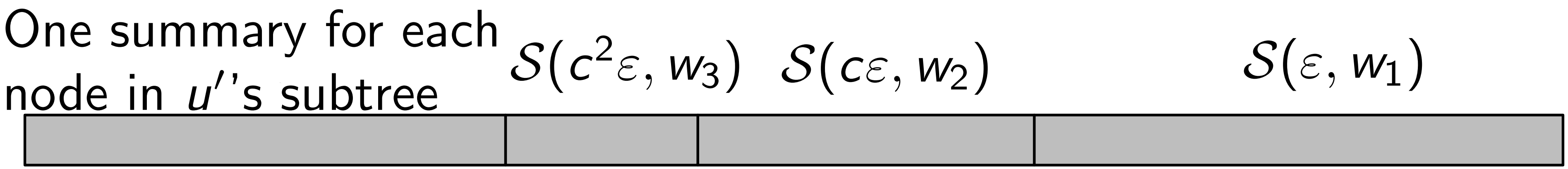

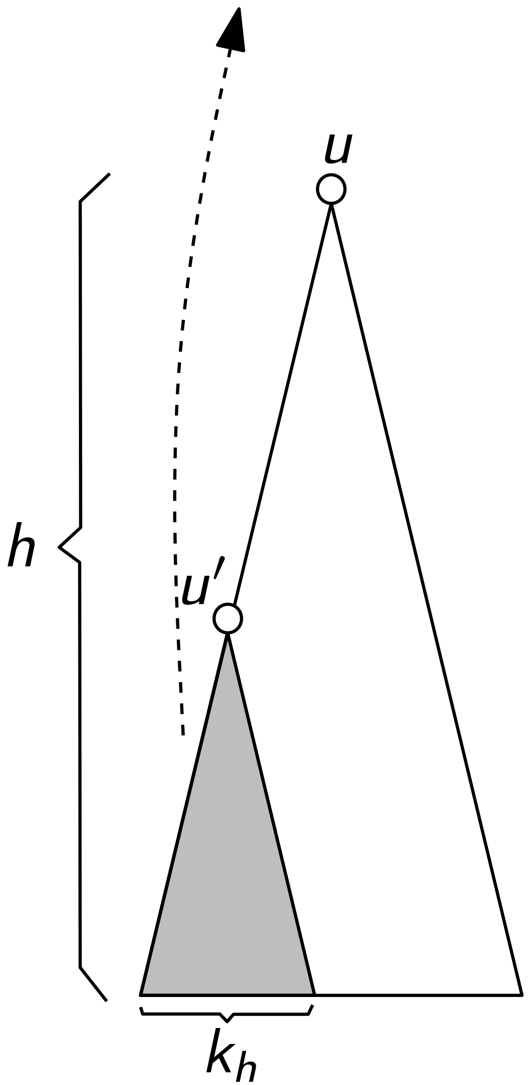

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Choose k_h such that (1) is $\Theta(s_\varepsilon)$.

The total size of the packed structures in $\mathcal{B}$ is bounded by

$$\sum_{h=1}^{\log B} Bs_\varepsilon / k_h \leq O(Bs_\varepsilon).$$

Optimal Data Structure - External Memory

Theorem

For any (1/2)-exponentially decomposable summary, a database $\mathcal{D}$ of N records can be stored in an external memory index of linear size so that a summary query can be answered in $O(\log_B N + s_\varepsilon/B)$ I/Os.

Exponentially Decomposable vs. Decomposable

- Exponentially decomposable summaries
  - Heavy hitters
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Space: $O(N)$

External Memory:
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$O(\log N / \log(B/s_\varepsilon))$ for $s_\varepsilon < B$
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Can we improve?

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- Joins? General SQL queries?

Thank you!