A Joint Power Control, Link Scheduling and Rate Control Algorithm for Wireless Ad hoc Networks

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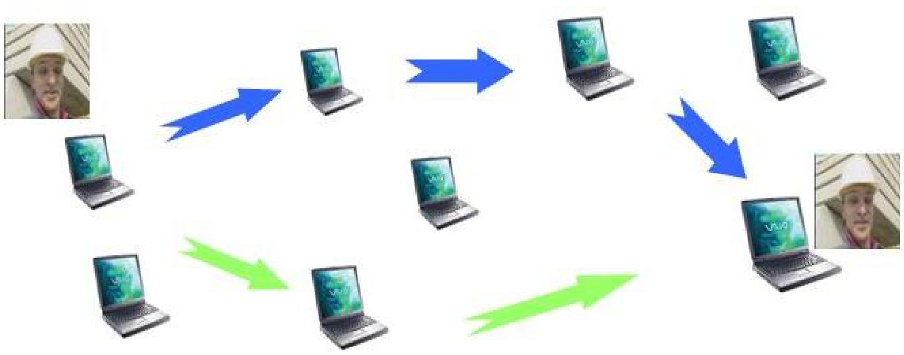
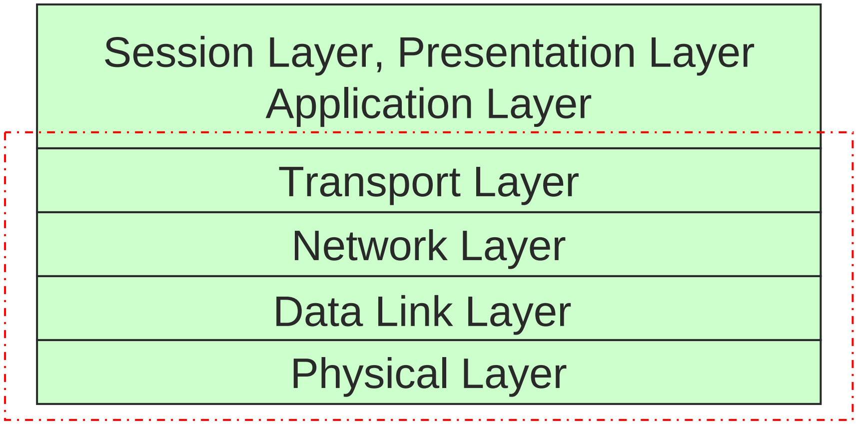
Outline

- Rate Control in Wireless Ad hoc Networks
  - Cross-layer Design
  - Joint Power, Link Scheduling and Rate Control
- PSR Algorithm
  - Rate Control Subproblem
  - Power Control & Link Scheduling Subproblem
  - Algorithm Description and Its Convergence
- Experimental Results
- Summary

Rate Control in Wireless Ad hoc Networks

- Rate control
  - Objectives: fully utilize available resource & ensure fairness
  - Resource allocation in wireless ad hoc networks is more complex than in wired network → requires a joint control among different layers (cross-layer design)
  - In this paper, we discuss the interference constraint in a band-limited Additive White Gaussian Noise channel, and integrate it with power control, link scheduling and rate control





Get Started from a Simple Example

- Assume data flows
  - $v_1 \rightarrow v_3 \rightarrow v_4 \rightarrow v_5$ and $v_2 \rightarrow v_3 \rightarrow v_4 \rightarrow v_6$
  - User node v_1 with rate x_1, user node v_2 with rate x_2
  - Half-duplex and use TDMA as medium access mechanism
  - Network topology → Flow contention graph

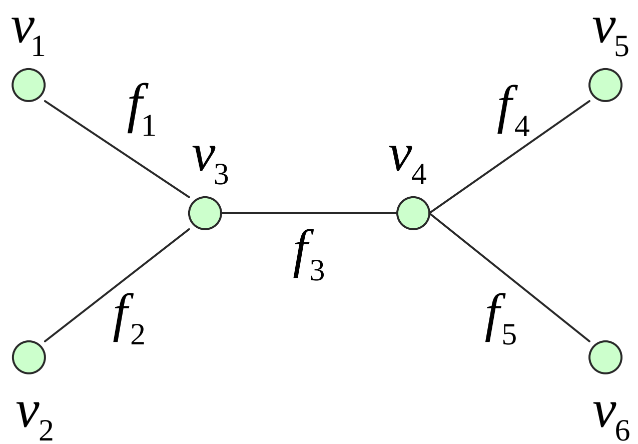


Fig1. Network topology graph

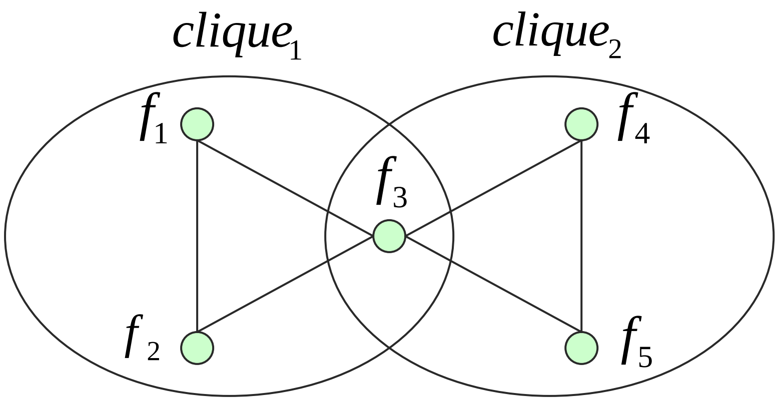


Fig2. Flow contention graph

Three Constraints

- Flow constraint

$$f_1 = f_4 = x_1, \quad f_2 = f_5 = x_2, \quad f_3 = x_1 + x_2$$

$\Rightarrow \quad F_{L\times 1} = R_{L\times S} X_{S\times 1}$ where route matrix $R_{L\times S} : R_{ls} = \begin{cases} 1, & if \quad l \in s \\ 0, & otherwise \end{cases}$

- Capacity constraint

$$\forall l \in L, \quad f_l \le c_l = W\log(1 + SINR_l) = W\log(1 + \frac{p_l h_l}{\sum_{l' \ne l} p_{l'} h_{l'} + \sigma^2})$$

$$\Rightarrow \quad F_{L\times 1} \le C_{L\times 1}(P_{L\times 1})$$

- Power/Scheduling constraint

$$\forall clique_m, \quad \sum_{l \in m} \frac{p_l}{p_l^{\max}} \le 1 \quad \Rightarrow \quad K_{M\times L} P_{L\times 1} \le 1$$

where power constraint matrix $K_{M\times L} : k_{m\times l} = \begin{cases} 1/p_l^{\max} & if \quad l \in m \\ 0 & otherwise \end{cases}$

Joint Power, Link Scheduling and Rate Control

- Problem formulation
  - Goal: maximize the utility function

$$\max \sum_s U_s(x_s)$$

$$subject\ to: F = RX \le C$$

$$KP \le 1$$

where $U_s(x_s)$ is the utility function of each station *s*, and we assume that U_s is increasing, strictly concave, continuously differentiable and unbounded as $x_s \rightarrow 0$

  - Methodology: convex optimization

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Convex Optimization

- Dual problem
  - Introduce an L-dimensional Lagrange multiplier $\lambda_{L\times 1}$

$$\max \sum_s U_s(x_s)$$
$$subject\ to: F = RX \le C$$
$$KP \le 1$$

$\Rightarrow$

$$\min_{P\ge 0} D(\lambda) \text{ with partial dual function}$$
$$\max_{X\ge 0, C\ge 0} \sum_s U_s(x_s) - \lambda^T(RX - C)$$
$$subject\ to: KP \le 1$$

- Decomposition to subproblems
  - Rate control $D_1(\lambda) = \max_{X\ge 0} \sum_s U_s(x_s) - \lambda^T RX$
  - Power/Scheduling control $D_2(\lambda) = \max_{C\ge 0} \lambda^T C = \max_{c_l \ge 0} \sum_l \lambda_l c_l$

$$subject\ to: KP \le 1$$

Rate Control Subproblem

- Subproblem $D_1(\lambda) = \max_{X \geq 0} \sum_s U_s(x_s) - \lambda^T RX$
- Solution
  - $D_1(\lambda)$ is differential to x_s → gradient method

    Rate control: $x_s(\lambda) = U_s'^{-1}(\sum_l \lambda_l R_{ls})$

  - $D_2(\lambda)$ is non-differential to λ → subgradient method

    Assume: $p(\lambda) \in \arg\max_{p_l \geq 0} \sum_l \lambda_l W \log(1 + \frac{p_l h_l}{I_{-l}})$

    $subject\ to: KP \leq 1$

    hence,

    $$\lambda_l(t+1) = \left[\lambda_l(t) + \alpha_t\left(\sum_s R_{ls} x_s(\lambda(t)) - W \log\left(1 + \frac{p_l(\lambda(t))h_l}{I_{-l}(t)}\right)\right)\right]^+$$

subgradient of $D(\lambda)$

Power Control & Link Scheduling Subproblem

- Subproblem $D_2(\lambda) = \max_{C \geq 0} \lambda^T C = \max_{c_l \geq 0} \sum_l \lambda_l c_l$

  $subject\ to: KP \leq 1$

  - Introduce an M-dimensional Lagrange multiplier $q_{M \times 1}$

    $$\min_{q \geq 0} \max_{p \geq 0} E(p,q) = \sum_l \lambda_l W \log(1 + \frac{p_l h_l}{I_{-l}}) - q^T (KP - 1)$$

  - Apply gradient method to p_l

    Power control: $p_l(t) = \left[\frac{\lambda_l(t) W}{\sum_m q_m K_{ml}} - \frac{I_{-l}(t)}{h_l} \right]^+$

  - Then, apply steepest descent method

    $$q_m(t+1) = \left[q_m(t) + \beta_m (\sum_l K_{ml} p_l(t) - 1) \right]^+$$

Algorithm Description

- At time t =0
  - Both the congestion price $\lambda(0)$ and clique control price $q(0)$ are initialized
- At time t > 0 with a constant step size α
  - Subproblem D_1: Each user calculates its transmission rate $x_s(\lambda)$
  - Subproblem D_2:
    - Search for an appropriate link scheduling set and a power allocation scheme with constant step size β
      - Each link updates its transmission power $p_l(t)$
      - Each link updates its related clique control price $q_m(t)$
      - Repeat until both $p_l(t)$ and $q_m(t)$ converge
    - In each clique, links collect info from each other and then elect one with maximum power to be the active link for next step's scheduling; meanwhile, the rest keep inactive with power reduced to zero
  - Each link updates its congestion price $\lambda_l(t)$

Algorithm Convergence

Proposition:

PSR algorithm can statistically converge to a small range of the optimal solution

Proof:

- Optimal congestion price λ^* and subgradient $g(t)$: $\|g(t)\|_2 \leq G$
- It can be proved that:

$$\|\lambda(t+1) - \lambda^*\|_2^2$$
$$\leq \|\lambda(1)-\lambda^*\|_2^2 - 2\alpha \sum_{\tau=1}^{t} (D(\lambda(\tau)) - D(\lambda^*)) + \alpha^2 \sum_{\tau=1}^{t} \|g(\tau)\|_2^2$$

hence,

$$\limsup_{t\to\infty} \frac{1}{t} \sum_{\tau=1}^{t} (D(\lambda(\tau)) - D(\lambda^*)) \leq \limsup_{t\to\infty} \frac{\|\lambda(1)-\lambda^*\|_2^2 + \alpha^2 t G^2}{2\alpha t} = \frac{\alpha G^2}{2}$$

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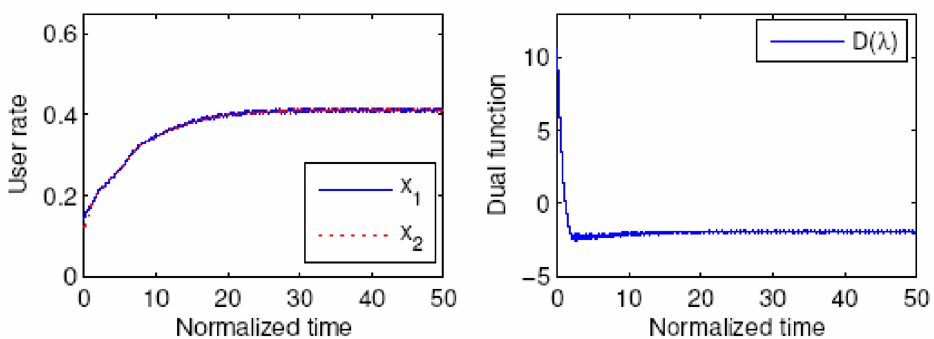
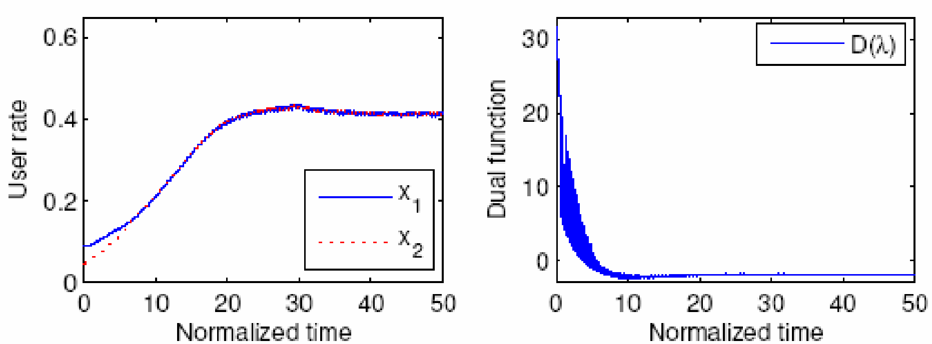
Experiment Setup

- Use the previous example
- Two parameters for initialization
  - Link congestion price $\lambda(0)$
  - Clique control price $q(0)$
- Four groups of experiment
  - Balanced link congestion price versus unbalanced clique control price
  - Balanced link congestion price versus balanced clique control price
  - Unbalanced link congestion price versus balanced clique control price
  - Unbalanced link congestion price versus unbalanced clique control price

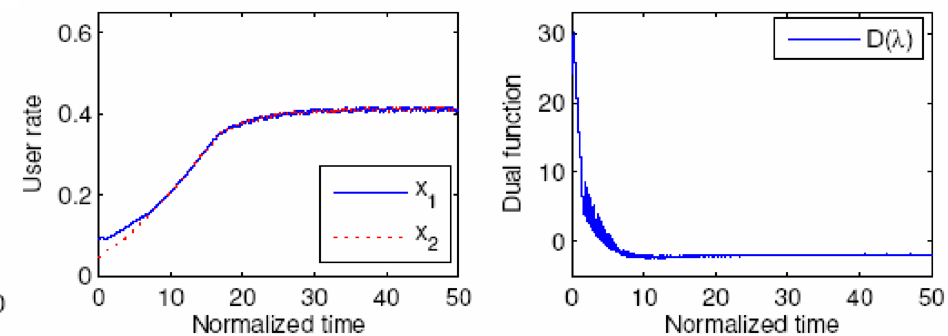
Experimental Results



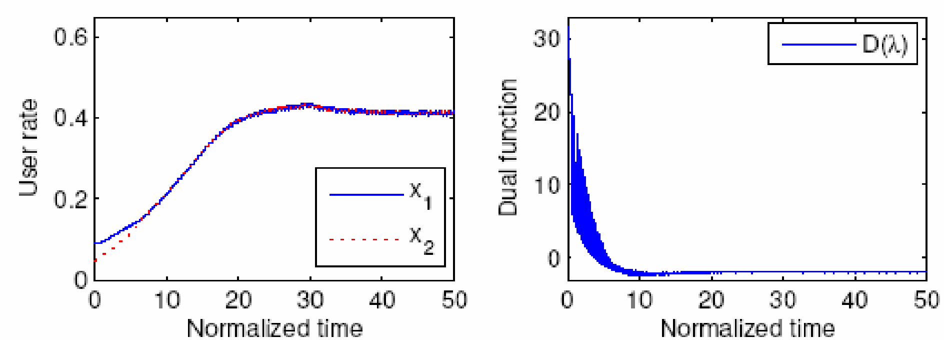
(a) $\lambda(0) = (2,4,2,3,2)^T, q(0) = (7,1)^T$



(b) $\lambda(0) = (2,4,2,3,2)^T, q(0) = (3,3)^T$



(c) $\lambda(0) = (1,9,7,3,5)^T, q(0) = (3,3)^T$



(d) $\lambda(0) = (1,9,7,3,5)^T, q(0) = (7,1)^T$

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- Future Work
  - More efficient way to determine maximal clique
  - Re-route & predict mobility

The End

Thank You

Questions?