THE HONG KONG UNIVERSITY OF SCIENCE & TECHNOLOGY

Department of Computer Science and Engineering

COMP344: Digital Image Processing

Fall 2007 Assignment 3

Due Date and Time: Dec 11, 2007, 11:59pm

NOTE: Your grade will be based on the correctness, efficiency and clarity.

Image Segmentation

Implement the EM algorithm to estimate parameters of a two-component Gaussian mixture model (GMM), and then use this model in gray image segmentation. Figure 1 gives an example, where the image is segmented into two parts based on the GMM.

(a) original image.

(b) segmentation result.

In general, a GMM with c components takes the form

$$f(x|\theta) = \sum_{k=1}^{c} p_k p(x|\theta_k),$$

where x is the pixel intensity, p_k is the mixing coefficient of the kth component, $p(x|\theta_k)$ is the k component which is assumed to be a normal distribution with mean μ_k and variance σ_k^2:

$$p(x|\theta_k) = \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left(-\frac{(x-\mu_k)^2}{2\sigma_k^2}\right).$$

The EM procedure proceeds by iterating the following two steps:

- E-step: compute

$$P_{ik}^{(t)} = \frac{p_k^{(t)} p(x_i|\theta_k^{(t)})}{\sum_{k=1}^{c} p_k^{(t)} p(x_i|\theta_k^{(t)})}.$$

- M-step: compute

$$\begin{aligned} p_k^{(t+1)} &= \frac{1}{n}\sum_{i=1}^{n} P_{ik}^{(t)}, \\ \mu_k^{(t+1)} &= \frac{\sum_{i=1}^{n} x_i P_{ik}^{(t)}}{\sum_{i=1}^{n} P_{ik}^{(t)}}, \\ \sigma_k^{(t+1)} &= \frac{\sum_{i=1}^{n} P_{ik}^{(t)} (x_i - \mu_k^{(t)})^2}{\sum_{i=1}^{n} P_{ik}^{(t)}}. \end{aligned}$$

After obtaining the parameters of the 2-component GMM, i.e., $\{p_k, \mu_k, \sigma_k\}$ for $k = 1, 2$, determine the segmentation threshold Z such that $p_1 p(z|\theta_1) = p_2 p(z|\theta_2)$, i.e.,

$$p_1 \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(Z-\mu_1)^2}{2\sigma_1^2}\right) = p_2 \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left(-\frac{(Z-\mu_2)^2}{2\sigma_2^2}\right). \quad (1)$$

You can determine Z by

1. analytically solve equation (1), or
2. simply obtain an approximate solution by searching in a set of grid points on x.

Details:

1. matlab function format

   Design the matlab function $\mathsf{label} = \mathsf{GMMEM}(\mathsf{I})$ for estimating the parameters of a GMM with 2 components. Here I is the $m \times n$ image matrix (not the name of the image file), and label is the $m \times n$ matrix where each entry is an integer in $(1, 2)$ specifying the cluster label of each pixel.

2. Initialization:

   (a) compute the mean of the pixel intensities;

   (b) use this mean value as a crude value of the segmentation threshold and obtain 2 clusters, each with size n_k;

   (c) use $\frac{n_k}{n}$ for initial p_k's, $k = 1, 2$.

   (d) use the mean/variance of each component as initial estimates of μ_k and σ_k^2.

3. Stopping criterion: For simplicity, iterate 10 times and stop.

Edge Detection

Perform edge detection on a given image by first applying a Laplacian of Gaussian (LoG) filter and then detect the zero-crossings on the filtered image. You need to design an 11×11 mask for the LoG filter, which is determined as

$$f(r) = -\left(\frac{r^2 - \sigma^2}{\sigma^4}\right) \exp\left(-\frac{r^2}{2\sigma^2}\right),$$

where r is the distance between each grid point in the mask from the center. The entries of the 11×11 mask can be determined by multiply the $f(r)$'s with 1000 and round it off to the closest integer. *To guarantee that the mask entries sum up to 0, you can further deduct from each entry the mean value of the mask.* The filtered image is thresholded into a binary image, and then edge pixels are determined.

Details:

1. matlab function format

   Design the matlab function F = LOGED(I, σ). Here I is the $m \times n$ image matrix (not the name of the image file), σ is the parameter in the LoG filter, and F is the $m \times n$ 0-1 matrix where 0 indicates non-edge pixel and 1 for edge pixel.

2. To avoid problem of filtering boundary pixels, simply ignore the pixels lying in the boundary band of width 5 pixels, i.e., the returned image is $(m - 10) \times (n - 10)$.

3. matlab function imfilter() can not be used here.

Put the two matlab functions, GMMGM.m and LOGED.m in a directory titled with your name, and then zip it for submission.