COMP344 Digital Image Processing
Fall 2007
Final Examination

Time allowed: 2 hours

Name	Student ID	Email

Question 1	Question 2	Question 3
Question 4	**Question 5**	**Question 6**

Total ________

With model answer

Course Code : COMP344 Section No.: ALL Time Allowed : 2 Hour(s)

Course Title : Digital Image Processing Total Number of Pages : 12

Time allowed: **2 hours**
Answer all questions.
The total mark for this quiz is 100.
This is a closed-book quiz.

1. a) Compute the Fourier transform ($F(\omega) = \int_D f(t)e^{-j\omega t}dt$ over a suitable domain D) of the step function $f(t) = \begin{cases} -1 & -1 \le t \le 0 \\ 1 & 0 < t \le 1 \end{cases}$ defined on $-1 \le t \le 1$. (5 marks)

1. b) Let f be a Gaussian function $f(x) = e^{-\frac{x^2}{2\sigma^2}}$. It is known that the Fourier transform of f is also a Gaussian function, i.e., $\mathcal{F}(f(x)) = F(\omega) = \sqrt{2\pi}\sigma e^{-\frac{1}{2}\omega^2\sigma^2}$ where ω is the angular frequency. Now consider the 2-dimensional filter with frequency domain representation

$$F(\omega_u, \omega_v) = e^{-\frac{\omega_u^2}{2\sigma_x^2} - \frac{\omega_v^2}{2\sigma_y^2}},$$

where ω_u, ω_v are the angular frequencies. Derive its spatial domain representation $f(x, y)$. (10 marks)

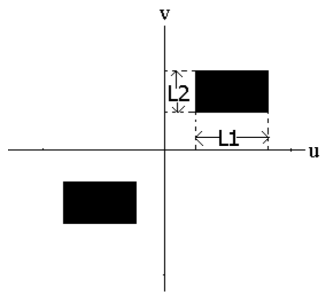
Answer to question 1:
1. a)

$$\begin{aligned}
F(\omega) &= \int_{-1}^{1} f(t)e^{-j\omega t}dt \\
&= \int_{-1}^{0} -e^{-j\omega t}dt + \int_{0}^{1} e^{-j\omega t}dt \\
&= \int_{0}^{1} \left(-e^{-j\omega t} - e^{j\omega t}\right) dt \\
&= -2j\int_{0}^{1} \sin(\omega t)dt \\
&= \frac{2j}{\omega}(\cos(\omega) - 1) = \frac{2 - 2\cos(\omega)}{j\omega}
\end{aligned}$$

1. b) Since $F(u, v)$ can be decomposed into the multiplication of two univariate Gaussian functions, we can consider each dimension separately. Based on the given information,

$$\begin{aligned}
\mathcal{F}^{-1}\left(\sqrt{2\pi}\sigma e^{-\frac{1}{2}\omega^2\sigma^2}\right) &= e^{-\frac{x^2}{2\sigma^2}} && (1) \\
\mathcal{F}^{-1}\left(\frac{\sqrt{2\pi}}{\sigma} e^{-\frac{\omega^2}{2\sigma^2}}\right) &= e^{-\frac{x^2\sigma^2}{2}} \qquad \left(\frac{1}{\sigma} \leftarrow \sigma\right) && (2) \\
\mathcal{F}^{-1}\left(e^{-\frac{\omega^2}{2\sigma^2}}\right) &= \frac{\sigma}{\sqrt{2\pi}} e^{-\frac{x^2\sigma^2}{2}} \qquad \left(\textit{multiply on both sides} \frac{\sigma}{\sqrt{2\pi}}\right) && (3) \\
\mathcal{F}^{-1}\left(e^{-\frac{\omega_u^2}{2\sigma_x^2} - \frac{\omega_v^2}{2\sigma_y^2}}\right) &= \mathcal{F}^{-1}\left(e^{-\frac{\omega_u^2}{2\sigma_x^2}}\right) \cdot \mathcal{F}^{-1}\left(e^{\frac{\omega_v^2}{2\sigma_y^2}}\right) && (4) \\
&= \frac{\sigma_x}{\sqrt{2\pi}} e^{-\frac{x^2}{2}\sigma_x^2} \cdot \frac{\sigma_y}{\sqrt{2\pi}} e^{-\frac{y^2}{2}\sigma_y^2} && (5) \\
&= \frac{\sigma_x\sigma_y}{2\pi} e^{-\frac{x^2\sigma_x^2}{2} - \frac{y^2\sigma_y^2}{2}} && (6)
\end{aligned}$$

2) The following figure shows the frequency domain representation of a notch filter. Here, the dark area has a value of 0 while the white area has value 1. The centers of the two dark rectangular (which are symmetric with respect to the origin) are (u_0, v_0) and $(-u_0, -v_0)$, respectively. The sides of the rectangle are of lengths L_1 and L_2, respectively. Is this a bandpass or bandreject filter? Explain. Write down the $H(u, v)$ of this notch filter. (5 marks)



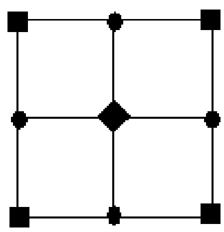
Answer to question 2:
This is a bandreject filter, and

$$H(u,v) = \begin{cases} 0 & u \in [u_0 - \frac{1}{2}L_1, u_0 + \frac{1}{2}L_1], v \in [v_0 - \frac{1}{2}L_1, v_0 + \frac{1}{2}L_2], \\ 0 & u \in [-u_0 - \frac{1}{2}L_1, -u_0 + \frac{1}{2}L_1], v \in [-v_0 - \frac{1}{2}L_1, -v_0 + \frac{1}{2}L_2], \\ 1 & otherwise \end{cases}$$

3) Here, we consider using the Laplacian (second-order derivative) of a Gaussian function to design a filter.

3. a) Compute $h(x,y) = \frac{\partial^2 G(x,y)}{\partial x^2} + \frac{\partial^2 G(x,y)}{\partial y^2}$, where $G(x,y)$ is the Gaussian function $e^{-\frac{1}{2\sigma^2}(x^2+y^2)}$. (5 marks)

3. b) Now, we use the $h(x,y)$ obtained in (a) to design a 3×3 mask (shown in the figure below). Here, the center of the mask is marked with a diamond, its immediate neighbors in the 3×3 mask marked with circles, and the others with squares. For simplicity, you can take the distance (corresponding to the expression $\sqrt{x^2+y^2}$ in (a)) between the diamond and the circle to be 1, and that between the diamond and the square to be $\sqrt{2}$. Assume that $\sigma = \sqrt{0.5}$, obtain the coefficients of the mask. Note that they do not need to be integers. (5 marks)



3. c) For the mask designed in (b), does it correspond to a lowpass or highpass filter? Explain.
(5 marks)

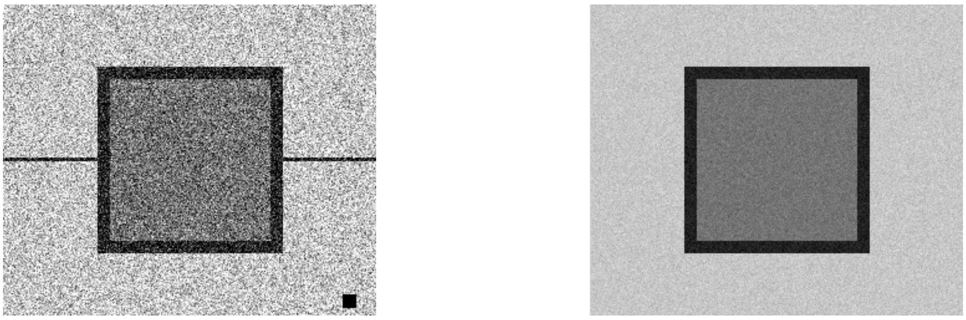
Answer to question 3:
3. a)

$$\nabla^2 g(x,y) \quad = \quad -e^{-\frac{x^2+y^2}{2\sigma^2}} \left(\frac{2\sigma^2 - x^2 - y^2}{\sigma^4} \right) \tag{7}$$

3. b) $4 \cdot \begin{bmatrix} e^{-2} & 0 & e^{-2} \\ 0 & -1 & 0 \\ e^{-2} & 0 & e^{-2} \end{bmatrix}$

3. c) It will be a high pass filter. Because the weights fluctuate between positive and negative values.

4) In the figure below, when the image on the left was filtered using a smoothing filter, the result was the image on the right. The filter used was one of these: 1. averaging filter; 2. ideal lowpass filter; 3. Gaussian lowpass filter; 4. median filter. The small black square on the lower right hand corner of the original image shows the size of the mask that was used. That small square is not part of the image.



(a) original image (b) filtered image

4. a) For each of the four possible filters listed above, give at least one reason why you think it was, or was not, the filter actually used. (10 marks)

4. b) If the size of the mask were tripled, and the same filter you selected in (a) used, how would the appearance of the image after filtering be changed? (10 marks)

Answer to question 4:
4. a) Can't be the averaging filter since the edges are not blurred. Can't be the ideal low pass filter, since most boundaries (high frequency component) are not blurred, and there is no ringing effect either; Can't be the Gaussian filter because the edges are not blurred. Must be the median filter, since it reduces noise, eliminates small objects and does not blur edges.
4. b) If the median filter mask were tripled, the part that will be influenced most would be the darkest square band in the figure: its 4 corners tend to shift towards the image center; other part of its edge will become much thinner or even disappear, depending on the noise level.

5. a) What will we obtain if the arithmetic mean filter is applied to an image again and again? What will happen if we use the median filter instead? Illustrate the difference with a simple example on an one-dimensional "image". (10 marks)

5. b) Write down the pseudo-code of the adaptive median filter. (10 marks)

5. c) The following figure shows an image that has been corrupted by either salt noise or pepper noise. Is it salt noise or pepper noise? Given a choice of (1) arithmetic mean filter; (2) harmonic mean filter; and (3) contraharmonic mean filter, which one is most appropriate for this task. Explain. (10 marks)

Answer to question 5:

5. a) For iterative arithmetic mean filtering, it will ultimately blur the whole image with constant gray level; for iterative median filtering, the image will only change once after the first round of filtering, and will remain stable from that moment on which is invariant to the filter. For example, for the signal $[0, 1, 3, 2, 4, 1, 0]$, where the mask used is of size 3. Then iterative mean filtering will result in $[\frac{4}{3}, \frac{4}{3}, 2, 3, \frac{7}{3}, \frac{5}{3}, \frac{5}{3}]$, $[\frac{10}{9}, \frac{10}{9}, \frac{19}{9}, \frac{22}{9}, \frac{7}{3}, \frac{4}{3}, \frac{10}{9}]$, ..., and ultimately $[\frac{31}{15}, \frac{31}{15}, \frac{31}{15}, \frac{31}{15}, \frac{31}{15}, \frac{31}{15}, \frac{31}{15}]$; while for iterative median filtering, after the first round that produces $[1, 1, 2, 3, 2, 1, 1]$, it will not change any longer.

5. b) Level A:
$A_1 = z_{med} - z_{min}$
$A_2 = z_{med} - z_{max}$
if $A_1 > 0$ and $A_2 < 0$, go to level B
else increase the mask size
if the mask size does not exceed S_{max}, repeat level A
else output z_{med}

Level B:
$B_1 = z_{xy} - z_{min}$
$B_2 = z_{xy} - z_{max}$
if $B_1 > 0$ and $B_2 < 0$, output z_{xy}
else output z_{med}

5. c) The noise is pepper noise, which should be removed by the contraharmonic mean filter with positive degree $Q > 0$. For the mean filter, the pepper noise will become blurred but still exists. For harmonic mean filter, it can remove salt noise well but not pepper noise.

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6. a) Use matlab to plot the curve $f(x) = e^{-x^2/2}$ for $x \in [-3, 3]$. Sampling x with interval length 0.01. (5 marks)

6. b) Design a matlab function G = ShiftedDFT(f). Here f is a discrete one-dimensional signal, ShiftedDFT(f) returns the Discrete Fourier Transform, G, where the zero-frequency component should be shifted to the middle point of the G.
Note: the only matlab built-in function you can use here is fft(). (10 marks)

Answer to question 6:

6. a)
```
x = -3:0.01:3;
f = exp(-x.*x/2);
plot(x, f);
```

6. b)
```
function G = ShiftedDFT(f);
n = length(f);
for i = 1:n;
f(i) = f(i) * (-1)^ i;
end;
G = fft(f);
```