Colorization by Patch-Based Local Low-Rank Matrix Completion

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Outline

1 Motivation
   - colorization
   - global approach

2 Proposed method (PaLLR)
   - grouping similar patches
   - local matrix completion with accelerated ADMM
   - PaLLR algorithm

3 Experimental Results
   - setup
   - results

4 Conclusion

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Colorization aims at recovering color of a gray image when given some few labeled color pixels.

- **(a) gray image** $G$
- **(b) given labels** $O$
- **(c) color image** $L$

- **color image (of size $m \times n$):** $L = [R, G, B] \in \mathbb{R}^{m \times 3n}$.
- **gray image** $G = LT \in \mathbb{R}^{m \times n}$: $T$ averages the channels.
- **labels** $O \in \mathbb{R}^{m \times 3n}$: observed pixel values are nonzero.
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state-of-the-art [Levin et al., 2004].
unlabeled gray pixels are colorized by a weighted average over its $K$ nearest color labels.
the similarity $w_{ij}$ between pixels is defined on:
1. spatial distance
2. difference on gray value
result in a least square minimization problem:

$$G_{ij} = \arg \min \sum_{k}^{K} (G_{ij} - w_{ij}O_k)^2$$
state-of-the-art [Wang et al., 2012].

models colorization as a Robust PCA problem

\[
\begin{align*}
\min_L & \quad \frac{1}{2} \|LT - G\|_F^2 + \lambda \|\Omega \odot (L - O)\|_1 + \mu \|L\|_* \\
\text{consistency with gray values} & \quad \text{sparse labeled errors} & \quad \text{low-rank}
\end{align*}
\]

use Alternating Direction Method of Multipliers (ADMM) [Boyd et al., 2011]

- introduce two extra parameters \((X\) and \(E\))

\[
\begin{align*}
\min_{L,X,E} & \quad \frac{1}{2} \|LT - G\|_F^2 + \lambda \|\Omega \odot E\|_1 + \mu \|X\|_* \\
\text{s.t.} & \quad O = L + E, \quad L = X
\end{align*}
\]

- convergence rate \(O(1/t)\): \(t\) is number of iterations
- low-rank assumption may not hold on natural images

- images need well-aligned repeating patterns
- but group of similar images are approximately low-rank
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split the image into overlapping patches
for patch \( P_{i,j} \) located at \((i, j)\), its distance to \( P_{i',j'} \) is

\[
d(P_{i,j}, P_{i',j'}) = \sqrt{\frac{1}{m^2} (i - i')^2 + \frac{1}{n^2} (j - j')^2} + \beta \left( \frac{1}{m^2} (i - i')^2 + \frac{1}{n^2} (j - j')^2 \right)
\]

- grouping is done by finding K-nearest neighbors.
- $P_{i,j}$ and its nearest neighbors: locally low-rank
- Single patch is not low rank.

(a) group of patches.  
(b) one patch.
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in each patch group, solve

\[
\min_{\tilde{L}} \frac{1}{2} \| \tilde{L} \tilde{T} - \tilde{G} \|^2_F + \frac{\lambda}{2} \| \tilde{\Omega} \odot (\tilde{L} - \tilde{O}) \|^2_F + \mu \| \tilde{L} \|_*
\]

consistency with gray values Gaussian noise local low-rank

change from $\ell_1$ to $\ell_2$ does not harm performance, but

1. leads to fewer optimization parameters
2. allows use of accelerated ADMM [Goldstein et al., 2012]

\[
\min_{\tilde{L}, X} \frac{1}{2} \| \tilde{L} \tilde{T} - \tilde{G} \|^2_F + \frac{\lambda}{2} \| \tilde{\Omega} \odot (\tilde{L} - \tilde{O}) \|^2_F + \mu \| X \|_* \quad \text{s.t.} \quad X = \tilde{L}
\]

only one new parameter $X$ is introduced
Accelerated ADMM, with a faster convergence rate $O(1/t^2)$

- augmented Lagrangian
- minimize w.r.t $X$
  \[
  X_t = \arg \min_X \frac{1}{2} \left\| X - \left( \tilde{L}_t + \frac{1}{\rho} \hat{Q}_t \right) \right\|^2_F + \frac{\mu}{\rho} |X|_* \\
  \]
- singular value thresholding (SVT) [Cai, et al. 2010]
- minimize w.r.t $L$
  \[
  \tilde{L}_t = \arg \min_{\tilde{L}} \frac{1}{2} \left\| \tilde{L} \tilde{T} - \tilde{G} \right\|^2_F + \frac{\lambda}{2} \left\| \tilde{\Omega} \odot (\tilde{L} - \tilde{O}) \right\|^2_F \\
  + \text{tr}(\hat{Q}_t^T (\tilde{L} - \hat{X}_t)) + \frac{\rho}{2} \left\| \tilde{L} - \hat{X}_t \right\|^2_F \\
  \]
- smooth problem
  \[
  R \text{vec}(\tilde{L}_t) = \text{vec}(C), \\
  C = \tilde{G} \tilde{T}^T + \lambda \left( \tilde{\Omega} \odot \tilde{O} \right) + \rho \hat{X}_t - \hat{Q}_t, \\
  R = (\tilde{T} \tilde{T}^T) \otimes I + \lambda \text{Diag} \left( \text{vec} \left( \tilde{\Omega} \right) \right) + \rho I \\
  \]
- can be solved by conjugate gradient descent
- we propose a faster solver based on divided-and-conquer
After colorization of groups, if a patch is covered by $K$ groups, it is combined by a weighted average:

$$P_{i,j} = \sum_{k}^{K} w_k P^k_{i,j}$$

where $P^k_{i,j}$ is corresponding colorized result in the group.

The final color image is obtained by rearranging overlapped patches back into a image.
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1: input: monochrome image; a small set of color pixels.
2: while there exists a patch $P$ not yet colored do
3: find $k - 1$ patches that are most similar to $P$;
4: obtain colorization for the group of $k$ patches by solving the optimization problem with accelerated ADMM;
5: end while
6: for each patch $P$ do
7: perform (weighted) average on the colorization results from all groups containing $P$;
8: end for
9: for each pixel in the image do
10: average the values from overlapping patches.
11: end for
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for each image, randomly sample a small proportion \( \{1\%, \ldots, 10\%\} \) of color pixels as labels

input: these labels and gray image

compare the proposed PaLLR with

1. local color consistency (LCC) [Levin et al. 2004];
2. global low-rank based (GLR) method in [Wang et al. 2012];
3. single patch based, local low-rank matrix approximation (LLORMA) [Lee et al. 2013].
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(a) castle. (b) koala. (c) mushroom. (d) woman.

(e) couple. (f) lake.

(g) landscape. (h) street.

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PSNR

(a) castle.
(b) couple.
(c) koala.
(d) lake.
(e) landscape.
(f) mushroom.
(g) street.
(h) woman.

Blue: PaLLR-ℓ₂(proposed), Red: PaLLR-ℓ₁, Black: GLR, Cyan: LCC, Magenta: LLORMA.

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Difference with ground truth.

For GLR, artifacts can be seen. Moreover, while the errors produced by GLR and PaLLR are localized, those by LCC are more diffused.
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Insensitiveness on parameters (castle).

(a) patch size.

(b) group size.

(c) $\mu$.

(d) $\lambda$.
low-rank assumption on a group of similar patches is more reasonable on natural images.

- optimization: accelerated ADMM can be used, and the subproblem can be efficiently solved by divide-and-conquer.
- experimental results demonstrate superiority with existing approaches.

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Thanks