On the Feasibility of Gradient-based Routing Mechanisms Using Bloom Filters

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***Abstract*—Gradient-based routing using Bloom filters is an effective mechanism to enable data-driven queries in multi-hop networks. A node compressively describes its data items as a Bloom filter, which is then diffused away to the other nodes with information decay. The Bloom filters form an information potential that eventually navigates queries to the source node by ascending the potential field. The existing designs of Bloom filters, however, have critical limitations with respect to the feasibility of gradient-based routing. The compressed routing entries appear to be noisy. Noise in unrelated routing entries is very likely to equal to even outweigh information in right routing entries, thus blinding a query to its desired destination. This work addresses the root cause of the mismatch between the idea and the practical performance of gradient-based routing using Bloom filters. We first investigate the impact of decaying model on the effectiveness of routing entries, and then evaluate the negative impact of noise on routing decisions. Based on such analytical results, we derive the necessary and sufficient condition of feasible gradient-based routing using Bloom filters. Accordingly, we propose a receiver-oriented design of Bloom filters, called *Wader*, which satisfies the above condition. The evaluation results demonstrate that *Wader* guarantees the correctness and efficiency of gradient-based routing with high probability.**

I. Introduction

Bloom filter (bf) is often deemed as a suitable tool to aid information discovery and navigation in multi-hop networks with intensive load of query processing. Many recent proposals employ Bloom filters [1], [2] to realize data-driven routing in overlay networks [3], [4], [5], [6], wireless sensor networks [7], ad hoc networks [8], [9], [10], [11], [12], and mesh networks [13]. The common idea among them is that each node uses a Bloom filter to describe the membership information of its data items, i.e. whether an item is stored at the node or not. Every node then broadcasts its Bloom filter to nodes within its *propagation range*, e.g. h hops. Each link, associated with all the received Bloom filters through it, is maintained as a routing entry. If a node needs to route a query to a destination residing within h hops away, it forwards the query over the link, which has at least one associated Bloom filter to satisfy the query.

With respect to the probability of successful routing, Bloom filter based routing outperforms those stateless routing schemes, such as flooding [14] and random walk [15], however, at the costs of large memory space to store Bloom filters and long delay to scan through the Bloom filters for a routing decision. These two problems become severe when the average node degree gets higher and the *propagation range* of Bloom filters increases. For example, in resource-constrained contexts such as wireless sensor networks [16], many Bloom filter based mechanisms suffer poor efficiency and scalability. The delay of routing decisions is relative long, which cannot meet the requirements of some delay-constrained networks [17].

Kumar et al. improve the previous mechanisms by proposing a gradient-based routing mechanism using Bloom filters [18]. The basic idea is to exponentially decay the information in each Bloom filter while propagating it within the given range. Meanwhile, in a routing entry, a link is associated with the union of all received Bloom filters through it. Note that each routing entry does not contain the complete membership information of any item. Hence, a query is sent via the link whose associated routing entry has the maximum amount of information of the queried item. Such a mechanism significantly saves storage space and shortens the delay of answering a query. X. Li et al. propose another similar scheme [19]. A Bloom filter is propagated without any loss within the first h_0-hops from the source, while decays exponentially or linearly outside the h_0-hops from the source.

A. Motivation and Contributions

For a gradient-based routing[1] mechanism, each node as a source creates an information gradient in a potential field. Hints about all data on a source are stored in routing entries of some nodes, and can be utilized to guide queries to the source. Thus the information gradients enable efficient decision of routing by ascending the potential field. Ideally, any query will be forwarded to desired destination once it enters the propagation region of the source. In practice, however, given a query for a data item at a remote source node, we find that *noise*[2] in unrelated routing entries is very likely to outweigh information of the queried item in right routing entries. As a result, gradient-based routing is blinded to the right routing decisions and has to forward the queries in a flooding-like manner. Sometimes it even fails to route queries to the desired destinations, due to prohibitively high traffic cost.

In this paper, we address the root cause of the mismatch between the ideal and the practical performance of gradient-based routing using Bloom filters, and explore approaches to guarantee the routing feasibility and efficiency. This basically involves the following ***two criterion***. First, once a query enters the potential field of a desired destination, the amount of information in right routing entries on the intermediate nodes

[1]For ease of presentation, we use the term gradient-based routing to represent "gradient-based routing using Bloom filters" in the rest of this paper.

[2]If a node V does not receive a decaying Bloom filter from the source node of x through link L, L is called a link on V that is unrelated to x. Then *noise* on L is defined as the amount of membership information in the corresponding routing entry of L, namely the number 1's among the k bits of Bfaddress(x) in the Bloom filter. Please refer to Section II for the definition of Bfaddress(x).

should keep increasing as the query are forwarded towards the destination. This criterion ensures that each node holds an information gradient in a certain potential field. Second, it should be guaranteed with high probability that noise in unrelated routing entries does not exceed the information strength of the queried item in right routing entries. That is, each node should appropriately suppress the strength of noise at its out-going links so that it can clearly distinguish right out-going links from other interfering ones. In this way, a query can be navigated by ascending the potential field along a single path. Bearing these points in mind, we propose the design of receiver-oriented decaying Bloom filters for gradient-based routing. Our contributions are summarized as follows.

1) To the best of our knowledge, we are the first to disclose the fact that the existing gradient-based routing mechanisms deteriorate to a flooding-like mechanism and even fail to route queries to the desired destinations. We then derive two criterion to ensure the feasibility of gradient-based routing.
2) With regard to gradient-based routing under a general decaying model, we analyze the strength of useful information in right routing entry and that of noise in unrelated routing entries. The results show that the existing gradient-based routing mechanisms satisfy the first design criterion under an appropriate constraint, while have critical limitations to meet the second criterion.
3) Based on such analytical results, we examine the negative impact of noise on one-hop routing decisions. We accordingly derive the necessary and sufficient condition of the second design criterion, which guarantees the feasibility of gradient-based multi-hop routing. Thus we propose a novel design of Bloom filters for the existing gradient-based routing mechanisms, called *Wader*. Our simulations demonstrate that *Wader* guarantees the routing feasibility with high probability.

B. Related Work

Information-guided routing has widely studied as a scalable approach for settings with high query frequency in ad hoc and wireless sensor networks [20], [21], [22], [23], [24], [25]. Most of these gradient-based approaches use the natural gradients of physical phenomena due to the fact that the spatial distribution of many physical quantities follows a natural diffusion law [20], [21], [22], [23]. However, these approaches become a random walk since the gradients imposed by natural laws exist local extreme or large plateau regions. In [26], the sensor readings are categorized into a set of high-level events. For any event detected by a random sensor, a potential field with harmonic functions is built such that greedy routing with the potential is guaranteed to reach the source. A similar approach builds a potential field with harmonic functions to guide the movement of mobile nodes, so as to achieve sweep coverage in wireless sensor networks [27]. The approaches in [26], [27], however, are not suitable for data-centric routing where each node holds large number of raw data not only several high-level events. In contrast, gradient-based routing using Bloom filter focuses on this type of network applications.

Another related research work revolves around the weak state routing using decaying Bloom filter, which has been first coined in [10], [12], where it has been used to perform routing in large scale and dynamic mobile ad-hoc networks. The weak state routing and gradient-based routing have the common idea, however, are different in the implementation methodologies and application scenarios.

C. Organization of Paper

The rest of this paper is organized as follows. In Section II, we briefly introduce Bloom filters, and analyze the effect of decaying models in gradient-based routing. Moreover, we examine the impact of noise on routing decisions. In Section III, we derive the necessary and sufficient condition which ensures a feasible gradient-based routing mechanism using Bloom filters, and then propose the design of *Wader*. Section IV presents the performance evaluation results. We conclude this work in Section V.

II. Quantitative Analysis of Gradient-based Routing Using Bloom Filters

We first analyze gradient-based routing under a general decaying model of Bloom filters. We then measure the strength of useful information in right routing entries and that of noise in those unrelated routing entries. By mathematical analysis and simulations, we find that the existing gradient-based routing mechanisms can satisfy the first criterion under a reasonable constraint, while have critical limitations to meet the second criterion. As a result, the information potential formed by each Bloom filter is very smooth, and hence, fails to navigate queries by ascending the potential field. Table I lists the symbols and notations used in the rest of this paper.

A. Preliminaries of Bloom filters

A set X of n items is represented by a Bloom filter using a vector of m bits which are initially set to 0. A Bloom filter uses k independent hash functions $R_1, R_2, \cdots, R_k$ with a range $\{1, ..., m\}$. When inserting an item x to X, all bits of $Bfaddress(x)$ (consisted of $R_i(x)$ for $1 \leq i \leq k$) will be set to 1. To answer a membership query for any item x, users check whether all bits $R_i(x)$ are set to 1. If not, x is not a member of X. If yes, we assume that x is a member of X, although we might be wrong in some cases. Hence, a Bloom filter may yield a *false positive* which suggests that the item x is in X even though it is not. A false positive is due to hash collisions, in which all bits of $Bfaddress(x)$ were set to 1 by other items in X [1].

Let p_0 be the probability that a random bit of a Bloom filter is 0, and let n be the number of items that have been added to the Bloom filter, then $p_0 = (1 - 1/m)^{n \times k} \approx e^{-n \times k/m}$, as $n \times k$ bits are randomly selected, with probability $1/m$ in the process of adding each item. Now we test membership of an element $x_1 \notin X$. Each of k bits of $Bfaddress(x_1)$ is 1 with a probability as above. The probability of all of k bits being 1, which would cause a false positive, is then

$$f = (1 - p_0)^k \approx (1 - e^{-k \times n/m})^k.$$

The minimum value of f is achieved when $k = \lfloor (m/n) \ln 2 \rfloor$.

TABLE I
SYMBOLS AND NOTATIONS

Term	Definition
X	a set represented by a bf
m	number of counters of a bf
n	cardinality of a set
k	number of hash functions used by a bf
$Bfaddress(x)$	bits of $h_i(x)$ for $1 \leq i \leq k$
f	false positive probability of a bf
d	decay factor
h	decay range (transmission range) of a bf
c	average node degree in an irregular network or the node degree in a regular network
bf_i	a bf resulted from the ith round decay of a bf
$\theta(bf_i)$	number of bits set to 1 in bf_i
$\theta(x, bf_i)$	amount of information in bf_i for $x \in X$
$\|link_j\|$	number of bfs a node receives through $link_j$
$bf(link_j)$	union of $\|link_j\|$ decay bfs through $link_j$
$bf_i(link_j)$	$bf(link_j)$ at a node receiving bf_i through $link_j$
$\theta(x, bf_i(link_j))$	amount of information in $bf_i(link_j)$ for $x \in X$
$E^z_{=i}$	a bit is set to i after throwing z balls into a bf
r_0	fraction of bits set to zero in a bf
r_1	fraction of bits set to one in a bf
Y	noise strength about $x \in X$ at unrelated links
σ	minimum probability of a valid multi-hop routing

B. Network model and decaying models of Bloom filters

For ease of presentation, we use a regular graph to model a multi-hop network. Let c denote the number of neighbors in a regular network or the average node degree in an irregular network. Although the analytical results presented in this paper assume the network is regular, the basic ideas and methodologies also apply to other irregular networks after minimal modifications, as discussed in Section III-D. As mentioned in Section I, a local bf at a random node should be diffused to a few nodes in order to establish an information potential that eventually navigates queries to the source node. The number of bits set to 1 in the bf decreases with the distance from the source. We enforce each local bf to travel isolated from origin to nodes within its propagation region and is not merged along the way with other filters. This effort is the precondition to conquer the duplicate decayed versions of bf, which reduces the accuracy of the gradient-based routing. Thus, if a random node receives many decayed versions of bf via each neighbor, it should only keep one of the filters, which travel the least number of nodes.

An accurate decaying model of Bloom filters is the dominating factor which affects the correctness and efficiency of the gradient-based routing mechanisms. In this paper, we concentrate our study on the scenario that all nodes employ a homogeneous decaying model. The scenario with heterogeneous decaying model is out of the scope of this paper.

Definition 1: Given a set X with n items and its Bloom filter bf, $\theta(x, bf)$ denotes the amount of information in bf for $\forall x \in X$, that is the number of bits being 1 in $Bfaddress(x)$. Let $\theta(bf)$ denote the expectation of the number of bits set to 1 in the bf, and equals to m multiply the probability p_1 that a random bit in the bf is set to 1. The p_1 is $1-(1-1/m)^{k\times n}$, and hence

$$\begin{aligned} \theta(bf) &= m \times (1-(1-1/m)^{k\times n}) \\ &\approx m \times (1-e^{-k\times n/m}) \end{aligned} \tag{1}$$

The value of $\theta(x, bf)$ equals to k for $\forall x \in X$. There are two models to reduce $\theta(x, bf)$ by decaying the bf. In *exponential* model, if a bit in $Bfaddress(x)$ is 1, it remains 1 at a constant probability $1/d$ during each round of decay. In *linear* model, number of d random bits which are 1 in $Bfaddress(x)$ become 0 during each decay. An approximate method to implement the linear decaying model is that number of $\lceil \frac{\theta(bf)d}{k} \rceil$ bits which are 1 in the bf are set to 0 during each round of decay. Note that d is a *decay factor* in both models and is a positive real number.

Definition 2: Let $Decay(bf, h, h_0, h_1, model, d)$ denote a general decaying model of a bf which is propagated to nodes within h hops from the source where $1 \leq h \leq h_0+h_1$. The bf does not decay within the first h_0-hops, while decays outside the h_0-hops by using the aforementioned decaying models, where h_1 is an upper bound on the hops in the second stage [19].

Definition 3: Let bf_i denote a new Bloom filter resulted from the ith round decay of a bf where $1 \leq i \leq h$. bf_i remains $\theta(bf_i)$ bits set to 1. If the *model* is *exponential*, then

$$\theta(bf_i)=\begin{cases} \theta(bf), & i \leq h_0 \\ \lceil \frac{\theta(bf_{i-1})}{d} \rceil, & h_0 < i \leq h_0+h_1 \end{cases} \tag{2}$$

If the *model* is *linear*, then

$$\theta(bf_i)=\begin{cases} \theta(bf), & i \leq h_0 \\ \theta(bf_{i-1})\text{-}\lceil \frac{\theta(bf)d}{k} \rceil, & h_0 < i \leq h_0+h_1 \\ 0, & \theta(bf_{i-1}) < \frac{\theta(bf)d}{k} \end{cases} \tag{3}$$

According to Definition 2, a Bloom filter a node produces can be received by $T_i{=}c(c{-}1)^{i-1}$ nodes in the i round, and a node should also receive T_i Bloom filters in their i round due to the symmetry. Thus, each node A can receive $(c{-}1)^{i-1}$ decaying Bloom filters in their i round via any link $link_j$. The received Bloom filters by node A are recorded as bf^l_i where $1 \leq i \leq h$ and $1 \leq l \leq (c{-}1)^{i-1}$. Thus, the number of decaying Bloom filters a node can receive from the whole system through $link_j$ is denoted as $|link_j|$, and

$$|link_j| = \sum_{i=1}^{h} (c{-}1)^{i-1}.$$

As mentioned in [28], the union of homogeneous Bloom filters can be realized by a logical *or* operation between their bit vectors. Thus, the union of $|link_j|$ decaying Bloom filters results in a joint Bloom filter $bf(link_j)$ for a link $link_j$ of node A. The $bf(link_j)$ acts as a probabilistic summary of all items which are reachable from node A along a routing path of at most h hops, and is given by

$$bf(link_j)) = \bigcup_{i=1}^{h} \bigcup_{l=1}^{(c-1)^{i-1}} bf^l_i. \tag{4}$$

Lemma 1: The number of bits set to 1 in any $bf(link_j)$ of each node is given by

$$\theta(bf(link_j)) = m(1-(1-1/m)^{\beta(link_j)}), \tag{5}$$

where

$$\beta(link_j) = \sum_{i=1}^{h} \sum_{l=1}^{(c-1)^{i-1}} \theta(bf^l_i). \tag{6}$$

Proof: Recall that $|link_j|$ decaying Bloom filters received by a node through $link_j$ will be merged to construct

$bf_i(link_j)$. During the union process, $\beta(link_j)$ balls are dropped into m bits of $bf(link_j)$ randomly, i.e., the location of each ball is independently and uniformly chosen from m possibilities. $\beta(link_j)$ denotes the total number of bits being 1 in those $|link_j|$ decaying Bloom filters. Let p_0 denote the probability that a random bit in $bf(link_j)$ is 0 after dropping all $\beta(link_j)$ balls. Clearly, $p_0{=}(1{-}1/m)^{\beta(link_j)}$. Let p_1 denote the probability that a random bit in $bf(link_j)$ is set to 1. Thus, $p_1{=}1-p_0$. Therefore the number of bits set to 1 in $bf(link_j)$ is given by $\theta(bf(link_j)){=}m(1{-}(1{-}1/m)^{\beta(link_j)})$. Thus proved. ∎

C. Quantification of membership information in right routing entries

Before examining whether the first criterion can be satisfied, we measure the strength of membership information after propagating each Bloom filter within the given range.

In general, $\theta(x,bf){=}k$ where an element x is represented by a bf. A general decaying model is possibly an exponential or linear decaying model. For these two types of models, we respectively measure the metric $\theta(x,bf_i)$, which denotes the amount of membership information of x in a decaying Bloom filter bf_i. Formula (7) and Lemma 2 examine that metric for the linear and exponential decaying models, respectively.

For the linear decaying model, we can derive the following result based on its definition.

$$\theta(x,bf_i)=\begin{cases}\theta(x,bf)=k, & i\le h_0\\ \theta(x,bf)-d(i-h_0), & h_0<i\le h_0+h_1\end{cases} \tag{7}$$

For the exponential decaying model, we can draw the following conclusion based on its definition.

Lemma 2: If $i \le h_0$, $\theta(x,bf_i)$ equals to k because $bf_i{=}bf$. Otherwise, $\theta(x,bf_i)$ is a discrete random variable, denoted as U_i. Its possible values are integers ranging from 0 to k. The probability mass function of U_i is

$$P(U_i=a)=\frac{\binom{k}{k-a}\binom{\theta(bf)-k}{\theta(bf)-\theta(bf_i)-k+a}}{\binom{\theta(bf)}{\theta(bf)-\theta(bf_i)}}, \tag{8}$$

where $\theta(bf_i)$ is given by Formula (2).

Proof: Assume a represents the possible value of U_i, and is an integer ranging from 0 to k. Let $U_i{=}a$ mean that the amount of bits being 1 in the $Bfaddress(x)$ is a. After i rounds of decay of bf, the number of $\theta(bf){-}\theta(bf_i)$ bits being 1 in bf are reset to 0 in bf_i. The number of possibilities that outcome bf_i is $\binom{\theta(bf)}{\theta(bf)-\theta(bf_i)}$. The number of possibilities that just $k{-}a$ bits in $Bfaddress(x)$ are reset to 0 during the i rounds of decay is $\binom{k}{k-a}\binom{\theta(bf)-k}{\theta(bf)-\theta(bf_i)-k+a}$. Then the probability that $\theta(x,bf_i)=a$ is given by Formula (8). Therefore, Lemma 2 holds. ∎

Corollary 1: If $h_0<i\le h_0+h_1$, the expectation of U_i can be calculated by

$$E[U_i]=\sum\nolimits_{a=0}^{k} a\times P(U_i=a)=k/d^{i-h_0}.$$

If the decay range i exceeds h_0, $\theta(x,bf_i)$ under the linear decaying model and the expectation of $\theta(x,bf_i)$ under the exponential decaying model decrease with the increasing i. Fig.1 plots an illustrative example of the propagation of a bf from node A. The color of propagation field becomes light from deep as the decay range increases. This result indicates that the number of membership information of $x\in X$ in bf reduces during the decay transmission of bf.

Practically, a node receiving bf_i through a link $link_j$ also collects other $|link_j|{-}1$ decaying Bloom filters through the same link. As shown in Fig.1, node E receives a decaying Bloom filer from nodes A, B, and C through the same link $C{\to}E$. Thus, the metric $\theta(x,bf_i)$ fails to support a gradient-based routing mechanism since each node uses the union of all received Bloom filters through a link as a correlated routing entry. To address this issue, we propose a metric $\theta(x,bf_i(link_j))$ which denotes the amount of information of x in a routing entry $bf_i(link_j)$ at the node receiving bf_i through $link_j$ where $1\le j\le c$.

Before measuring the metric in Lemmas 3 and 4, we first define two events used frequently in the rest of this paper. Given any bit in an empty Bloom filter, an event $E^z_{=i}$ means that the bit is set to i after throwing z balls into the Bloom filter. The probability of $E^z_{=0}$ can be calculated by

$$P(E^z_{=0})=(1-1/m)^z.$$

The probability of $E^z_{=1}$ is given by

$$P(E^z_{=1})=1-P(E^z_{=0}).$$

Lemma 3: In the context of exponential decaying model, the metric $\theta(x,bf_i(link_j))$ is a discrete random variable, denoted as V_i. Its possible values are integers ranging from 0 to k. The probability mass function of V_i is

$$Pr(V_i{=}v)=\sum\nolimits_{a=0}^{v} Pr(U_i{=}a)\cdot Pr(W_i{=}v-a|U_i{=}a). \tag{9}$$

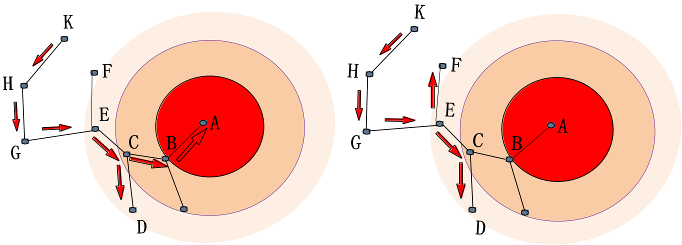
Proof: In bf_i, let us consider an event $\theta(x,bf_i){=}a$ that a bits in $Bfaddress(x)$ are set to 1 while other $k{-}a$ bits are set to 0, where $0\le a\le k$. The probability of this event is given by Formula (8). To achieve $bf_i(link_j)$, other $|link_j|-1$ decaying Bloom filters merge with bf_i based on the union operation of Bloom filters. In other words, the number of $\alpha(link_j)$ balls are thrown into bf_i randomly, where $\alpha(link_j){=}\beta(link_j)-\theta(bf_i)$. Let us consider another event that $\theta(x,bf_i){=}a$ and there exists b bits in $Bfaddress$ which are 0 in bf_i but are hit after throwing $\alpha(link_j)$ balls into bf_i, where $0\le b\le k-a$. The probability of this event is denoted as $P(W_i{=}b|U_i{=}a)$, and is

$$\binom{k-a}{b}P\big(E^{\alpha(link_j)}_{=1}\big)^b\cdot P\big(E^{\alpha(link_j)}_{=0}\big)^{k-a-b}.$$

Assume v represents the possible value of V_i, and is an integer ranging from 0 to k. An event $V_i=v$ means that the amount of bits set to 1 in $Bfaddress(x)$ of $bf_i(link_j)$ is v. The probability of this event is given by Formula (9). Thus proved. ∎

Lemma 4: In the context of linear decaying model, the metric $\theta(x,bf_i(link_j))$ is a discrete random variable, denoted as V_i. Its possible values are integers ranging from 0 to k. The probability mass function of V_i is

$$\binom{k-\theta(x,bf_i)}{v-\theta(x,bf_i)}\cdot P\big(E^{\alpha(link_j)}_{=1}\big)^{v-\theta(x,bf_i)}\cdot P\big(E^{\alpha(link_j)}_{=0}\big)^{k-v}. \tag{10}$$



(a) Valid gradient-based routing. (b) Invalid gradient-based routing.

Fig. 1. Illustrative examples of gradient-based routing.

Proof: In bf_i, $\theta(x, bf_i)$ bits in $Bfaddress(x)$ remain 1 and other $k - \theta(x, bf_i)$ bits are reset to 0. The value of $\theta(x, bf_i)$ is given by Formula (7). According to the definition of $bf_i(link_j)$, number of $\alpha(link_j)$ balls will be thrown into the m bits of bf_i randomly during the construction process of $bf_i(link_j)$, where $\alpha(link_j) = \beta(link_j) - \theta(bf_i)$. The number of bits in $Bfaddress(x)$ which are set to 0 in bf_i but are hit at least once after throwing $\alpha(link_j)$ balls into bf_i is a discrete random variable, denoted as Q_i. Its possible values range from 0 to $k - \theta(x, bf_i)$. The probability that $Q_i = a$ is

$$\binom{k - \theta(x, bf_i)}{a} \cdot P(E_{=1}^{\alpha(link_j)})^a \cdot P(E_{=0}^{\alpha(link_j)})^{k - \theta(x, bf_i) - a}.$$

The bits being 1 in $Bfaddress(x)$ of $bf_i(link_j)$ includes those bits being 1 in bf_i and other bits set to 1 after throwing $\alpha(link_j)$ balls into bf_i. Therefore, the number of this kind of bits is a discrete random variable, denoted as V_i. Its possible values range from $\theta(x, bf_i)$ to k. The probability that $V_i = v$ is equivalent to the probability that $Q_i = v - \theta(x, bf_i)$, and is given by Formula (10). Thus proved. ■

D. Quantification of noise on unrelated routing entries

Before examining whether the second criterion can be satisfied, we measure the strength of noise in unrelated routing entries at any node for an arbitrary query.

For the gradient-based routing mechanism, a node receiving a query for an item x selects $link_j$ so that $bf_i(link_j)$ contains the largest amount of membership information of x among all the filters. In other words, the node receiving bf_i through $link_j$ will send the query over $link_j$ if x belongs to a set represented by bf_i. Meanwhile, the *noise* at other links do not affect the decision of routing and thus can be neglected. It is *the second criterion* mentioned in Section I. Before examining whether the second criterion can be satisfied, we should first measure the strength of noise at any unrelated links for any queries.

Given an item x represented by a bf and a node A receiving bf_i through its link $link_j$, let $\theta(x, bf_i(link_j^{'}))$ denote the amount of information of x in a routing entry $bf_i(link_j^{'})$ at another link $link_j^{'}$. Given any Bloom filter, we use r_0 and r_1 to denote the fraction of bits set to zero and one in it, and use them as the probability that any one bit is set to zero and one, respectively. If node A did not receive a decayed version of bf through the link $link_j^{'}$, $\theta(x, bf_i(link_j^{'}))$ denotes the strength of noise on the information of x at that link, and is a discrete random variable, denoted as Y. Its possible value, denoted as u, is an integer ranging from 0 to k. The probability mass function of Y is defined as

$$P(Y = u) = \binom{k}{u} r_1^u r_0^{k-u}. \tag{11}$$

Corollary 2: The expectation of Y can be calculated by

$$E[Y] = \sum_{u=0}^{k} u \times P(Y = u).$$

E. Examinations of the two criterion

In this section, we show that the existing gradient-based routing mechanisms satisfy the first criterion under a reasonable constraint, while have critical limitations to meet the second criterion. Those gradient-based routing mechanisms use the traditional designs of Bloom filters, as discussed in Section III-C.

According to *the first criterion* mentioned in Section I, a feasible mechanism of gradient-based routing should ensure that the value of $\theta(x, bf_i(link_j))$ increases together with $\theta(x, bf_i)$ when i decreases. As shown in Fig.1(a), the value of $\theta(x, bf_i(link_j))$ should increase along a path $E \rightarrow C \rightarrow B \rightarrow A$. Such a criterion essentially determines the feasibility of the gradient-based routing mechanism using Bloom filters. The metric is a function of i and $\alpha(link_j)$, but not a monotonic decreasing function of i because $\alpha(link_j)$ is a discrete random variable with uncertain distribution. Under Lemmas 3, 4, and a reasonable constraint on $\beta(link_j)$, we may derive Theorem 1 to show that the *first criterion* of gradient-based routing can be satisfied.

Theorem 1: Given an item x represented by a bf, consider two nodes receiving bf_i and bf_{i+1} through $link_j$ and $link_j^{'}$, respectively. The expectation of $\theta(x, bf_i(link_j))$ decreases as the value of i increases if $\beta(link_j) \approx \beta(link_j^{'})$ and $h_0 \leq i < h_0 + h_1$, irrespective the exponential or linear decaying model.

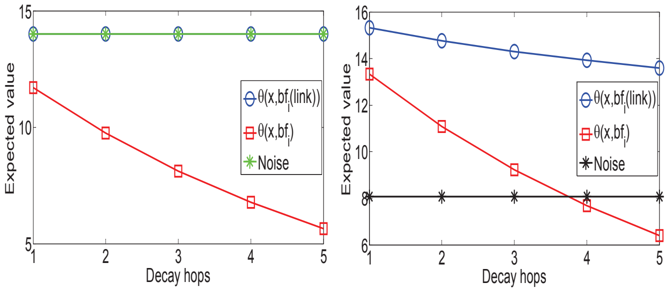
Proof: After i rounds of decay, bf becomes a new version bf_i, the number of membership information of x becomes $\theta(x, bf_i)$ from $\theta(x, bf_i)$, and $\theta(x, bf) - \theta(x, bf_i)$ bits being 1 in bf are reset to 0 in bf_i. For the linear decaying model, Formula (7) shows that the value of metric $\theta(x, bf_i)$ decreases as the decay hop i increases. To construct $bf_i(link_j)$, number of $\alpha(link_j)$ balls are thrown into the m bits of bf_i. The probability that any bit in the m bits is hit by at least one ball is $1 - (1 - 1/m)^{\alpha(link_j)}$. Therefore, the number of bits which belongs to those $\theta(x, bf) - \theta(x, bf_i)$ bits and are hit is

$$f_1 = \big(\theta(x, bf) - \theta(x, bf_i)\big) \cdot \big(1 - (1 - 1/m)^{\alpha(link_j)}\big).$$

Actually, the number of f_1 bits and remainder bits being 1 in $Bfaddress(x)$ of bf_i represent the membership information of x in $bf_i(link_j)$. Hence, we can infer that

$$E(x, bf_i(link_j)) = \theta(x, bf_i) + \big(\theta(x, bf) - \theta(x, bf_i)\big) \cdot \big(1 - (1 - 1/m)^{\alpha(link_j)}\big). \tag{12}$$

Note that $\alpha(link_j) = \beta(link_j) - \theta(bf_i)$ and $\alpha(link_j^{'}) = \beta(link_j^{'}) - \theta(bf_{i+1})$. The difference between $\theta(bf_i)$ and $\theta(bf_{i+1})$ is trivial by comparing to $\beta(link_j)$ or $\beta(link_j^{'})$. Therefore the component $(1 - (1 - 1/m)^{\alpha(link_j)}$ in Formula



(a) The existing gradient-based routing mechanisms, where m=2000, k=14, and f=0.0001. (b) The gradient-based routing based on Wader , where m=60000 and k=16.

Fig. 2. The expected values of $\theta(x, bf_i(link))$, $\theta(x, bf_i)$, and noise, where n=100, k=16, d=1.2, and h=5.

(12) becomes a constant factor since $\beta(link_j) \approx \beta(link_j^{'})$, and can be denoted as $0<g<1$. Formula (12) can be expressed as

$$E\big(\theta(x, bf_i(link_j))\big) = (1-g)\times\theta(x, bf_i) + g\times\theta(x, bf).$$

It is clear that $E\big(\theta(x, bf_i(link_j))\big)$ and $\theta(x, bf_i)$ are monotonic decreasing functions of i if $h_0 \le i \le h_0 + h_1$. Thus, Theorem 1 holds in the case of the linear decaying model. For the exponential decaying model, we first achieve the expectation of $\theta(x, bf_i)$ according to Corollary 1, and replace $\theta(x, bf_i)$ with $E\big(\theta(x, bf_i)\big)$ in Formula (12). For the same reason, Theorem 1 also holds for the exponential decaying model. Thus proved. ■

We further conduct simulations to evaluate the two criterion, especially the second one, in the following two scenarios. The simulations use the same configuration as that in Section IV.

In the scenario of existing gradient-based routing mechanisms, the configuration of Bloom filter is defined in Section IV-E. We can see from Fig.2(a) that the expected value of $\theta(x, bf_i(link_j))$ is very close to that of noise and decreases slowly as the decay hop increases. The root case is presented in Sections III-C and IV-E. Consequently, the information potential formed by each Bloom filter is very smooth, and hence, cannot navigate queries by ascending the potential field. Finally, the gradient-based routing is blinded to the right routing decisions and has to forward the queries in a flooding-like manner.

We address the root cause of the above mismatch between the idea and the practical performance of gradient-based routing by proposing wader, a receiver-oriented design of Bloom filters in the next section. In the scenario of wader, Fig.2(b) shows that the expected value of $\theta(x, bf_i(link_j))$ decreases as the decay hop increases, whereas always significantly outperforms that of noise. Consequently, the information potential formed by each Bloom filter works well to navigate queries by ascending the potential field. This demonstrates that Wader guarantees the correctness and efficiency of the gradient-based routing. The next section presents the details of Wader.

III. Feasibility of Gradient-based Routing

Based on the above measurements, we first examine the impact of noise on one-hop routing decisions of gradient-based routing mechanism. We then derive the necessary and sufficient condition of the *second criterion*, which guarantees the feasibility of gradient-based multi-hop routing. We further propose *Wader*, a novel design of Bloom filters, to satisfy this condition. We also improve the adaptivity of *Wader* in regular as well as irregular networks and address redundant queries caused by the gradient-based routing mechanism.

A. Impact of noise on routing decisions

Recall that V_i=$\theta(x, bf_i(link_j))$ denotes the amount of information of x in right routing link $link_j$ at node A and is a discrete random variable whose possible value, denoted as v, is an integer ranging from 0 to k. The probability mass function of V_i is given by Formulas (9) and (10) for the exponential and linear decaying models, respectively. In addition, Y denotes the strength of noise in other unrelated routing links $link_j'$ at node A and is a discrete random variable whose possible value, denoted as u, is an integer ranging from 0 to k. The probability mass function of Y is given by Formula (11). Before a query for an item x enters the decay range of a destination node, a routing decision is made randomly. As shown in Fig.1, all routing decisions along a path $K{\to}H{\to}G{\to}E$ are made randomly. Otherwise, a routing decision is made according to the following strategies.

1) The value of u is less than v for any unrelated routing link $link_j^{'}$, so that node A can distinguish $link_j$ from others and forward the query for x through $link_j$. This is called an ***unicast*** strategy of gradient-based routing. For example, a query towards node A is only forwarded to node C by node E, as shown in Fig.1(a).
2) The value of u is equal to v for some unrelated routing links, however, is less than v for others. In this condition, node A cannot distinguish $link_j$ from other links $link_j^{'}$ where u=v, and hence forwards the query through $link_j$ and such links together. This is called a ***multicast*** strategy of gradient-based routing. For example, a query towards to node A is forwarded to node B as well as node D by node C, as shown in Fig.1(a).
3) The value of u is larger than v for a link or links except $link_j$. The strength of noise about x at such links is higher than the strength of information about x at $link_j$. Therefore, the query will be wrongly forwarded to a link or links except $link_j$. This is called an ***invalid*** strategy of gradient-based routing. For example, a query towards to node A is wrongly forwarded to node D by node C, as shown in Fig.1(b).

We will prove the probability of each aforementioned decision in theory once a query enters the propagation field of a destination. Note that each node has c links averagely and each is associated with a Bloom filter as its routing entry.

Theorem 2: A node forwards a query for an item x using the *unicast* strategy if it receives bf_i from a destination of the query. The probability of this event is

$$f_{unicast}(V_i){=}\sum\nolimits_{v=1}^{k} P(V_i{=}v)\cdot\Big(\sum\nolimits_{u=0}^{v-1} P(Y{=}u)\Big)^{c-2}. \quad (13)$$

Proof: Recall that the information of x in the Bloom filter associated with $link_j$ through which current node receives bf_i is a discrete random variable, denoted as V_i. Its probability mass function has been given by Formulas (9) and (10), depending on the decaying model. For any possible value v of V_i where $0 \leq v \leq k$, consider an event that the information of x in the Bloom filter associated with another link is less than v. The probability of this event is given by $\sum_{u=0}^{v-1} P(Y{=}u)$. Further, we consider an event that the information of x in the Bloom filter associated with each link except $link_j$ and the link the query came from is less than v. The probability of this event is given by $(\sum_{u=0}^{v-1} P(Y=u))^{c-2}$. Thus, the probability that the query is forwarded using the unicast strategy can be calculated by Formula (13). Thus proved. ■

Theorem 3: A node forwards a query for an item x using the *multicast* strategy if it receives bf_i from the destination. The probability of this event is

$$f_{multicast}(V_i) = f_{valid}(V_i) - f_{unicast}(V_i). \tag{14}$$

Proof: Recall that the information of x in the Bloom filter associated with $link_j$ through which current node receives bf_i from the destination is a discrete random variable, denoted as V_i. Its probability mass function has been given by Formulas (9) and (10), depending on the decaying model. For any possible value v of V_i where $0 \leq v \leq k$, consider an event that the information of x in the Bloom filter associated with another link is less than or equal to v. The probability of this event is given by $\sum_{u=0}^{v} P(Y{=}u)$. Further, let us consider an event that the information of x in the Bloom filter associated with each link except $link_j$ and the link the query came from is less than or equal to v. The probability of this event is given by $(\sum_{u=0}^{v} P(Y{=}u))^{c-2}$. Thus, the probability that the query can be forwarded successfully using the unicast or multicast strategy can be calculated by

$$f_{valid}(V_i){=}\sum_{v=1}^{k} P(V_i{=}v) \cdot \left(\sum_{u=0}^{v} P(Y{=}u)\right)^{c-2}. \tag{15}$$

The probability that the query is forwarded using the unicast strategy has been given by Formula (13). Therefore, we can infer that the probability of the event defined in this theorem is the difference between Formulas (15) and (13), that is given by Formula 14. Thus proved. ■

Theorem 4: A node forwards a query for an item x using the *invalid* strategy if it receives bf_i from the destination. The probability of this event is

$$f_{invalid}(V_i) = 1 - f_{valid}(V_i). \tag{16}$$

Proof: As discussed above, the probability that queries for x are forwarded successfully according to the unicast or multicast strategy is given by Formula (15). It is easy to infer that the probability of the event defined in this theorem is given by Formula (16). Thus proved. ■

B. The necessary and sufficient condition for gradient-based multi-hop routing

In the above Section, we have discussed the conditions of ***unicast***, ***multicast***, and ***invalid*** strategies for one-hop routing decisions. Only one of such strategies will be chosen to deal with a query at each node. Theorems 2, 3, and 4 have proved the probability that each strategy is chosen. Among the three strategies, the *unicast* is a valid and desired gradient-based routing mechanism. In this strategy, a query for an item x is only biased at an intermediate node which receives a decaying Bloom filter from the destination and is closer to the destination than current node. The benefit of the *unicast* strategy is that it can ensure the correctness of routing whereas does not produce redundant queries (forwarding a query to additional intermediate nodes). The *multicast* is another valid gradient-based routing mechanism at the cost of sending a query to some neighbors which do not receive a decaying Bloom filter from the destination. A gradient-based routing decision is *valid* if it ensures an *unicast* or a *multicast* routing decision by preventing an *invalid* decision at nodes which reside within the decay range of the destination.

Note that the gradient-based routing using Bloom filters is essentially a probabilistic routing. Thus, it is impossible and there is no need to achieve an absolutely valid routing decision for each query. What we need is a valid gradient-based routing decision for any query with high probability. For any query, we can infer from Theorem 3 that the node which received bf_i from the destination of the query can make a valid gradient-based routing decision with probability $f_{valid}(V_i)$, and an unicast routing decision with probability $f_{unicast}(V_i)$.

So far, we consider the valid gradient-based routing decision in the scenario of one hop transmission of queries. In practice, only potential destinations of a very few queries reside one hop away from the sources of queries. Thus, we consider a general scenario in which a query traverses multiple intermediate nodes along a multi-hop path before it reaches its destination. In this scenario, a query can be sent to its destination with high probability only if each intermediate node achieves a valid routing decision for the query with high probability.

Definition 4: **(Gradient-based routing for multi-hop queries)** Given a multi-hop query, a *valid* routing can ensure that the query is sent to its destination by a sequence of *valid* routing decisions made at intermediate nodes once it enters the decay range of its destination. An *unicast* routing for the query requires all unicast routing decisions at intermediate nodes. An *invalid* routing for the query means that the routing decision at any intermediate node is invalid. Figures 1(a) and 1(b) plot a valid and an invalid routing for a multi-hop query, respectively.

Let σ denote a lower bound, depending on applications, on the probability that each query is sent to its destination by a valid routing mechanism. According to Theorems 2 and 3, we can infer that the necessary and sufficient condition of a valid gradient-based routing mechanism for a multi-hop query is

$$\prod_{i=1}^{h} f_{valid}(V_i) \geq \sigma. \tag{17}$$

If we further seek all unicast routing decisions, the necessary and sufficient condition should be

$$\prod_{i=1}^{h} f_{unicast}(V_i) \geq \sigma. \tag{18}$$

Recall that the expectation value of the metric $\theta(x, bf_i(link_j))$ decreases as the value of i increases as shown in Theorem 1. It is easy to infer that

$$P(V_i{=}v) > P(V_{i+1}{=}v) \text{ for } \theta(x, bf_i) \leq v \leq k, 1 \leq i < h.$$

On the other hand, the noise distribution is similar in Bloom filters associated with neighbor links at each node. In summary, for any query,

$$f_{unicast}(V_i) > f_{unicast}(V_{i+1}) \text{ and}$$
$$f_{valid}(V_i) > f_{valid}(V_{i+1}).$$

By now, Formulas (17) and (18) become Formulas (19) and (20), respectively, if we replace $f_{unicast}(V_i)$ and $f_{valid}(V_i)$ with $f_{unicast}(V_h)$ and $f_{valid}(V_h)$, respectively.

$$\big(f_{valid}(V_h)\big)^h \geq \sigma. \tag{19}$$

$$\big(f_{unicast}(V_h)\big)^h \geq \sigma. \tag{20}$$

Inequality (19) or (20) acts as the necessary and sufficient condition of a feasible gradient-based multi-hop routing. Note that this necessary and sufficient condition is also hold for a single-hop and gradient-based routing. In the remainder of this paper, we will use inequality (19) or (20) to instruct the novel design of Bloom filters to satisfy this condition.

C. Wader

In many distributed applications, all nodes are required to adopt the same configuration of m, k, and hash functions in order to guarantee the compatibility and inter-operability of Bloom filters. In this work, the union operation of decaying Bloom filters requires the same configuration between all Bloom filters. Thus, the initial and decaying Bloom filters of each node should adopt the same configuration, and so does the joint Bloom filter associated with each link. Many efforts have been made to optimize Bloom filters [28], [29], [30], [31] from different aspects. The common idea is to minimize the false negative probability or the size of an individual Bloom filter, which only represents all data at a single node.

Such efforts, however, don't address the fact that each node uses the union of all received decaying Bloom filters through a link as a routing entry of that link. Although the fraction of bits set to one in each individual Bloom filter might be low, that in each routing entry becomes high due to the union of many decaying Bloom filters. Thus, given a query for any item at a random node, noise about the item in unrelated routing entries is very likely equal to or even stronger than the useful information in right routing entries. The above analytical as well as experimental results in Section IV-E demonstrate that the existing designs of Bloom filters fail to support the gradient-based routing mechanism. To address this issue, we propose a novel design of Bloom filters for each routing entry, the union of many individual Bloom filters. The main idea, called ***Wader***, is to derive the optimal configuration of each individual Bloom filter under the constraint of inequality (19) or (20), so as to satisfy the second criterion of gradient-based routing.

Besides the well-known metrics of Bloom filters (the number of items n, the size of Bloom filter m, and the number of hash functions k), the decay factor d and decay range h are two additional dependent factors which impose constraints on inequalities (19) and (20).

Based on a given decaying model with parameters d and h, we first calculate $\theta(bf)$ and $\theta(bf_i)$ according to Formulas (1), (2) and (3) for $1 \leq i \leq h$. Note that $\theta(bf)$ is a function of variables m, n, and k, whereas $\theta(bf_i)$ is a function of variables m, n, k, and d. We then estimate the fraction of bits set to one, r_1, in each joint Bloom filter according to Formula (5) and m, and finally obtain the distribution of noise strength at each neighbor link based on Formula (11). Note that r_1 is a function of variables m, n, k, d, and h. Similarly, According to Formulas (9) and (10), we can achieve the distribution of information of any item x in a joint Bloom filter associated with a link through which a decaying Bloom filter is received from a destination. We calculate the probability of an *unicast* and a *valid* gradient-based routing decision by Formulas (13) and (15) which are functions of m, n, k, d, and h. Finally, inequality (19) or (20) is used to restrict the value of m, n, k, d, and h under a constraint of the lower bound σ.

The parameters n, d and h should be assigned with appropriate values with regard to several factors, such as the topological properties, data distribution in the network, and query popularity, etc. Many efforts have been made to investigate the data distribution and query popularity, depending on applications. Thus, it is reasonable to assume that we are given n, d and h. In this case, inequalities (19) and (20) merely depend on parameters m and k, and hence we can optimize the number of hash functions k to maximize $f_{unicast}(V_h)$ and $f_{valid}(V_h)$. Accordingly, inequalities (19) and (20) can be satisfied with m as small as possible. It is well-known that a single Bloom filter is optimal when $k{=}(m/n)\ln 2$. Such an optimal result, however, cannot ensure an optimal joint Bloom filter. After optimizing $f_{unicast}(V_h)$ or $f_{valid}(V_h)$, we can calculate the optimal value of m and k by solving inequalities (19) and (20), respectively. So far, the parameters m, k, d, h, and n are configured. Consequently, these parameters of each individual Bloom filter each node proposes can ensure the fraction of bits set to one in each routing entry is low, and hence the second criterion of gradient-based routing can be satisfied.

D. Implementation issues with Wader

According to the design approach in Section III-C, the parameters m, k, d, h, and n can be optimized to ensure the second criterion of gradient-based routing in theory. With those parameters, the number of bits set to 1 in any routing entry can be calculated by Formula (5). The following practical issues, however, directly affect the performance of *Wader*. The distributions of node degree and received BFs through every link are usually non-uniform. Thus, for the majority of links, the number of bits set to 1 in a routing entry usually does not equal to the estimated value $r_1 \times m$. To make *Wader* be adaptive to dynamic network conditions, a practical implementation way is to let each node monitor the number of bits set to 1 in each routing entry. Once the number of bits set to 1 in a routing entry exceeds $r_1 \times m$, it denies all the Bloom filters received afterwards. Such a method makes inequalities (19) and (20) always satisfied, and hence ensures the correctness and efficiency of *Wader* in practice.

We use a regular graph to model a multi-hop network for ease of mathematical analysis and presentation, the theoretical design and practical implementation of *Wader* can guarantee the correctness and efficiency of the gradient-based routing with high probability. We further reconsider the gradient-based routing in more general networks such as random networks and sensor/geographic networks. The basic ideas and methodologies mentioned above also apply to other general networks.

An intuitive way is to revise the analytical results in Section II and the theoretical design of *Wader* in Section III-C, given the average node degree c and the distribution of node degree. This way, however, is very complex to derive the desired analytical results. The second way is to borrow the theoretical design of *Wader* for regular networks, whose node degree is appropriated to the average node degree of an irregular network. The evaluation results in Section IV show that the second way can ensure the feasibility of gradient-based routing with high probability. The root reason is that the practical implementation of *Wader* is adaptive to dynamic network conditions, and can deal with the mismatch between the topological properties of an irregular network and an appropriate regular network.

In the case of *Wader*, the size of each individual Bloom filter might be too large to represent items hosted by each node. On the other hand, a Bloom filter a node proposes must be diffused away as messages, so as to establish an information potential. To reduce transmission size and save bandwidth, a Bloom filter can be compressed before transmission. A Bloom filter designed by existing approaches, however, cannot gain any compression gain. The reason is that under good random hash functions, each bit of Bloom filter is 0 or 1 independently with probability 1/2 [32]. The Bloom filter designed by *Wader* can achieve high compression gain since each bit in it is 1 with a lower probability than 1/2.

E. On redundant queries

Given a multi-hop query, the optimal m and k for the joint Bloom filter which ensures inequalities (19) are not necessary to guarantee inequality (20) since $f_{unicast}(V_i) < f_{valid}(V_i)$ under the same m and k. Therefore, the joint Bloom filter for the unicast routing always consumes more bits than that for the valid routing. The advantage of the unicast routing is that the query does not traverse any nodes which do not participate the routing path, and hence does not create any redundant query. In contrast, the *valid* routing consumes less bits than the *unicast* routing, however, usually produces redundant queries due to the potential use of *multicast* routing decision. Those replicas are sent along other paths deviating from the destination, and are redundant. Here, we first examine the number of redundant queries, and then tackle those redundant queries.

In the case of a multicast routing decision for a query at a node receiving bf_i, let S_i denote the number of neighbors to which the query is forwarded by the current node, except the neighbor receiving a bf_{i-1}. On the other hand, the query cannot be sent back to the neighbor it came from. In words, S_i measures the number of redundant queries produced by routing a query at a node which resides within the decay range of a destination. A multicast routing decision occurs if the query is sent to at least one of the remainder $c-2$ neighbors. Therefore, S_i is a discrete random variable. Its possible values, denoted as s, are integers ranging from 0 to $c-2$.

Theorem 5: In the case of a multicast routing decision of a query for an item x, the probability mass function of S_i is

$$P_m(S_i{=}s) = \binom{c-2}{s}\sum_{v=1}^{k} P(V_i{=}v)\cdot P(Y{=}v)^s \cdot \Big(\sum_{u=0}^{v-1} P(Y{=}u)\Big)^{c-2-s} \quad (21)$$

Proof: Recall that the information of x in the Bloom filter associated with $link_j$ through which current node receives bf_i is a discrete random variable, denoted as V_i. Its probability mass function has been given by Formulas (9) and (10), depending on the decaying model. For any possible value v of V_i where $0 \le v \le k$, consider an event that the information of x in the Bloom filter associated with one link except $link_j$ is less than v. The probability of this event is given by $\sum_{u=0}^{v-1} P(Y{=}u)$. For any possible value v of V_i, the event $S_i{=}s$ means that the information of x in Bloom filters associated with s links among the $c-2$ links is equal to v, whereas that in Bloom filters associated with the other $c-2-s$ links is less than v. The probability of S_i under a given v of V_i is given by

$$\binom{c-2}{s} P(V_i{=}v)\cdot P(Y{=}v)^s \cdot \Big(\sum_{u=0}^{v-1} P(Y{=}u)\Big)^{c-2-s},$$

and that under all possible values of V_i is given by Formula (21). Thus proved. ■

So far, we only consider the event S_i caused by a multicast routing decision. Actually, such S_i might also occur under an invalid routing decision. That is, multiple links except $link_j$ have the highest information of x in their routing entries, and hence the query are wrongly forwarded to such links except $link_j$. We will discuss the probability mass function of S_i under an invalid routing decision in Theorem 6.

Theorem 6: In the case of an invalid routing decision of a query for an item x, the probability mass function of S_i is

$$P_i(S_i{=}s) = \binom{c-2}{s}\sum_{v=0}^{k} P(V_i{=}v)\cdot \sum_{u=v+1}^{k}\Big(P(Y{=}u)^s \cdot \Big(\sum_{r=0}^{u-1} P(Y{=}r)\Big)^{c-s-2}\Big). \quad (22)$$

Proof: For any possible value v of V_i where $0 \le v \le k$, let consider the event that s of the $c-2$ links (not including the link the query came from and the link $link_j$) have the highest information of x in their routing entries. The probability of this event under a given v of V_i is given by

$$\binom{c-2}{s} P(V_i{=}v)\cdot \sum_{u=v+1}^{k}\Big(P(Y{=}u)^s\cdot\Big(\sum_{r=0}^{u-1} P(Y{=}r)\Big)^{c-s-2}\Big),$$

and that under all possible values of V_i is given by Formula (22). Thus proved. ■

We further consider S_i in the case of a gradient-based routing decision which covers the two independent cases we discussed in Theorems 5 and 6, respectively. In this case, the probability mass function and expectation value of S_i are given by

$$\begin{aligned} P(S_i = s) &= P_m(S_i = s) + P_i(S_i = s) \text{ and} \\ E[S_i] &= \sum_{s=1}^{c-2} s \cdot P(S_i = s). \end{aligned}$$

So far, we only consider the variable S_i in the scenario of one-hop transmission of a query from a node receiving bf_i to a node receiving a bf_{i-1}. Here, we consider a general query whose source node is out of the decay region of its nearest destination. In this case, the query suffers number of h one-hop transmissions in the decay region of the destination, and causes number of $E[S_i]$ redundant queries due to a routing decision for each transmission. The average number of such kind of redundant queries caused by a multi-hop query after it enters the decay range of its destination is given by

$$\sum_{i=1}^{h} \sum_{s=1}^{c-2} s \cdot P(S_i = s). \tag{23}$$

Nodes receiving such kind of redundant queries take additional computations for making decisions on routing those queries. Although each query only produces a very few redundant queries before reaching a destination as given by Formula (23), these replicas can incur non-trivial negative impact if they keep on propagating in the network. Fortunately, we find that such redundant queries can be terminated after their first transmissions with high probability as shown in Theorem 7.

Theorem 7: Given a gradient-based routing decision of a query for an item x at a random node, receivers of S_i resulting redundant queries stop forwarding such queries with high probability as given by

$$\sum_{v=1}^{k} P(E_v^i|E^i) \cdot \Big(\sum_{u=0}^{v-1} P(Y{=}u)\Big)^{c-1}. \tag{24}$$

Proof: Recall that V_i is a discrete random variable, and denotes the information of x in the Bloom filter associated with $link_j$ through which node C receives bf_i from a destination node A, as shown in Fig.1. For any neighbor node D which does not receives a bf_{i-1} from node A, let E^i denote an event that node D receives a redundant query from node C. Let E_v^i denote an event that node C forwards a redundant query to node D since the noise strength at the link $C{\to}D$ is at least the same as a value v of V_i.

The probability of the event E_v^i and E^i is given by

$$\begin{aligned} P(E_v^i) &= P(V_i{=}v) \cdot \sum_{u=v}^{k} P(Y{=}u) \text{ and} \\ P(E^i) &= \sum_{v=0}^{k} \big(P(V_i{=}v) \cdot \sum_{u=v}^{k} P(Y{=}u)\big). \end{aligned}$$

Thus, it is easy to infer the conditional probability of E_v^i given E^i is given by

$$P(E_v^i|E^i) = P(E_v^i)/P(E^i).$$

For any possible value v of V_i where $0 \le v \le k$, consider an event that the information of x in the Bloom filter associated with each link is less than v at node D, except the link through which a redundant query came from. This event means that the gradient-based routing mechanism fails to find a neighbor of node D which holds higher level of information about x than v, and hence node D cannot keep on forwarding the query. The probability of this event under all possible values of V_i is given by Formula (24). Thus proved. ■

IV. Performance evaluation

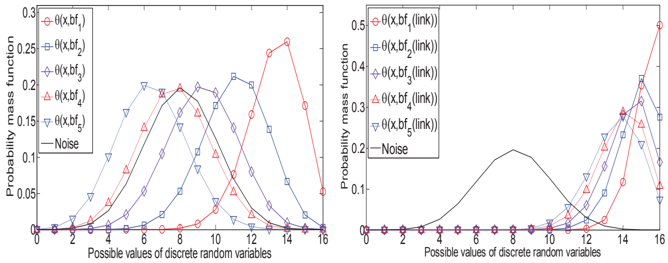
We implement *Wader* in a random network using the approach proposed in Section III-D to demonstrate that only *Wader* can guarantee the feasibility of gradient-based routing. The simulation settings are as follow. The node degree ranges from 3 to 7 and the average node degree is c=5. The average number of items hosted by each node is n=100. All Bloom filters are decayed from the first round of propagation. That is, h_0=0. For a large h_0, we have to enlarge the value of m to satisfy the same lower bound σ, which results in unnecessarily higher space cost. Although we only report the results under the exponential decaying model, the similar results are achieved under the linear decaying model.

A. Effect of decaying operation on membership information

Assume the decay factor is set to be d=1.2 and the decay range is set to be h=5, depending on a given application. Then, we can derive that an optimal number of bits for each Bloom filter is m=60000 and the number of hash functions is k=16 from the aspect of receiver. Given a bf which represents a set X, we have analyzed the amount of information of any item $x \in X$ in a decay version of bf in Lemma 2. The possible values of $\theta(x, bf_i)$ are integers ranging from 0 to k=16. Fig.3(a) shows the probability mass function of $\theta(x, bf_i)$ for $1{\le}i{\le}5$ and noise. The results match well with Formula (8). As we can see from the figure, when the possible value increases, the probabilities of $\theta(x, bf_i)$ first goes up and then goes down for $1{\le}i{\le}5$. On the other hand, the probabilities of $\theta(x, bf_i)$ for the large possible values decrease as the value of i increases, whereas that for those small possible values increase as the value of i increases. The experimental results exactly conform to the analytical results.

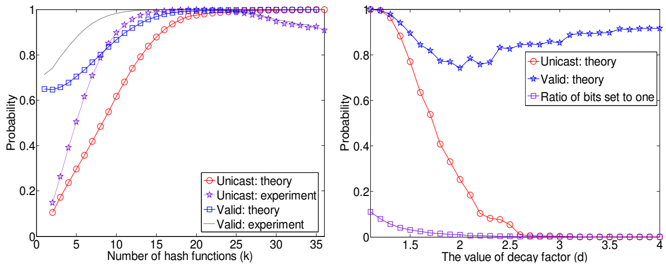
Fig.3(b) shows the probability mass functions of noise and $\theta(x, bf_i(link_j))$.The simulation results follow a similar trend as the theoretical results given by Formula (9). As we can see from the figure, when the possible value increases, the probabilities of $\theta(x, bf_i(link_j))$ first goes up and then goes down where $1{\le}i{\le}5$. On the other hand, the probabilities of the $\theta(x, bf_i(link_j))$ for the large possible values decrease as the value of i increases, whereas that for those small possible values increase as the value of i increases.

Fig.3(b) also shows that the expectation of $\theta(x, bf_i(link_j))$ decreases as the decay hop i increases, and thus the *first criterion* proposed in Section I is satisfied by *Wader*. In addition, the expectation of $\theta(x, bf_i(link_j))$ is larger than that of noise for $1{\le}i{\le}h$. This reveals the reason why a node holding bf_i can forward a query for an item x to a node holding a bf_{i-1} with high probability, and thus satisfy the second criterion proposed in Section I. On the other hand, the simulation results conform to Theorem 1 in terms of the expectation value of $\theta(x, bf_i(link_j))$ for $1{\le}i{\le}h$.



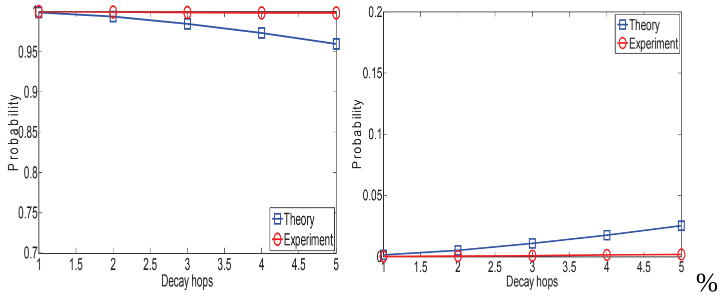
(a) $\theta(x, bf_i)$ and noise. (b) $\theta(x, bf_i(link))$ and noise

Fig. 3. Probability mass functions of $\theta(x, bf_i)$, $\theta(x, bf_i(link))$ and noise, where $m = 60000$, $n = 100$, $k = 16$, $d = 1.2$, and $h = 5$.

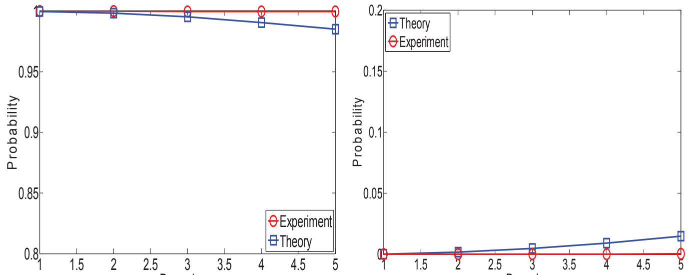


(a) (b)

Fig. 4. Effect of the parameters k and d on the probability of an unicast routing and a valid routing, where m=60000, n=100, and $h = 5$.



(a) Unicast decision (b) Multicast decision



(c) Valid decision (d) Invalid decision

Fig. 5. Probability of four types of routing decisions, where $m = 60000$, $n = 100$, $k = 16$, $d = 1.2$, and $h = 5$.

B. Effect of decay parameters on the performance of Wader

Now we examine the effect of the parameters k and d on the probability that each query is sent to its destination through an unicast routing or a valid routing. As shown in Fig.4(a), given a fixed m and d=1.2, k is the only dependent factor of all four curves which follow a similar trend. They first ascend as k increases and quickly reach the peak, and then descend as k increases. The reason is that $\theta(x, bf_i(link))$ and noise strength increase for any x and $1 \le i \le h$ as k increases, and $\theta(x, bf_i(link))$ is more likely higher than noise relatively. As discussed in Section III-C, inequalities (19) and (20) are the benchmarks to optimize the parameters of Bloom filters. Given a lower bound σ on the probability of an unicast routing or a valid routing for each query, we can find the optimal value of k under each scenario. Similarly, we can achieve the optimal k under varying value of m, and can finally find the global optimal k and m. As shown in Fig.4(a), there is minor difference between the theoretical and experimental results. The root reason is that we use a regular network to appropriate an irregular network as discussed in Section III-D.

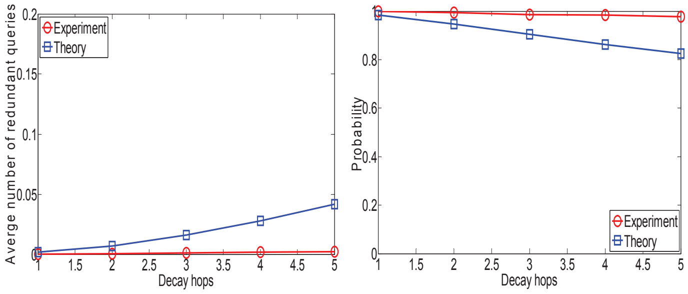
As shown in Fig.4(b), given a fixed m and k=16, the probability of an unicast routing for any query decreases as the decay factor d increases in theory, and reaches almost zero after the decay factor exceeds a threshold. The reason is that $\theta(x, bf_i(link))$ and noise strength decrease as the decay factor increases for $\forall x \in X$ and $1 \le i \le h$, and the noise strength is more likely higher than $\theta(x, bf_i(link))$. We can also see that the probability of the valid routing first decreases, and then increases as the decay factor increases. It is worth noticing that a small decay factor should be adopted in order to ensure the unicast routing with high probability, although a large decay factor can always guarantee the valid routing with high probability. For a large decay factor, we have to enlarge the value of k in order to satisfy the same lower bound σ, which results in unnecessarily higher computation cost.

C. Performance of gradient-based routing with Wader

We examine the impact of noise on a gradient-based routing decision when each node adopts an optimal Bloom filters based on *Wader*. A gradient-based routing decision for a single-hop query can be valid (unicast or multicast) or invalid under the interference of noise in unrelated links once the query enters the decay range of a destination. The probabilities of the aforementioned routing decisions have been proved in Theorems 2, 3 and 4. Fig.5 shows the probabilities of those routing decisions from aspect of both theory and practice.

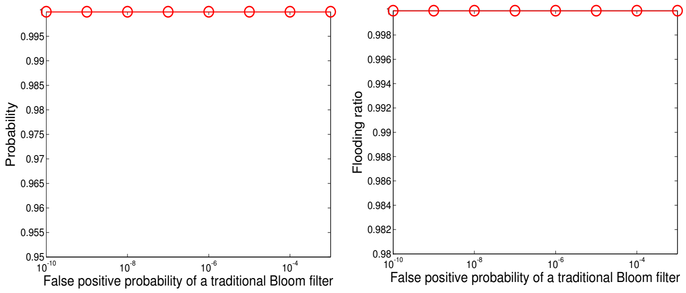
We can see that the probability of an unicast routing decision decreases with the increasing of the decay hop, whereas the probability of a multicast routing decision increases with the increasing decay hop. The reason is that the expectation value of metric $\theta(x, bf_i(link_j))$ decreases as the decay hop increases. Thus, the noise strength is more likely higher than $\theta(x, bf_i(link_j))$, and queries might suffer invalid or multicast routing decision. Fig.5(c) shows that the probability of a valid routing decision decreases as the decay hop increases. The reason is that the negative effect of decreasing unicast routing decision outperforms the positive effect of increasing multicast routing decision. Fig.5(d) shows that the probability of an invalid routing decision increases as the decay hop increases.

It is worth noticing that the probabilities of the unicast and valid decisions for routing a single-hop query is high for $1 \le i \le h$. Thus, a multi-hop query can reach a destination through a sequence of valid even unicast routing decisions with high probability. As shown in Fig.5, the curve of practical probability follows the same trend as the curve of the theoretical probability for each type of routing decision. The



(a) Redundant queries (b) Termination probability

Fig. 6. Number of redundant queries and the probability that they will be terminated by receivers, where m=60000, n=100, k=16, d=1.2, and h=5.



(a) Valid decision (b) Flooding ratio at any node

Fig. 7. Probability of valid decision and flooding ratio at any node in the case of EDBF, where n=100, k=16, d=1.2, and h=5.

practical probability, however, is larger than the theoretical value for the unicast and valid routing decisions. In addition, *Wader* achieves lower probabilities of the multicast and invalid routing decisions than the theoretical values. The root reason is that we use a regular network to appropriate an irregular network as discussed in Section III-D. In summary, the theoretical and practical results demonstrate that *Wader* guarantees the correctness and efficiency of gradient-based routing for multi-hop queries with high probability.

D. Design Effectiveness against redundant queries

A node possibly suffers the multicast or invalid decision, and then sends very few redundant queries (according to Formula (23)), to neighbors which deviate from the potential destination. We have proved the probability that such queries can be terminated by receivers with high probability in Theorem 7. As shown in Fig.6(a), the average number of redundant queries caused by routing one query increases as the decay hop increases in both theory and practice. As shown in Fig.6(b), the termination probability of those redundant queries by receivers decreases as the decay hop increases in theory as well as in practice, but the termination probability still remains at a high level in theory as well as in practice. Note that the simulation results outperform the theoretical results. Formula (23) provides an upper bound on the number of redundant queries resulted from routing any query at a random node. Formula (24) provides a lower bound on the termination probability of any redundant query by a random receiver. In summary, the practical results of *Wader* conform to the theoretical analysis, and thus the negative effect of redundant queries can be controlled at a low level. This is very helpful to ensure the feasibility and usability of the gradient-based routing.

E. Comparisons

In this section, we conduct extensive simulations to evaluate the one-hop routing decisions of the gradient-based routing using an existing design EDBF [18]. In our examinations, n=100, d=1.2, and h=5. Recall that f denotes an upper bound on the false positive probability of the Bloom filter. Given f and n, we can optimize k and m with $m=\lceil n \times \log(f)/\log(0.6185)\rceil$ and $k=\lceil (m/n)\ln 2\rceil$ [33]. The experimental results, as shown in Fig.7, demonstrate that an arbitrary node always forwards any query to almost all of its neighbors expect the one the query comes from, when f ranges from 10^{-10} to 10^{-3}. The fundamental reason is that the ratio of bits set 1 in each routing entry is very close to 1. Thus, noise in unrelated routing entries and the useful information in right routing entries approximate to k. We have analyzed the root cause of this result in Section III-C.

The message complexity of the gradient-based routing is n for EDBF, and is $\log n$ or $\sqrt{n}$ for *Wader*, where n denotes the network size, $\log n$ and $\sqrt{n}$ denote the approximate diameter in a random network or sensor/geographic network, respectively. Actually, the gradient-based routing using existing designs of Bloom filters deteriorate to the well-known flooding mechanism. It is worth noticing that the termination rules of redundant queries mentioned in Section III-E are not invoked since it can terminate all query replicas and make queries fail to reach any destination.

In contrast, the gradient-based routing using *Wader* ensures that a query, which enters the decay range of a desired destination, will be routed to the destination by a sequence of unicast or multicast routing via the intermediate nodes. So far, only *Wader* can guarantee the correctness and efficiency of gradient-based routing using Bloom filter with high probability.

V. Conclusion

We disclose the fact that the existing gradient-based routing mechanisms deteriorate to a flooding-like mechanism and even fail to route queries to the desired destinations. We address this issue by deriving two criterion to ensure the feasibility of gradient-based routing and proposing a novel design of Bloom filters, called *Wader*, to satisfy the two criterion. The evaluation results demonstrate that only *Wader* ensures the correctness and efficiency of the gradient-based routing mechanism with high probability. In our future work, we plan to explore the way to enhance the efficiency of routing outside the decay region of the destination, by using a random replication mechanism.

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