DSAA 5012
Advanced Data Management for Data Science

LECTURE 12
INDEXING: INTRODUCTION

INDEXING: OUTLINE

Indexing Basic Concepts

Ordered Index

- Dense vs. Sparse
- Clustering vs. Non-clustering

B^+-tree Index

Hash Index

- Static Hashing
- Dynamic Hashing

Bitmap Index

INDEXING BASIC CONCEPTS

8,000,000 records of Hong Kong residents.

8 records/page $\Longrightarrow$ 1,000,000 pages!

Find me the record of the person with hkid A634569.

Records: 8,000,000
Records/page: 8
Pages: 1,000,000

INDEXING BASIC CONCEPTS (cont'd)

How would you arrange the records in the catalog?

- Your goal is to minimize the cost (i.e., effort) of finding records.
- We measure this cost as the number of pages you have to "access" before finding the record.

Solution 1: Random order

If the catalog records are in random order of hkid, then in the worst case you must search the entire catalog (cost = 1,000,000 page accesses) before finding a record, or to determine that the hkid does not exist in the catalog. **What would be the average case page access cost?**

Solution 2: Records ordered on hkid

What would be the average case page access cost?

Records: 8,000,000
Records/page: 8
Pages: 1,000,000

INDEXING BASIC CONCEPTS (cont'd)

How would you arrange the records in the catalog?

- Your goal is to minimize the cost (i.e., effort) of finding records.
- We measure this cost as the number of pages you have to "access" before finding the record.

- The same considerations apply when we use computers; instead of paper pages, we have disk pages of a fixed size.
  - Every time we read something from the disk (i.e., do a page I/O), we need to bring an entire page into main memory.
  - The major cost is how many pages we read because disk operations are much more expensive than CPU operations.

☞ **Can we reduce the cost even more?**

INDEXING BASIC CONCEPTS (cont'd)

- Continuing with our catalog example, let's keep the ordered file, but also build an additional index (e.g., at the front of the catalog).
  - Each index entry is a small record, that contains a hkid and the page where you can find this hkid.
  - For example, <A634569, 259> means that hkid A634569 is on page 259 of the catalog.

☞ hkid is the <u>search key</u> of the index.

<u>Recall</u>: A search key is <u>not</u> the same as a primary key or a candidate key!

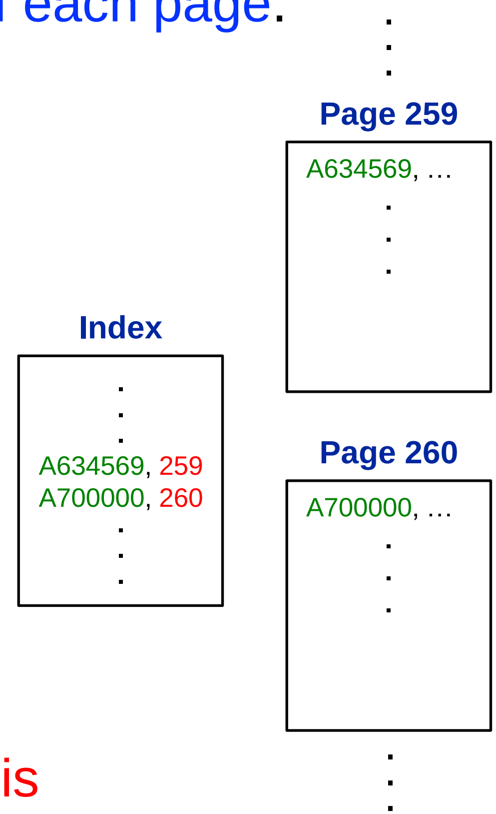
  - Each index entry is much smaller than the actual record.
  - Let's assume that we can fit 100 index entries per paper page.

☞ **The index entries are also ordered on hkid.**

INDEXING BASIC CONCEPTS (cont'd)

Do we need an index entry for each of the 8,000,000 records?

No We only need an entry for the <u>first record</u> of each page.



<u>Example</u>

If there are two consecutive entries <A634569, 259>, <A700000, 260> in the index, then we know that every hkid starting from A634569 and *up to, but not including*, A700000 must be on page 259.

Therefore, we need only 1,000,000 index entries (one for each page of the main catalog).

Since we can fit 100 index entries per page, and we have 1,000,000 index entries, the index is $\lceil 1{,}000{,}000/100 \rceil$ = 10,000 pages (i.e., 10^4 pages).

index entries/page: 100
index pages: 10,000 (10^4)

INDEXING BASIC CONCEPTS (cont'd)

How can we use the index to speed up search for a record?

– Use binary search on the index to find the index page containing the largest hkid value that is smaller or equal to the search hkid value.

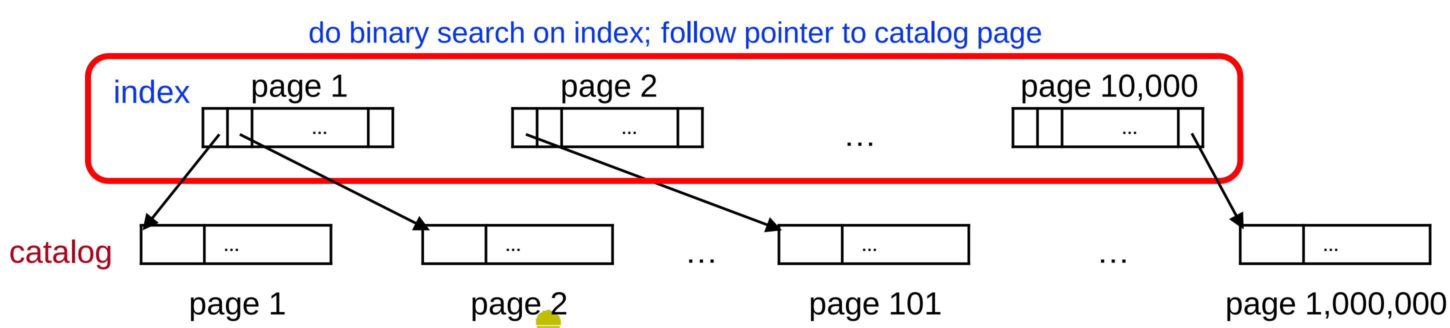
- The cost for this search is $\lceil \log_2 10^4 \rceil$ = 14 page accesses.

– Then, follow the pointer from that index entry to the actual catalog page.

- The cost for this is 1 page access.

Total cost: 14 + 1 = 15 page accesses.

(Page accesses reduced from 20 → 15)



index entries/page: 100
index pages: 10,000 (10^4)

INDEXING BASIC CONCEPTS (cont'd)

Can we reduce the cost even further?

Yes Build an index on the index (i.e., a second level index)!

– The second level index contains 10,000 index entries, one for each page of the first index, and requires $\lceil 10{,}000/100 \rceil$ = 100 (10^2) pages.

– Use binary search on the second level index to find the index page containing the largest hkid that is smaller or equal to the search hkid.

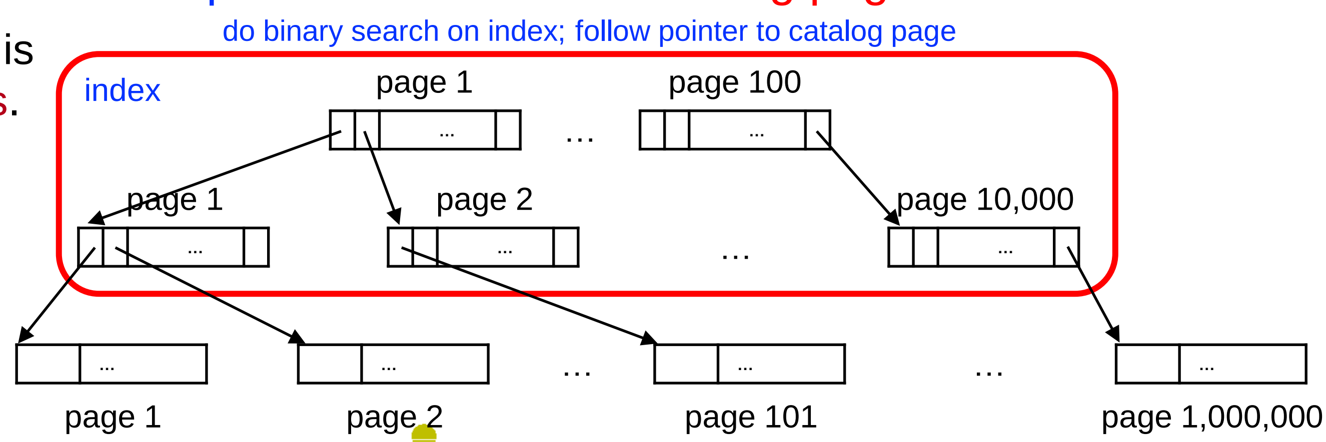
➢ The cost is $\lceil \log_2 10^2 \rceil$ = 7 page accesses.

– Then, follow the pointer from that index entry to the first level index and finally follow the pointer to the actual catalog page.

➢ The cost for this is 2 page accesses.

Total cost:
7 + 2 = 9
page accesses.

(Page accesses reduced from 20 → 15 → 9.)



index entries/page: 100
index pages: 10,000 (10^4)

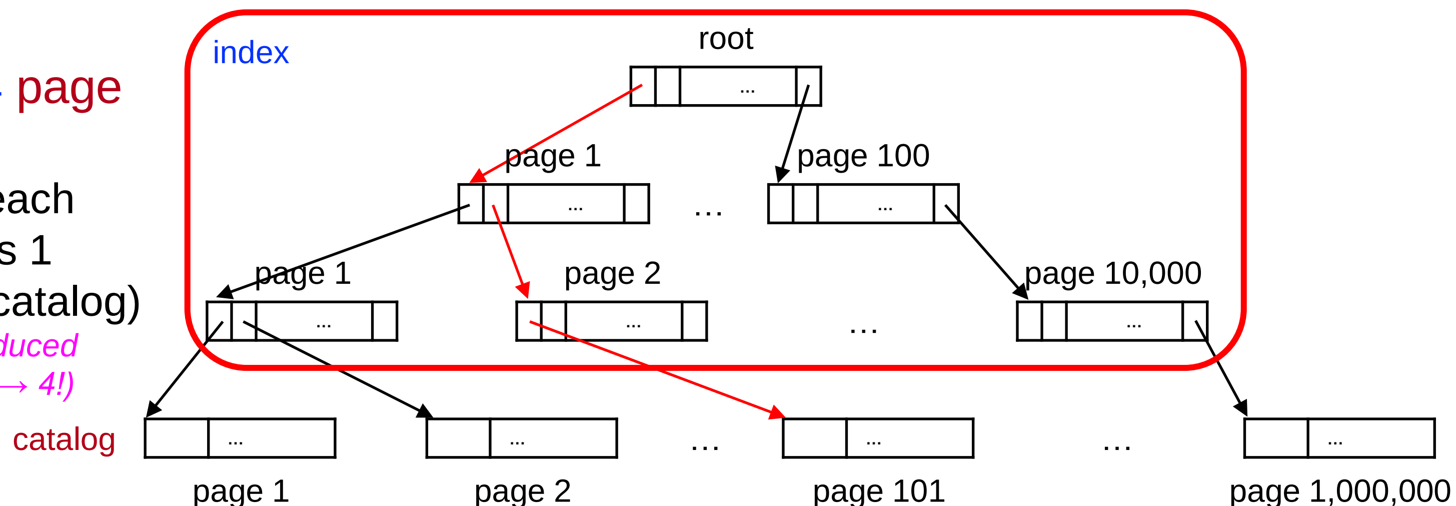
INDEXING BASIC CONCEPTS (cont'd)

Can we reduce the cost even further?

Yes Build a third level index!

- The third level index contains 100 index entries, one for each page of the second level index, and requires $\lceil 100/100 \rceil$ = 1 page.
- Read this page to find the largest hkid that is smaller or equal to the search hkid, and follow the pointer to the second level index, then follow the pointer to the first level index and finally follow the pointer to the actual catalog page.

Total cost: 4 page accesses!
(1 access for each index level plus 1 access to the catalog)
(Page accesses reduced from 20 → 15 → 9 → 4!)



INDEXING BASIC CONCEPTS (cont'd)

Search key The attribute, or set of attributes, used to search for records in a file.

☞ **Do not confuse with the concept of primary or candidate key.**

- A primary key is always also a search key.
- A search key is not necessarily a primary key (it can be any table attribute).

– In the preceding example, the search key was hkid since records were found given a value for hkid.

– To find records given the name (or another attribute) additional indexes need to be constructed.

Index file A file consisting of records (called index entries) of the form <search key, pointer>.

– Index files are typically much smaller than the original file as they do not store all the attributes, but only search-key values and pointers.

INDEXING: INTRODUCTION

EXERCISE 1

CALCULATING FILE SIZE AND SEARCH COST

- The blocking factor, *bf*, of a file is the number of records that fit in a page and is equal to

  $\lfloor \text{\# bytes per page / \# bytes per record} \rfloor$

- The number of pages needed to store a file is equal to

  $\lceil \text{\# records} / bf \rceil$

Operation	Heap File	Sequential File	Hash File
Scan all records	B	B	1.25[1] B
Equality search[2]	0.5 B	$\log_2$ B	1
Range search	B	$\log_2$ B + # of pages with matches	1.25[1] B

B is the number of pages in a file.
1 Assumes 80% occupancy of pages to allow for future additions. Thus 1.25B pages are needed to store all records.
2 Assumes the search is on the key value.

$bf = \lfloor \text{\# bytes per page / \# bytes per record} \rfloor$
$\text{\# pages} = \lceil \text{\# records} / bf_r \rceil$

Film records: 30,000
Actor records: 100,000
Page size: 512 bytes
Pointer size: 6 bytes

EXERCISE 1

A movie database has the following files and sizes of each field:

84 bytes/record — Film(title: 40 bytes, director: 20 bytes, releaseYear: 4 bytes, company: 20 bytes)

28 bytes/record — Actor(id: 4 bytes, name: 20 bytes, dateOfBirth: 4 bytes)

There are 30,000 film and 100,000 actor records.
Each page is 512 bytes. Each pointer is 6 bytes.

a) What is the blocking factor bf_F for the Film file and bf_A for the Actor file?

bf_F: $\lfloor 512$ bytes per page / 84 bytes per Film record$\rfloor$ = 6 records/page

bf_A: $\lfloor 512$ bytes per page / 28 bytes per Actor record$\rfloor$ = 18 records/page

$bf = \lfloor$# bytes per page / # bytes per record$\rfloor$
pages = $\lceil$# records / $bf_r\rceil$

Film records: 30,000
Actor records: 100,000
Page size: 512 bytes
Pointer size: 6 bytes
Film record size: 84 bytes; $bf_F = 6$
Actor record size: 28 bytes; $bf_A = 18$

EXERCISE 1 (cont'd)

b) Assuming the Film file is ordered on title and there is no index, what is the page I/O cost for:

i. Finding the film with title "Titanic"?

pages needed: $\lceil$30,000 Film records / 6 Film records per page$\rceil$ = 5000

page I/O cost: $\lceil \log_2 5000 \rceil$ = 13 (binary search)

ii. Finding all the films directed by "John Woo"?

page I/O cost**:** 5000 **Why?**

Explanation: A sequential scan is needed since the file is not ordered based on director.

INDEXING: OUTLINE

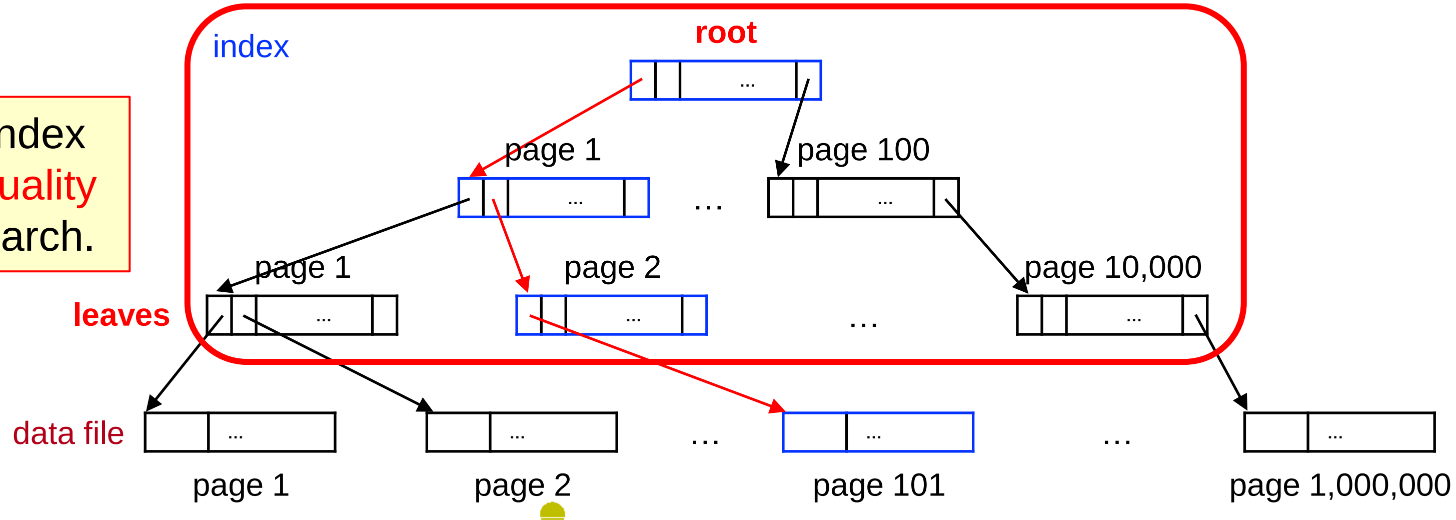
- ✓ Indexing Basic Concepts
- ➡ **Ordered Index**
  - **Dense vs. Sparse**
  - **Clustering vs. Non-clustering**
- B^{+}-tree Index
- Hash Index
  - Static Hashing
  - Dynamic Hashing
- Bitmap Index

ORDERED INDEX

- The index constructed for hkid is an ordered (or tree) index.
  - The index entries are ordered (sorted) on the search key (e.g., hkid).
  - Searching for a record always starts from the root and follows a single path to the leaf that contains the search key of the record.
  - An additional access is then required to retrieve the record from the data file.

Page I/O cost: height of the tree (i.e., number of index levels) plus 1.

An ordered index is good for equality and range search.



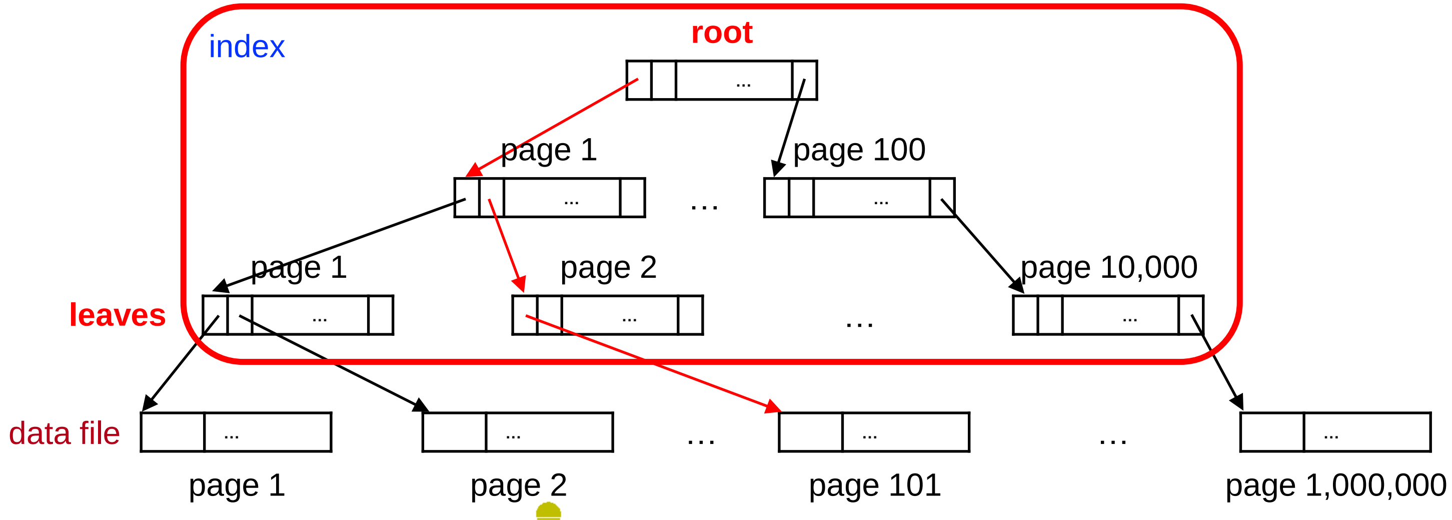
ORDERED INDEX (cont'd)

- An index page is also called an index node.
- The number of children (pointers) of an index node is called the fan-out.

☞ **In our example, the fan-out is 100.**

- The height of the tree is $\lceil \log_{\text{fan-out}}(\text{\# of leaf index entries}) \rceil$.

☞ **In our example, the height of the tree is $\lceil \log_{100}(10^6) \rceil = 3$.**



DENSE VS. SPARSE INDEX

Dense Index Contains an index entry for *every* search-key value.

Sparse Index Contains an index entry for *only some* search-key values.

Example: The hkid index only has index entries for the first record in each page of the file.

- In general, there is an index entry for every data file page corresponding to the minimum search-key value in the page.
- To locate a record with search-key value *K* (single-level index):
  - Find the index entry with largest search-key value ≤ *K*.
  - Follow the pointer to the data file page.
  - Starting at the first record on this page, search the data file sequentially until the search key value is found or the end of the data file is reached.
- Sparse indexes require less space and less maintenance overhead for insertions and deletions than dense indexes.

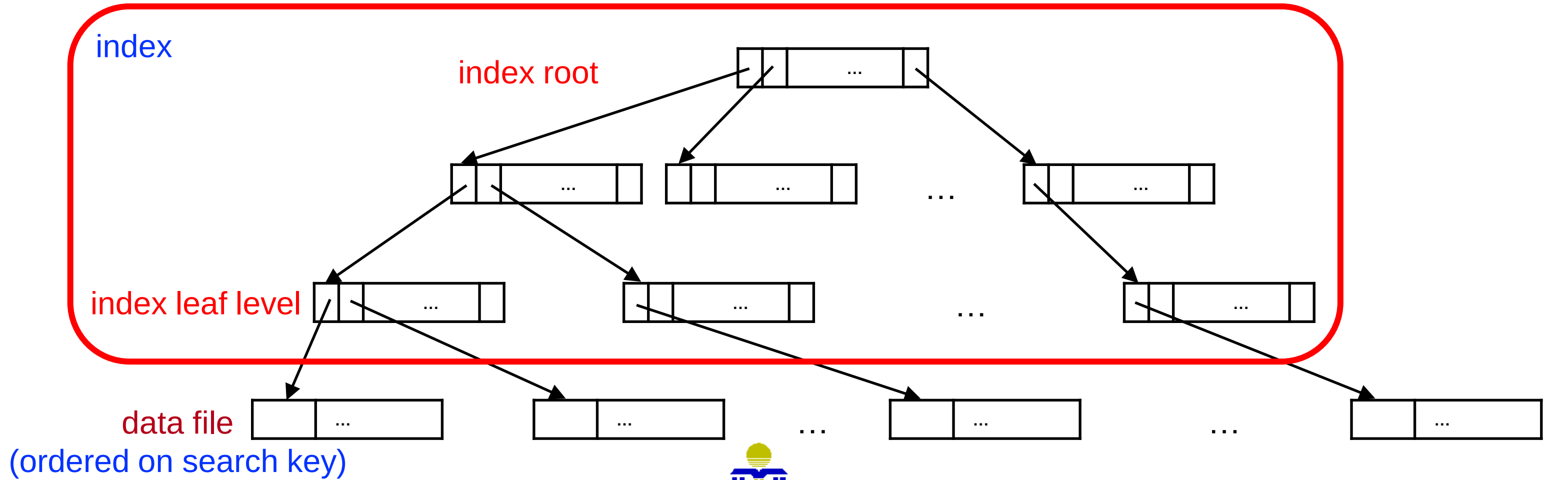
CLUSTERING/PRIMARY INDEX

- A clustering index is an index for which the data file is ordered on the search key of the index (e.g., the index on hkid).
- If a clustering index search key is the primary key, then the index is called a primary index.

  ☞ **There can only be one primary index for a data file.**

  ☞ **A primary index is usually sparse.**

- Index-sequential file: An ordered, sequential file with a primary index (also called ISAM - indexed sequential access method).

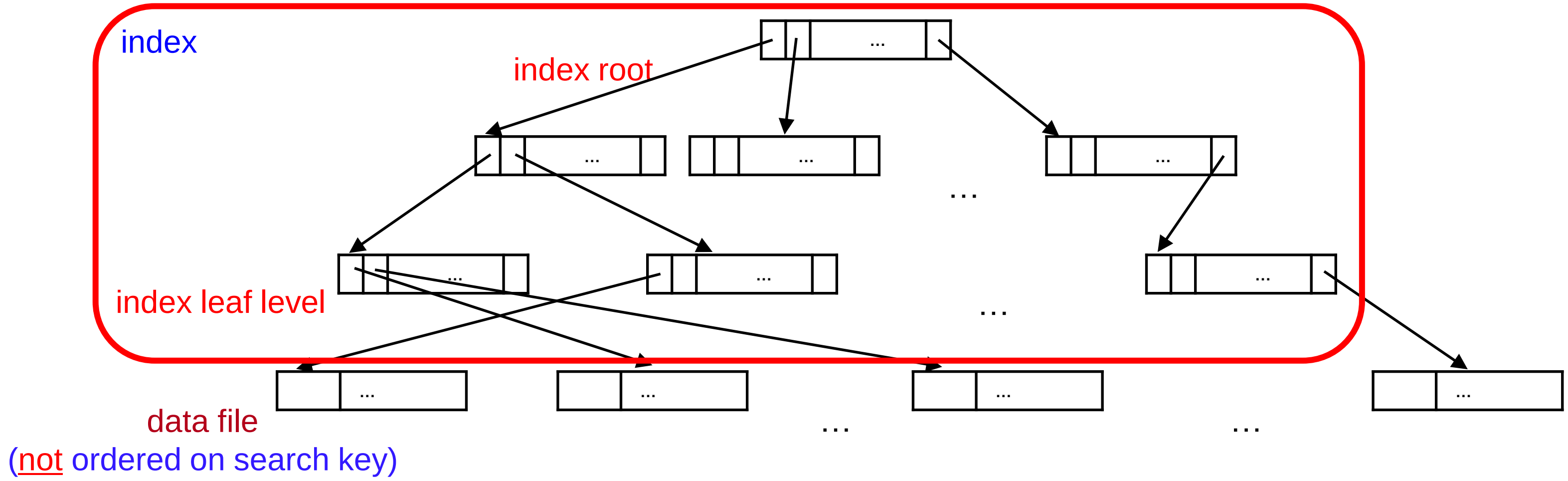


NON-CLUSTERING/SECONDARY INDEX

– A non-clustering/secondary index is an index for which the data file is not ordered on the search key of the index.

☞ **There can be several secondary indexes for a data file.**

☞ **A secondary index must be dense.**



index entries/page: 100

SECONDARY INDEX EXAMPLE

For the catalog of Hong Kong residents, we also want to be able to find records given a name. **How to find the record fast?**

Solution: Build another index on the name

- Since the file is ordered on hkid, the new index must be secondary (since the file is not ordered on the search key) and dense (there is one entry for every search-key value).
- Assuming that all names are distinct (*not realistic!*), the index will contain 8 million entries.
- Assuming that the fan-out is again 100, the cost of finding a record given the name is $\underbrace{\lceil \log_{100}(8{,}000{,}000) \rceil}_{\text{height of the index}} + 1 = 4 + 1 = \underline{5}$ page I/Os.

• A secondary index is almost as good as a primary index (in terms of cost) when retrieving a single record.

- However, it may be very expensive when retrieving many records (e.g., for range queries) and it requires more storage space.

INDEX ON NON-CANDIDATE SEARCH KEY

We want to build an index on name, but there may be several people with the same name.

⟹ Zero, one or more records are retrieved.

☞**Not a problem if the index is clustering and sparse.**

How would you do it?

INDEX ON NON-CANDIDATE SEARCH KEY

If the index is non-clustering (secondary) and dense.

Option 1: Use variable length index entries

– Each entry contains a name and pointers to all records with this name.

Example: <Jackie Chan, *pointer*$_1$, *pointer*$_2$, …., *pointer*$_n$>

Problem: Complicated implementation as a file organization that supports records of variable length is needed.

Option 2: Use multiple index entries per name

– There is an entry for every person, if he/she shares the same name with other people.

Example: <Jackie Chan, *pointer*$_1$>, <Jackie Chan, *pointer*$_2$>, …, <Jackie Chan, *pointer*$_n$>

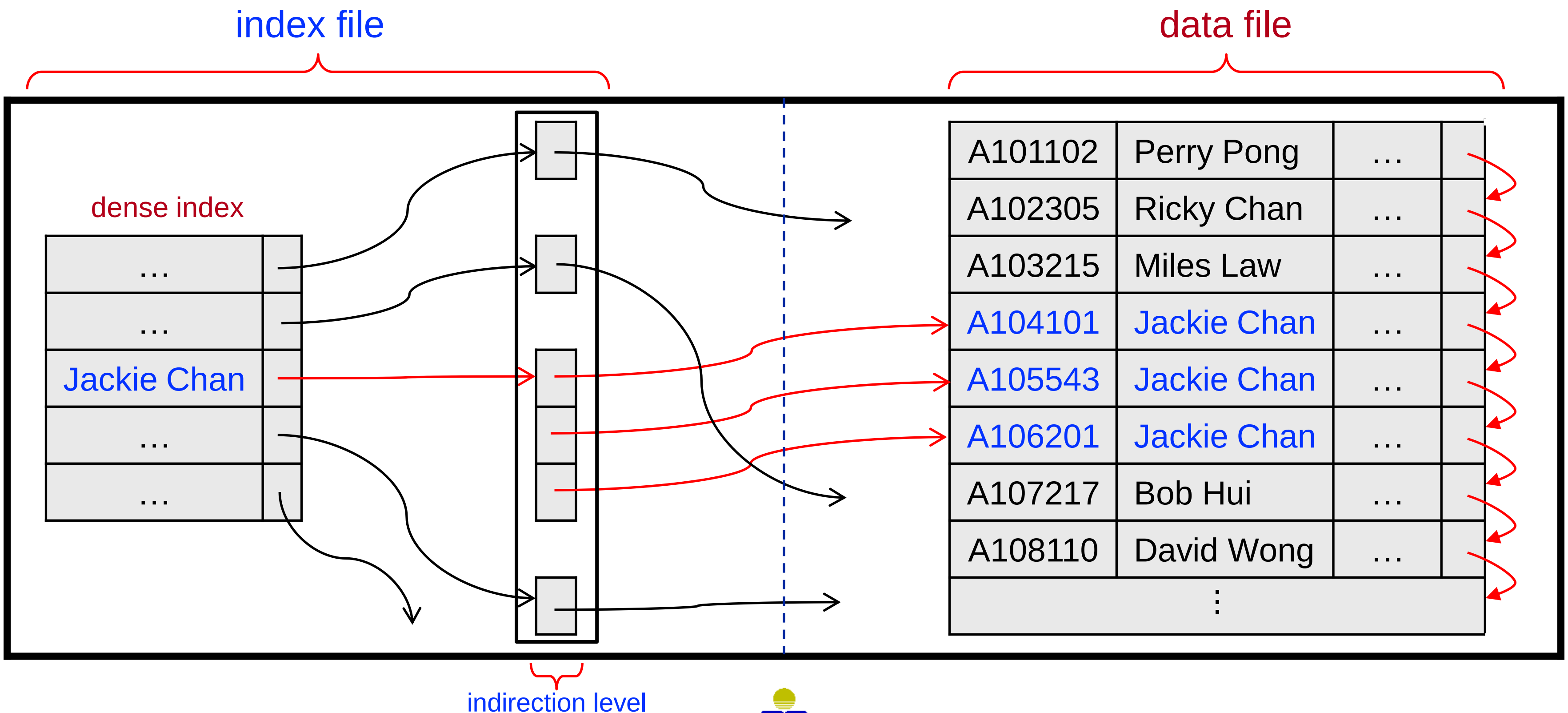
Problem: Redundancy – the name repeats many times.

INDEX ON NON-CANDIDATE SEARCH KEY (cont'd)

Option 3: Use an extra level of indirection (most common approach)

– An index entry points to a list that contains the pointers to all the records with the same name ⟹ requires one additional page access.

☞ **Also called an inverted file.**



INDEX ON COMPOSITE SEARCH KEY

- If a query often uses certain combinations of attributes together (e.g., hkid, age), then creating an index on this attribute combination can speed up retrieval.

  ☞ **A composite search key is a search key that consists of more than one attribute.**

- The index structure for a composite search key is the same as that for a single attribute search key.

- For a composite search key, the ordering of search key values is the lexicographic ordering.

  Example: For two search keys (a_1, a_2) and (b_1, b_2):

  $(a_1, a_2) < (b_1, b_2)$ if *either* $a_1 < b_1$ *or* $a_1 = b_1$ and $a_2 < b_2$

  – This is basically the same as alphabetic ordering of words.

INDEXING: INTRODUCTION

EXERCISE 2

$bf = \lfloor$# bytes per page / # bytes per record$\rfloor$
pages = $\lceil$# records / $bf_r\rceil$

Film records: 30,000
Actor records: 100,000
Page size: 512 bytes
Pointer size: 6 bytes
Film record size: 84 bytes; $bf_F = 6$
Actor record size: 28 bytes; $bf_A = 18$

EXERCISE 2

Assume the Actor file is ordered on name and we want to create an ordered index on id (4 bytes) where each index entry has the form <id, *pointer*>.

a) What is bf_{Aindex} if the index is single-level?

$\lfloor$512 bytes per page / (4 + 6) bytes per index entry$\rfloor$ = 51

bf_{Aindex}:

b) How many index entries are needed? (Briefly explain your answer.)

100,000 **Why?**

index entries:

Explanation: A dense index is needed (i.e., one entry per Actor record) since the file is ordered on name, not on id.

$\lceil$100,000 Actor records / 51 index entries per page$\rceil$ = 1961

c) How many pages are required for the Actor index entries?

pages needed:

$bf = \lfloor$# bytes per page / # bytes per record$\rfloor$
pages = $\lceil$# records / $bf_r\rceil$

Film records: 30,000
Actor records: 100,000
Page size: 512 bytes
Pointer size: 6 bytes
Film record size: 84 bytes; $bf_F = 6$
Actor record size: 28 bytes; $bf_A = 18$

EXERCISE 2 (cont'd)

d) What is the page I/O cost of retrieval based on a single id value using the Actor index (e.g., “Find actor with id 100”)?

Actor file ordered on name.

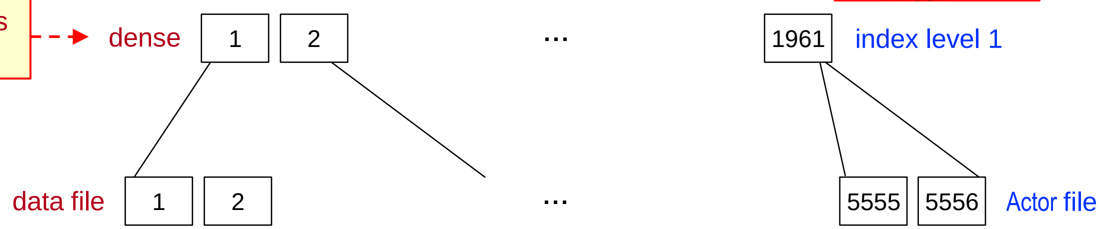
page I/O cost: $\lceil \log_2 1961 \rceil + 1 = \underline{12}$

Actor single-level index

$bf_{Aindex} = 51$

100,000 index entries, 51 entries/page ⟹ 1961 pages
(need to point to every record)

100,000 records, 18 records/page ⟹ 5556 pages



$bf = \lfloor \text{\# bytes per page / \# bytes per record} \rfloor$
$\text{\# pages} = \lceil \text{\# records} / bf_r \rceil$

Film records: 30,000
Actor records: 100,000
Page size: 512 bytes
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EXERCISE 2 (cont'd)

e) If the single-level index is converted into a multi-level index, how many levels are needed (assuming full pages)? (Briefly explain your answer.)

index levels: 3

Explanation:

39 index entries, 51 entries/page ⟹ 1 page (only need to point to every page)

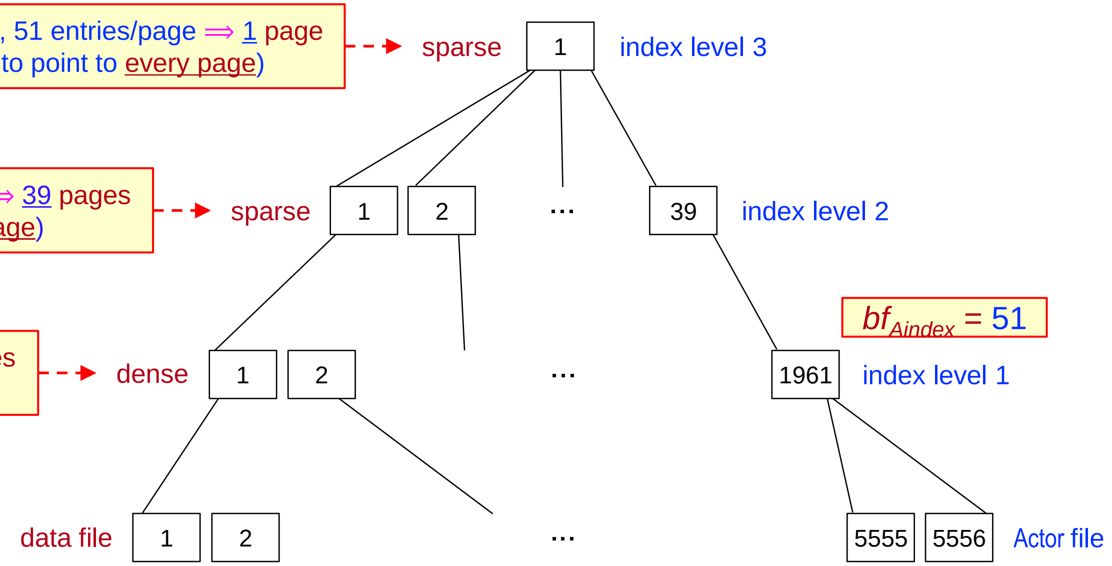
1961 index entries, 51 entries/page ⟹ 39 pages (only need to point to every page)

100,000 index entries, 51 entries/page ⟹ 1961 pages (need to point to every record)

100,000 records, 18 records/page ⟹ 5556 pages

Actor multi-level index

$bf_{Aindex} = 51$



$bf = \lfloor \text{\# bytes per page / \# bytes per record} \rfloor$
$\text{\# pages} = \lceil \text{\# records} / bf_r \rceil$

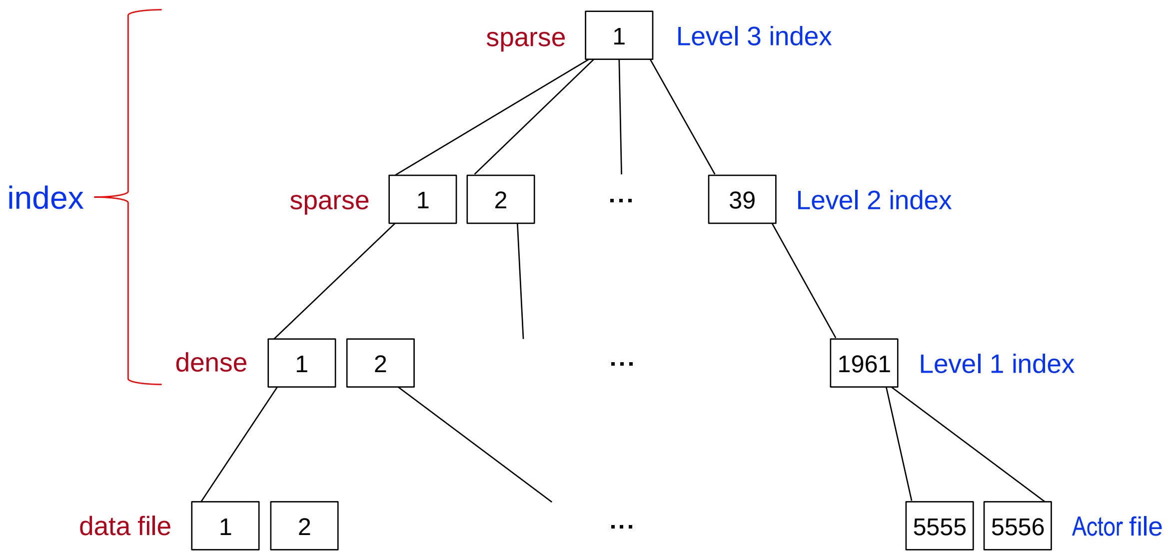
Film records: 30,000
Actor records: 100,000
Page size: 512 bytes
Pointer size: 6 bytes
Film record size: 84 bytes; $bf_F = 6$
Actor record size: 28 bytes; $bf_A = 18$

EXERCISE 2 (cont'd)

f) Using the multi-level index, what is the page I/O cost of answering the query "Find the actor with id 100"?

page I/O cost: 4 **Why?**

Explanation: 3 page I/Os for the index plus 1 page I/O to retrieve the record.



INDEXING 1

EXERCISES 3, 4

Upload your completed exercise worksheet to Canvas by March 12^{th} **11 p.m.**