Exercise 1: Assume that this table contains the only set of tuples that may appear in a relation \( R(X, Y, V, W) \). Which of the following FDs hold in \( R \)?

<table>
<thead>
<tr>
<th>tuple</th>
<th>X</th>
<th>Y</th>
<th>V</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x₁</td>
<td>y₁</td>
<td>v₁</td>
<td>w₁</td>
</tr>
<tr>
<td>2</td>
<td>x₁</td>
<td>y₁</td>
<td>v₂</td>
<td>w₂</td>
</tr>
<tr>
<td>3</td>
<td>x₂</td>
<td>y₁</td>
<td>v₁</td>
<td>w₃</td>
</tr>
<tr>
<td>4</td>
<td>x₂</td>
<td>y₁</td>
<td>v₃</td>
<td>w₄</td>
</tr>
</tbody>
</table>

- \( X \rightarrow X \) ☐ Yes ☐ No
- \( Y \rightarrow X \) ☐ Yes ☐ No
- \( X \rightarrow Y \) ☐ Yes ☐ No
- \( V \rightarrow X \) ☐ Yes ☐ No
- \( X \rightarrow V \) ☐ Yes ☐ No
- \( Y \rightarrow X \) ☐ Yes ☐ No
- \( W \rightarrow X \) ☐ Yes ☐ No
- \( X \rightarrow W \) ☐ Yes ☐ No
- \( Y \rightarrow X \) ☐ Yes ☐ No

Exercise 2: In Exercise 1, we assumed that we know all possible records in the table, which is not usually true. In general, by looking at an instance of a relation, we can only tell FDs that are not satisfied. List 5 FDs that are not satisfied in the table.

- \( X \rightarrow A \)
- \( Y \rightarrow B \)
- \( V \rightarrow C \)
- \( W \rightarrow D \)
- \( X \rightarrow E \)

Exercise 3: Given relation schema \( R(X, Y, U, V, W) \) and \( F = \{ X \rightarrow Y, UV \rightarrow W, V \rightarrow X \} \)

a) Determine the closure of each attribute.

- \( X^+ = \) (generated attribute)
- \( U^+ = \) (generated attribute)
- \( W^+ = \) (generated attribute)
- \( Y^+ = \) (generated attribute)
- \( V^+ = \) (generated attribute)

b) What are the candidate keys of \( R \)?

Exercise 4: Given relation schema \( R(A, B, C, G, H, I) \) and \( F = \{ A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H \} \)

a) Is \( AG \) a (super)key of \( R \) given \( F \)?

b) Is \( AG \) a candidate key?

c) Does \( A^+ \rightarrow R \) hold?

d) Does \( G^+ \rightarrow R \) hold?
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Exercise 5: Given relation schema R(A, B, C, D, E) and \( F = \{ A \rightarrow B, AB \rightarrow C, D \rightarrow AC \} \)

a) Determine the following attribute closures.
   \[
   A^+ = C^+ = E^+ = \]
   \[
   B^+ = D^+ = \]

b) What are the candidate keys of R?

c) Find a canonical cover of \( F \).

Exercise 6: We want to create the database for a bank that contains accounts (A), branches (B) and customers (C). We are given the following constraints.

i. An account cannot be shared by multiple customers.
ii. Two different branches do not have the same account.
iii. Each customer can have at most one account in a branch (but different accounts in different branches).

a) What are the functional dependencies implied by the above constraints?

b) What are the candidate keys?

Exercise 7: Given relation schema R(A, B, C). Assume we do not know the keys of the relation. Write a valid SQL query whose result can be used to determine if A is a potential candidate key. Explain how to interpret the query result to determine if A is a potential candidate key.