EXERCISE 1

Given: \( R(A, B, C, D, E) \) \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad F = \{A \rightarrow BC\}

Decomposition: \( R_1(A, B, C) \) and \( R_2(A, D, E) \)

a) Is the decomposition lossless? Why? \quad (iff \( R_1 \cap R_2 \rightarrow R_1 \) or \( R_1 \cap R_2 \rightarrow R_2 \))

Yes \quad The common attribute A is a key for \( R_1 \).

b) Is the decomposition dependency preserving? Why? \quad (iff \( (\bigcup F_i)^+ = F^+ \))

Yes \quad A\rightarrow BC is preserved in \( R_1 \).

c) Is the decomposition \( R_1(A, B, C) \) and \( R_2(C, D, E) \) lossless? Why?

No \quad C is not a key for any table.
EXERCISE 2

Given: \( R(A, B, C, D, E) \) \( F = \{A \rightarrow BC, CD \rightarrow E, B \rightarrow D, E \rightarrow A\} \)

Decomposition: \( R_1(A, B, C) \) and \( R_2(A, D, E) \)

a) Is the decomposition lossless? \( (iff \ R_1 \cap R_2 \rightarrow R_1 \ or \ R_1 \cap R_2 \rightarrow R_2) \)
   
   Yes  The common attribute A is a key for \( R_1 \).

b) Is the decomposition dependency preserving? \( (iff \ (\cup F_i)^+ = F^+) \)
   
   No   We lose \( CD \rightarrow E \) and \( B \rightarrow D \).
**EXERCISE 3**

a) Given: \( R(A, B, C, D) \quad F = \{AB \rightarrow CD, B \rightarrow C\} \)

Is \( R \) in 2NF? Why?

Key: \( AB \quad AB^+ = \{A, B, C, D\} \quad B^+ = \{B, C\} \)

No For \( B \rightarrow C \), \( B \) is a proper subset of the key \( AB \) and \( C \) is non-prime. So, \( R \) is not in 2NF.

b) Given: \( R(A, B, C, D) \quad F = \{AB \rightarrow CD, C \rightarrow D\} \)

Is \( R \) in 2NF? Why?

Key: \( AB \quad AB^+ = \{A, B, C, D\} \quad C^+ = \{C, D\} \)

Yes For \( C \rightarrow D \), \( C \) is not a proper subset of the key, so \( R \) is in 2NF.
EXERCISE 4

Identify the candidate key(s) and the current highest normal form for each of the following relation schemas given their corresponding FDs.

a) \( R(A, B, C, D, E) \)

\[ F = \{A \rightarrow B, C \rightarrow D\} = F^+ \]

\( A^+ = \{A, B\} \quad C^+ = \{C, D\} \)

Candidate keys: ACE

⇒ For \( A \rightarrow B \) and \( C \rightarrow D \)
  i. A and C are proper subsets of the candidate key ACE (both FDs fail 1st 2NF test).
  ii. both B and D are not prime attributes of R (both FDs fail 2nd 2NF test).

☞ Both FDs violate 2NF.

Normal form: 1NF
EXERCISE 4 (cont’d)

Identify the candidate key(s) and the current highest normal form for each of the following relation schemas given their corresponding FDs.

b) $R(A, B, C)$

$F = \{AB \rightarrow C, C \rightarrow B\} = F^+$

$AB^+ = \{A, B, C\}$  
$C^+ = \{C, B\}$

Candidate keys: $AB, AC$

⇒ For $AB \rightarrow C$,
  $C$ is a prime attribute of $R$  
  (FD passes 2nd 2NF and 3NF tests).

⇒ For $C \rightarrow B$,
  $B$ is a prime attribute of $R$  
  (FD passes 2nd 2NF and 3NF tests).

☞ Both FDs satisfy 3NF.

Normal form: 3NF
Identify the candidate key(s) and the current highest normal form for each of the following relation schemas given their corresponding FDs.

c) \( R(A, B, C, F) \)

\[ F = \{AB \rightarrow C, C \rightarrow F\} = F^+ \]

\[ AB^+ = \{A, B, C, F\} \quad \text{C}^+ = \{C, F\} \]

**Candidate keys:** \( AB \)

\( \Rightarrow \) For \( AB \rightarrow C \)

i. \( AB \) is **not** a proper subset of a candidate key (FD passes 1st 2NF test);

ii. \( AB \) is a superkey for \( R \) (FD passes 1st 3NF test).

\( \Rightarrow \) For \( C \rightarrow F \)

i. \( C \) is **not** a proper subset of a candidate key (FD passes 1st 2NF test);

ii. \( C \) is not a superkey of \( R \) (FD fails 1st 3NF test);

iii. \( F \) is **not** a prime attribute (FD fails 2nd 3NF test).

\( \Rightarrow \) Both FDs satisfy 2NF.

Normal form: 2NF
Exercise 5: Decompose $R(A, B, C, D, E, F, G)$ into 3NF relations for the FD set $F = \{AB \rightarrow CD, C \rightarrow EF, G \rightarrow A, G \rightarrow F, CE \rightarrow F\}$.

Exercise 6: Decompose $R(A, B, C, D)$ into 3NF and BCNF relations for each of the following FD sets.
   a) $F = \{B \rightarrow C, D \rightarrow A\}$  
   b) $F = \{ABC \rightarrow D, D \rightarrow A\}$

Exercise 7: Given: $Sale(\text{customer, store, product, price})$ and the constraints:
   A customer buys from only one store.
   There is a unique price for each product in a store, but the same product can have a different price in different stores.
   a) What are the FDs implied by the above description?
   b) What are the candidate keys?
   c) Explain why $Sale$ is not in 3NF.
   d) Decompose $Sale$ into 3NF relation schemas.
   e) Is the decomposition dependency preserving? Why?
EXERCISE 5

Decompose $R(A, B, C, D, E, F, G)$ into 3NF relations for the FD set $F = \{AB \rightarrow CD, C \rightarrow EF, G \rightarrow A, G \rightarrow F, CE \rightarrow F\}$.

Attribute closures

$AB^+ = \{A, B, C, D, E, F\}$  
$C^+ = \{C, E, F\}$  
$G^+ = \{G, A, F\}$  
$CE^+ = \{C, E, F\}$

Candidate key: $BG$  
From $G \rightarrow A$ we can infer $BG \rightarrow AB$ using IR2.  
$BG^+ = \{B, G, A, C, D, E, F\}$  
All FDs violate 3NF!

Canonical cover: $F_C = \{AB \rightarrow CD, C \rightarrow EF, G \rightarrow AF\}$

3NF decomposition:  
$R_1(A, B, C, D)$  
$R_2(C, E, F)$  
$R_3(G, A, F)$  
$R_4(B, G)$  
$\Rightarrow$ due to the candidate key
EXERCISE 6

Decompose $R(\text{A, B, C, D})$ into 3NF and BCNF relations for each of the following FD sets.

a) $F = \{\text{B} \rightarrow \text{C}, \text{D} \rightarrow \text{A}\}$

Attribute closures: $B^+ = \{\text{B, C}\}$ $D^+ = \{\text{D, A}\}$

Candidate keys: BD

Canonical cover: $F_C = \{\text{B} \rightarrow \text{C}, \text{D} \rightarrow \text{A}\}$

3NF Decomposition

$R_1(\text{B, C})$

$R_2(\text{D, A})$

$R_3(\text{B, D}) \implies$ due to the candidate key

BCNF Decomposition

$R_1(\text{B, C})$

$R_2(\text{D, A})$

$R_3(\text{B, D})$
Decompose $R(A, B, C, D)$ into 3NF and BCNF relations for each of the following FD sets.

b) $F = \{ABC \rightarrow D, D \rightarrow A\}$

Attribute closures: $ABC^+ = \{A, B, C, D\}$  \(D^+ = \{D, A\}\)

Candidate keys: $ABC, BCD$  \textbf{Using IR2:} $BCD \rightarrow ABC$

Canonical cover: $F_C = \{ABC \rightarrow D, D \rightarrow A\}$

**3NF Decomposition**

- Already in 3NF
- $R(A, B, C, D)$

Dependency preserving? \hspace{1cm} Yes

Is this BCNF decomposition OK? \hspace{1cm} $R_1(A, B, C)$ and $R_2(D, A)$ \textbf{No} Why?

**BCNF Decomposition**

- Only $D \rightarrow A$ violates BCNF
- $R_1(B, C, D)$  \(R_2(D, A)\)

Dependency preserving? \hspace{1cm} No The FD $ABC \rightarrow D$ is lost.

The FD $ABC \rightarrow D$ is lost, and we need to join $R_1$ and $R_2$ on $A$, but $A$ is not a key of $R_2$.

\rightarrow \textbf{Not a lossless decomposition!}
EXERCISE 7

Given: Sale(customer, store, product, price) and the constraints:
   A customer buys from only one store.
   There is a unique price for each product in a store, but the same product can have a different price in different stores.

a) What are the FDs implied by the above description?
   customer → store
   store, product → price

b) What are the candidate keys?
   {customer, product}

Since customer, product → store, product (IR2)
and customer, product → price (IR3)

Also \{customer, product\}^+ = \{customer, product, store, price\}
EXERCISE 7 (cont’d)

Given: Sale(customer, store, product, price)
A customer buys from only one store.
There is a unique price for each product in a store, but the same product can have a different price in different stores.

c) Explain why Sale is not in 3NF.
Both FDs violate 3NF. 

\[ F = \{ \text{customer} \rightarrow \text{store}; \ \text{store, product} \rightarrow \text{price} \} \]

The LHS of the FDs are not superkeys; the RHS are not prime attributes of Sale.

d) Decompose Sale into 3NF relation schemas.
\[ R_1(\text{customer, store}) \quad R_2(\text{store, product, price}) \]

e) Is the decomposition dependency preserving? Why?
Yes Each FD is preserved in a relation schema, BUT ... (next page).
The decomposition $R_1(\text{customer, store}), R_2(\text{store, product}, \text{price})$ is **lossy** because the common attribute $\text{store}$ is not a key of any table.

The two decomposed relations do not generate the original one if joined (on the common $\text{store}$ attribute). The join result contains 4 records instead of 3 as in the original relation.

**What is the problem?** None of the fragments contains the candidate key $(\text{customer, product})$.

**Solution?** Include an additional table $R_3(\text{customer, product})$ containing the candidate key in the decomposition.