5. Functional Dependencies

Exercises
Assume the following table contains the 
only set of tuples that may appear in a 
table R. Which of the following FDs hold 
in R:

<table>
<thead>
<tr>
<th>tuple</th>
<th>X</th>
<th>Y</th>
<th>V</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x_1</td>
<td>y_1</td>
<td>v_1</td>
<td>w_1</td>
</tr>
<tr>
<td>2</td>
<td>x_1</td>
<td>y_1</td>
<td>v_2</td>
<td>w_2</td>
</tr>
<tr>
<td>3</td>
<td>x_2</td>
<td>y_1</td>
<td>v_1</td>
<td>w_3</td>
</tr>
<tr>
<td>4</td>
<td>x_2</td>
<td>y_1</td>
<td>v_3</td>
<td>w_4</td>
</tr>
</tbody>
</table>

• \{X\}→\{X\}
Yes – trivial (holds in any table)

• \{X\}→\{Y\}
Yes – all values of Y are identical

• \{X\}→\{V\}
No – see first two rows

• \{X\}→\{W\}
No – same as before

• \{Y\}→\{X\}
No (same for \{Y\}→\{V,W\})

• \{W\}→\{X\}
Yes – all W values are different

• \{X,V\}→\{Y\}
Yes – X alone determines Y

• \{Y,V\}→\{X\}
No – see rows 1 and 3
• In the previous example, we assumed that we know all possible records in a table, which is not usually true.
• In general by looking at an instance of a relation, we can only tell FDs that are NOT satisfied.
• List 5 FDs that are not satisfied in the table:

\[
\{B\} \rightarrow \{A\} \\
\{B\} \rightarrow \{C\} \\
\{C\} \rightarrow \{A\} \\
\{A\} \rightarrow \{C\} \\
\{A,B\} \rightarrow \{C\}
\]
Exercise

• In reality, FDs are given implicitly in the form of constraints when designing a database.
• Let a relation R(Title, Theater, City) where title is the name of a movie, theater is the name of a theater playing the movie and city is the city where the theater is located.
• We are given the following constraints:
  – Two different cities cannot have theaters with the same name.
  – Two different theaters in the same city cannot play the same movie.
  – A theater can play many movies (e.g., cineplex).
• Write the set of functional dependencies implied by the above assumptions.

{Theater} → {City} (if we know the theater, we know the city where it is located – the opposite is not true as a city can have many theaters)

{City, Title} → {Theater}

Can you identify the candidate key(s)

City, Title and Theater, Title
Exercise

• We want to create the database for a bank that contains accounts (A), branches (B) and customers (C). We are given the following constraints:
  • An account cannot be shared by multiple customers.
  • Two different branches do not have the same account.
  • Each customer can have at most one account in a branch (but different accounts in different branches).
• Write the functional dependencies implied by the above constraints

  \{\text{Account}\} \rightarrow \{\text{Customer}\}
  \{\text{Account}\} \rightarrow \{\text{Branch}\}
  \{\text{Customer, Branch}\} \rightarrow \{\text{Account}\}

Write the candidate key(s):
Customer, Branch and Account
Exercise

Let R(A,B,C). Assume that we do not know the keys of the table.

How would you test if A is a candidate key of R with a SQL query?

SELECT A
FROM R
GROUP BY A
HAVING COUNT(*) > 1

If this query gives a non-empty result, then A is not a key.
If the result is empty we cannot be sure.

What about testing if the dependency \{A\} \rightarrow \{B\} holds in R?
Same as before but replace the last line with HAVING COUNT(DISTINCT B) > 1
Which are the FDs implied in the above ER diagram?

A→B
A→C
C→A
D→C
EC→F
Exercise

• Let the rule: if X → Z and Y → Z, then X → Y
Show that this rule is not sound (correct) with a counter-example.

Let's use R(X,Y,Z). We want to find an instance of R where the rule is violated.

Let's say that R contains just two tuples:
(x1, y1, z1)
(x1, y2, z1)

{X} → {Z} and {Y} → {Z} hold, but {X} → {Y} is not true.
Consider a relation R(X,Y,U,V,W) with the following set of dependencies:

\{X \rightarrow Y, \{U,V\} \rightarrow \{W\}, \{V\} \rightarrow \{X\}\}

Find the closure of each attribute:

- \(X^+ = \{X,Y\}\)
- \(Y^+ = \{Y\}\)
- \(U^+ = \{U\}\)
- \(V^+ = \{V,X,Y\}\)
- \(W^+ = \{W\}\)

What is the primary key of R?:

- UV
• \( R = (A, B, C, G, H, I) \)
  
  \( F = (A \implies B \quad A \implies C \quad CG \implies H \quad CG \implies I \quad B \implies H} \)

• Is \( AG \) a (super)key of \( R \) given \( F \)?

\( (AG^+) \)
1. Result= AG
2. Result= ABCG \((A \implies C; A \implies B \text{ and } A \subseteq AG)\)
3. Result= ABCGH \((CG \implies H \text{ and } CG \subseteq AGBC)\)
4. Result= ABCGHI \((CG \implies I \text{ and } CG \subseteq AGBCH)\)

• Is \( AG \) a candidate key?
1. \( AG \implies R \)
2. Does \( A^+ \implies R \)?
3. Does \( G^+ \implies R \)?
Exercise

Let the relation schema $R(A,B,C,D,E)$ and the set of functional dependencies: $F = \{\{A\} \rightarrow \{B\}, \{A,B\} \rightarrow \{C\}, \{D\} \rightarrow \{A,C\}\}$:

Find the canonical cover of $F$

\{A \rightarrow B, AB \rightarrow C, D \rightarrow AC\}
\{A \rightarrow B, A \rightarrow C, D \rightarrow AC\}
\{A \rightarrow BC, D \rightarrow A\}

Compute the attribute closures

$A^+ = \{A,B,C\}$
$B^+ = \{B\}$, $C^+ = \{C\}$
$D^+ = \{D,A,B,C\}$
$E^+ = \{E\}$

What is the primary key of $R$?

DE
Exercise

- Let \( R(A,B,C,D,E) \), the FD \( \{ A \} \rightarrow \{ B,C \} \) and the decomposition:
  \( R1(A,B,C) \) and \( R2(A,D,E) \)

- **Is the decomposition lossless?**
  Yes because the common attribute \( A \) is a key for \( R1 \)

- **Is the decomposition dependency preserving?**
  Yes, \( \{ A \} \rightarrow \{ B,C \} \) is preserved in \( R1 \)

- **Is the decomposition \( R1(A,B,C) \) and \( R2(C,D,E) \) lossless?**
  No – \( C \) is not a key for any table
Exercise

Let $R = (A, B, C, D, E)$, $F = \{A \rightarrow BC, CD \rightarrow E, B \rightarrow D, E \rightarrow A\}$ and the decomposition $R_1 = (A, B, C)$ $R_2 = (A, D, E)$

• **Is the decomposition lossless?**
  Yes because the common attribute $A$ is a key for $R_1$

• **Is the decomposition dependency preserving?**
  No: we loose $CD \rightarrow E$ and $B \rightarrow D$