

We are given two functions h_1 and h_2 as two admissible heuristic functions, such that $h_1(n) > h_2(n)$ for all nodes n in the searching problem domain.

Let C^* be the optimal cost found to the goal state, therefore we have the following three observations.

- For all nodes n that have $f(n) < C^*$, A* algorithm will surely expand these nodes.
- For some nodes n that have $f(n) = C^*$, A* algorithm may not expand these nodes.
- For all nodes n that have $f(n) > C^*$, A* algorithm will surely not expand these nodes.

Based on the above observation, if a node n was expanded using the heuristic function h_1 , it means that $f(n) < C^*$, therefore $h_1(n) < C^* - g(n)$, since $h_1(n) > h_2(n)$, so we also have $h_2(n) < C^* - g(n)$, so this node will also be expanded by h_2 . In other words, all nodes that have been expanded by h_1 will surely be expanded by h_2 .

Q.E.D.