COMP 271 Design and Analysis of Algorithms
2003 Spring Semester
Questions for Ninth Tutorial – Week 0f May 14, 2003.
All problems are from Question Bank 5 – Solutions available there

1. Prove that P $\subseteq$ Co-NP.

2. Prove that if NP $\neq$ Co-NP then P $\neq$ NP.

3. For each of the following assertions, indicate whether it is **True**: known to be true, **False**: known to be false, or **Unknown**: unknown based on our current scientific knowledge. In each case provide a short explanation for your answer.

   (i) No problems in NP can be solved in polynomial time.

   (ii) Every NP-complete problem requires at least exponential time to be solved.

   (iii) X is in NP and X $\leq_P$ SAT. Then X is NP-complete.

4. Given an undirected graph $G = (V, E)$, a *feedback vertex set* is a subset of vertices such that every simple cycle in G passes through one of these vertices. The feedback vertex set problem (FVS) is: Given a graph G and an integer k, does G contain a feedback vertex set of size at most k?

   Show that FVS is in NP. That is, given a graph G that has a FVS of size k, give a certificate, and show how you would use this certificate to verify the presence of a FVS of size k in polynomial time.

   (Hint: The certificate should be a set $V' \subseteq V$ with $|V'| = k$. You need to show a polynomial time algorithm that tests whether V' is a FVS or not. Note that since a graph can hace exponentially many cycles you can not just do the simple thing of checking every cycle.)

5. The *set cover problem* is: Given a finite set X and a collection of sets F whose elements are chosen from X, and given an integer k, does there exist a subset $C \subseteq F$ of k sets such that

$$X = \bigcup_{S \in C} S.$$

   Prove that the set cover problem is NP-complete. (*Hint:* Reduce from Vertex-Cover.)