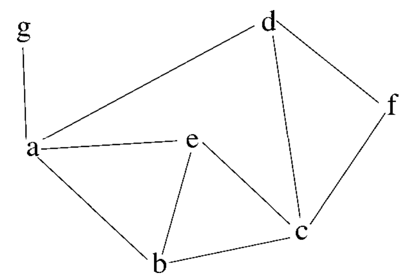
COMP 271 Design and Analysis of Algorithms
2003 Spring Semester
Questions for Fifth Tutorial – March 14, 2003.

1. The adjacency list representation of a graph G, which has 7 vertices and 10 edges, is:

$$a :\rightarrow d,\ e,\ b,\ g \qquad b :\rightarrow e,\ c,\ a$$
$$c :\rightarrow f,\ e,\ b,\ d \qquad d :\rightarrow c,\ a,\ f$$
$$e :\rightarrow a,\ c,\ b \qquad f :\rightarrow d,\ c$$
$$g :\rightarrow a$$



   (a) Show the tree produced by depth-first search when it is run on the graph G, using vertex a as the source. You *must* use the adjacency list representation given above. (Recall that the DFS tree can depend on the order of vertices in the adjacency lists; for this problem you are required to use the adjacency lists as given above.) Note taht in this case you are running DFS on an *undirected* graph and not a directed one so you will have to slightlu modify the algorithm you learnt in class.

   (b) In the DFS tree of item (a), show the edges of the graph G which are not present in the DFS tree by *dashed* lines.

2. Show that depth-first search of an undirected graph G can be used to identify the connected components of G. More precisely, show how to modify depth-first search so that each vertex v is assigned an integer label $cc[v]$ between 1 and k, where k is the number of connected components of G, such that $cc[u] = cc[v]$ if and only if u and v are in the same connected component.

3. Prove that if G is a connected undirected graph, then each of its edges is either in the depth-first search tree or is a back edge.

4. Give a simple example of a directed graph with negative-weight edges for which Dijkstra's algorithm produces incorrect answers. Why does the correctness proof of Dijkstra's algorithm not go through when negative-weight edges are allowed?

5. Another way to perform topological sorting on a directed acyclic graph $G = (V, E)$ is to repeatedly find a vertex of in-degree 0 (*why does such a vertex always exist?*) output it, and remove it and all of its outgoing edges from the graph. Explain how to implement this idea so that it runs in time $O(V + E)$. What happens to this algorithm if G has cycles?