COMP 271 Design and Analysis of Algorithms
2003 Spring Semester
Questions for Fourth Tutorial – March 7, 2003.
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1. Given an undirected graph $G = (V, E)$, recall that the *complement*, $\overline{G}$, is a graph (V, E') such that for all $u \neq v$, $\{u, v\} \in E'$ if and only if $\{u, v\} \notin E$. Prove that either G or $\overline{G}$ is connected.

2. Let $G = (V, E)$ be a connected undirected graph. Let s be any vertex of V and run the BFS algorithm on G starting at s. Show that if $\{u, v\}$ is any edge of E then $|d(u) - d(v)| \leq 1$.

3. An (undirected) graph $G = (V, E)$ is *bipartate* if there exists some $S \subset V$ such that, for every edge $\{u, v\} \in E$, either (i) $u \in S$, $v \in V - S$ or (ii) $v \in S$, $u \in V - S$.

   Let $G = (V, E)$ be a connected graph. Design an $O(n + e)$ algorithm that checks whether G is bipartite. *Hint: Run BFS.*

   How can you modify your algorithm so that it also works for unconnected graphs?

4. Give an example of a directed graph G in which a vertex u of G ends up in a depth-first tree containing only u, even though u has both incoming and outgoing edges. Your example graph should have no self-loops.