COMP 271 Design and Analysis of Algorithms
2003 Spring Semester
Questions for Third Tutorial – 28 Feb, 2003

- **Problem 1. CLRS p. 192 prob 9.3-8.** Professor Olay is consulting for an oil company, which is planning a large pipeline running east to west through an oil field of n wells. From each well, a spur pipeline is to be connected directly to the main pipeline along a shortest path (either north or south). Given the x, y coordinates of the wells, how should the professor pick the optimal location of the main pipeline (the one that minimizes the total length of the spurs). Show that the optimal location can be determined in linear time.

- **Problem 2. CLRS p. 193 prob 9-1.** *Largest i numbers in sorted order.*

  Given a set of n numbers, we wish to find the i largest in sorted order using a comparison-based algorithm. Find the algorithm that implements each of the following methods with the best asymptotic worst-case running time and analyze the running times of the algorithms in terms on n and i.

  1. Sort the numbers and list the i largest.
  2. Build a max-priority queue (i.e., a heap) from the numbers and call EXTRACT-MAX i times.
  3. Use a selection algorithm to find the ith largest number, partition around that number and sort the i largest numbers.

- **Problem 3. CLRS p193, prob 9-2**. *Weighted Median*

  For n distinct elements $x_1, x_2, \ldots, x_n$ with positive weights $w_1, w_2 \ldots, w_n$ such that $\sum_{i=1}^{n} w_i = 1$, the *weighted (lower) median* is the element x_k satisfying

  $$\sum_{x_i < x_k} w_i < \frac{1}{2} \quad \text{and} \quad \sum_{x_i > x_k} w_i \le \frac{1}{2}.$$

  1. Argue that the median of $x_1, x_2, \ldots, x_n$ is the weighted median of the x_i weights $w_i = 1/n$ for $i = 1, 2, 3 \ldots, n$.
  2. Show how to compute the weighted median of n elements in $O(n \log n)$ worst case time using sorting.
  3. Show how to compute the weighted median in $\Theta(n)$ worst-case time using a linear-time median finding algorithm such as SELECT.