COMP 271 Design and Analysis of Algorithms
2003 Spring Semester
Questions for First Tutorial

Problem 1. Suppose $T_1(n) = O(f(n))$ and $T_2(n) = O(f(n))$. Which of the following are true? Justify your answers.

(a) $T_1(n) + T_2(n) = O(f(n))$

(b) $\frac{T_1(n)}{T_2(n)} = O(1)$

(c) $T_1(n) = O(T_2(n))$

Problem 2. For each pair of expressions (A, B) below, indicate whether A is O, Ω, or Θ of B. Note that zero, one, or more of these relations may hold for a given pair; list all correct ones. Justify your answers.

(a) $A = n^3 + n \log n$; $B = n^3 + n^2 \log n$.

(b) $A = \log \sqrt{n}$; $B = \sqrt{\log n}$.

(c) $A = 2^n$; $B = 2^{n/2}$.

Problem 3. Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Using the basic definition of Θ-notation, prove that $\max(f(n), g(n)) = \Theta(f(n) + g(n))$.

Problem 4. Give asymptotic upper bounds for $T(n)$. Make your bounds as tight as possible. You may assume that n is a power of 2.

(a)

$$\begin{aligned} T(1) &= 1 \\ T(n) &= T(n/2) + 1 \qquad \text{if } n > 1 \end{aligned}$$

(b)

$$\begin{aligned} T(1) &= 1 \\ T(n) &= T(n/2) + n \qquad \text{if } n > 1 \end{aligned}$$

(c)

$$\begin{aligned} T(1) &= 1 \\ T(n) &= 2 \cdot T(n/2) + 1 \qquad \text{if } n > 1 \end{aligned}$$

(d)

$$\begin{aligned} T(1) &= 1 \\ T(n) &= 2 \cdot T(n/2) + n \qquad \text{if } n > 1 \end{aligned}$$

(e)

$$\begin{aligned} T(1) &= 1 \\ T(n) &= 3T\left(\frac{n}{2}\right) + n^2 \qquad \text{if } n > 1 \end{aligned}$$