COMP 271 Design and Analysis of Algorithms
2003 Spring Semester
Question Bank 5

There is some overlap between these question banks and the tutorials. Solving these questions (and those in the tutorials) will give you good practice for the midterm. Some of these questions (or similar ones) will definitely appear on your exam! Note that you do not have to submit answers to these questions for grading. Your TAs will discuss answers to selected questions in the tutorials.

1. Prove that P $\subseteq$ Co-NP.

2. Prove that if NP $\neq$ Co-NP then P $\neq$ NP.

3. Aliens from another world come to Earth and tell us that the 3-SAT problem is solvable in $O(n^8)$ time.

   Which of the following statements follow as a consequence? (List **all** that are true.)

   (i) All NP-complete problems are solvable in polynomial time.

   (ii) All NP-complete problems are solvable in $O(n^8)$ time.

   (iii) All problems in NP, even those that are not NP-complete, are solvable in polynomial time.

   (iv) No NP-complete problem can be solved faster than in $O(n^8)$ time in the worst case.

   (v) P = NP.

4. For each of the following assertions, indicate whether it is **True**: known to be true, **False**: known to be false, or **Unknown**: unknown based on our current scientific knowledge. In each case provide a short explanation for your answer.

   (i) No problems in NP can be solved in polynomial time.

   (ii) Every NP-complete problem requires at least exponential time to be solved.

   (iii) X is in NP and X $\leq_P$ SAT. Then X is NP-complete.

5. Given an undirected graph $G = (V, E)$, a *feedback vertex set* is a subset of vertices such that every simple cycle in G passes through one of these vertices. The feedback vertex set problem (FVS) is: Given a graph G and an integer k, does G contain a feedback vertex set of size at most k?

   Show that FVS is in NP. That is, given a graph G that has a FVS of size k, give a certificate, and show how you would use this certificate to verify the presence of a FVS of size k in polynomial time.

6. Prove that the clique problem is polynomial-time reducible to the vertex cover problem.

7. Give a polynomial time reduction from the set partition problem (SP) to the subset sum problem (SS). Here are the problem definitions.

   Set Partition (SP): Given a finite set S of positive integers, can the set S be partitioned into two sets A and $\bar{A} = S - A$ such that $\sum_{x \in A} x = \sum_{x \in \bar{A}} x$.

   Subset Sum (SS): Given a finite set S of positive integers, and a positive integer t, is there a subset $S' \subseteq S$ which sums exactly to t.

8. The *subgraph isomorphism problem* takes two graphs G_1 and G_2 and asks whether G_1 is a subgraph of G_2. Prove that the subgraph isomorphism problem is NP-complete. (*Hint:* Reduce from a problem we have shown in class to be NP-complete.)

9. The *set cover problem* is: Given a finite set X and a collection of sets F whose elements are chosen from X, and given an integer k, does there exist a subset $C \subseteq F$ of k sets such that
$$X = \bigcup_{S \in C} S.$$
   Prove that the set cover problem is NP-complete. (*Hint:* Reduce from a problem we have shown in class to be NP-complete.)

10. The *set partition problem* takes as input a set S of integers. The question is whether the integers can be partitioned into two sets A and $\overline{A} = S - A$ such that $\sum_{x \in A} x = \sum_{x \in \overline{A}} x$. Show that the set partition problem is NP-complete. You may use the fact that the subset sum problem is NP-complete.

11. Given an integer m-by-n matrix A and an integer m-vector b, the *0-1 integer-programming problem* asks whether there is an integer n-vector x with elements in the set $\{0, 1\}$ such that $Ax \leq b$. prove that 0-1 integer programming is NP-complete. (*Hint:* Reduce from 3-SAT.)

12. Show that the Hamiltonian path problem (HP) is NP-complete. You may assume the fact that the Modified Hamiltonian path problem (MHP) is NP-complete. Here are the problem definitions.

    Hamiltonian Path (HP): Given an undirected graph $G = (V, E)$, is there a simple path that visits all the vertices of G exactly once.

    Modified Hamiltonian Path (MHP): Given an undirected graph $G = (V, E)$ and two distinct vertices u and v, is there a simple path starting at u and ending at v that visits all the vertices of G exactly once?