

# Topology Optimization for Wireless Mesh with Directional Antennas

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**Abstract**—A wireless mesh employing directional antenna, termed *DMesh* in this paper, can greatly extend coverage and improve spatial reuse of wireless channels. As beaming direction of the antennas changes network topology which in turns affects routing and channel decisions, we address in this work how to jointly optimize topology (in terms of beaming directions of antennas), routing and channel assignment so as to maximize network throughput. Specifically, we use a model based on SINR which captures much more realistically network interference than the traditional conflict graph approach. Using the model, we then formulate the NP-hard optimization problem for a general DMesh with multiple gateways, possibly heterogeneous number of antennas in routers, and routers generating traffic to any of the gateways or routers. As the problem is NP-hard, we propose a simple and implementable joint optimization heuristic called TORCA (topology control, routing and channel assignment). TORCA is based on iterative LP rounding guaranteed to converge. Extensive simulation based on NS3 shows that TORCA is closely optimal and highly efficient, performing significantly better than recent approaches by wide margins in terms of loss rate, delay, fairness and throughput.

**Index Terms**—directional antenna; topology control; iterative rounding; joint optimization; routing; channel assignment

## I. INTRODUCTION

A wireless mesh network (WMN) is a multi-hop communication infrastructure consisting of connected mesh routers to provide Internet access for mobile clients. The mesh routers, usually fixed in place, cooperatively route the client traffic to gateways (routers directly connected to the Internet) in a multi-hop manner. Traditionally, WMN is designed with omni-directional antenna, termed *OmniMesh* in this paper. A wireless mesh designed with directional antennas, the so-called *DMesh* in this paper, can potentially improve network performance due to its following strengths: (1) Directional antennas reduce the geographical scope of signal interference in the network; (2) Two antennas beaming to different directions but crossing each other may operate on the same channel at the same time. This greatly increases the spatial reuse of wireless channels; and (3) Due to the concentration of energy, directional antennas can greatly extend the coverage of the network.

This work was supported, in part, by the HKUST Special Research Fund Initiative (SRF11EG15) and Hong Kong Research Grant Council (RGC) General Research Fund (610713).

In a DMesh, each node is equipped with a certain number of directional antennas, typically ranging from two to eight.<sup>1</sup> The antenna number may be *heterogeneous* in the network. Due to its narrow beam width, an antenna usually beams, and hence connects, to a neighboring node. Despite this, the transmission to the node may still interfere with other on-going transmission(s) at that node. We consider a DMesh where the beaming angle of each antenna may be adjusted independently at network setup.

In OmniMesh, a node may be connected or interfere with *all*, and hence arbitrarily many, of its neighbors within a certain distance. DMesh, on the other hand, has much sparser connectivity because the degree of a node is constrained by its number of directional antennas. Furthermore, its connection topology, and hence the resultant interference pattern, may be adjusted or optimized through beaming to a selected few of the neighbors of a node. Therefore, *topology control*, i.e., which mesh routers a node beams to, becomes an important determining factor in DMesh performance.

We study in this paper topology control for DMesh by adjusting antenna orientation (as opposed to some previous works which studied topology control via power adjustment [1]). Besides topology control, channel assignment to each of the antenna beams, and hence neighboring connection, also affects network performance. The goal is to have good spatial reuse on the orthogonal channels to reduce network interference. Routing decision is also important to reduce congestion in the network, leading to an overall improvement in system capacity. In DMesh, topology control, channel and routing assignments are clearly inter-dependent decisions.

We consider a multi-interface multi-channel WMN formed by stationary routers. Each router is equipped with several low-cost non-steerable directional antennas (i.e., the beaming direction, once set, cannot be changed continuously or frequently). Some of these routers are gateway routers connected to the Internet via wired links. Some of the non-gateway routers are associated with wireless users, termed *source* routers. The source routers aggregate traffic generated by users and route it to any one of the gateway routers for Internet access, or to one of the mesh routers as destination. Routers without any

<sup>1</sup>In this paper, we will interchangeably use nodes and routers, and edges and links.

associated users provide connectivity to help forming routing paths by relaying traffic towards destinations.

Denote the number of antennas router  $j$  has as  $I_j$ . In the case that a router has more neighbors than antennas, the router needs to decide which  $I_j$  of its neighbors should its antennas beam to. This is the so-call topology control problem for DMesh. Once the beaming directions are determined, the mesh topology is formed. Given the locations of the routers, it is hence critical to address the challenging problem of how to *jointly* optimize the topology, routing assignment and channel assignment to maximize the network throughput. Due to the degree constraints of each router given by its number of antennas, our problem is different from and much more difficult than the OmniMesh case.

We propose TORCA (**T**opology control, **r**outing and **c**hannel assignment), a highly efficient topology, routing and channel optimization for DMesh. To the best of our knowledge, this is the first piece of work on joint optimization of such nature. TORCA may be implemented at a server at network setup to configure node beaming offline. Our simulation shows that TORCA outperforms other recent approaches in terms of loss rate, delay, fairness and throughput.

We briefly review previous work as follows. There has been much work on the optimization of WMN with *omni*-directional antennas. The optimization of channel assignment, routing and link scheduling has been discussed in [2]–[5]. While these works are impressive, the results cannot be extended to DMesh with topology control (i.e., nodes with degree constraints due to the number of directional antennas). Another body of works studies the optimization of DMesh [6], [7] without optimizing antenna orientations. There has been work on topology control issue for DMesh [8]. However all these works have not studied their joint optimization. To the best of our knowledge, this is the first piece of work addressing their joint optimization by proposing an efficient algorithm that is closely optimal.

The rest of the paper is organized as follows. We first discuss the the system and formulate the problem in Section II. We then present TORCA, an efficient heuristic for topology control, channel assignment and routing in Section III. In Section IV, we present illustrative simulation results with NS3. We conclude in Section V.

## II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

### A. System and Interference Models

Let us consider the transmission from  $u$  to  $v$ . SINR at  $v$  is usually modeled as  $SINR = P_{uv}/(\eta + Q_v)$ , where  $P_{uv}$  is the power of signal from transmitter  $u$  received at node  $v$ ,  $Q_v$  is the received interference power at node  $v$  due to other simultaneous transmissions, and  $\eta$  is the background noise power. In order to correctly decode the received signal, the SINR must be no less than certain threshold  $\gamma$ . For DMesh, the received signal strength  $P_{uv}$  at receive antenna is related to not only sending power  $P$  and  $d(u, v)$ , but also transmit antenna gain and receive antenna gain. Let  $G_t^u$  be the transmit antenna gain of node  $u$  and  $G_r^v$  be the receive antenna gain of node  $v$ . Then, received signal strength can be modeled

as  $(G_t^u \cdot G_r^v \cdot P)/d^\alpha(u, v)$ . In the cosine antenna model, the antenna gain  $G_t$  is expressed as  $G_t = \cos^n(\theta/2)$ , for some  $n \geq 1$  which determines the beamwidth. Where  $\theta$  is the angle between the connection of two nodes and the antenna orientation (central beam axis).

We show an example in Figure 1. Let  $u$  be the sender,  $v$  be the receiver, and vector  $uy$  be the central beam axis for the sender's antenna orientation. Thus,  $\theta$  is the angle between the vector  $uv$  and the vector  $uy$ . The antenna gain  $G_r$  can be computed in a similar way. We consider that the central beam axis aligns with the connection of two nodes, as shown in Figure 2 where the antenna of sender  $u$  beams to the same direction of vector  $uv$  (i.e.,  $uy$  is proportional to  $uv$ ). In the same way, the receive antenna beams to direction  $vu$  to achieve the best signal power. Let  $G_t^{max}$  denote the maximum transmit antenna gain that can be achieved and  $G_r^{max}$  denote the maximum receive antenna gain. We define that two node  $u$  and  $v$  are neighbors if  $(G_t^{max} \cdot G_r^{max} \cdot P)/d^\alpha(u, v) \geq \gamma$ . The set of neighbors of node  $v$  is denoted as  $N(v)$ .

We define channel gain of transmitter  $u$  towards receiver  $v$  as:  $A_{uv} = G_t^u \cdot G_r^v / (d^\alpha(u, v))$ . As we already require the transmit antenna and receive antenna of  $u$  to beam to the same direction of vector  $uv$  and  $vu$  respectively. Moreover, we consider the antennas are using the same antenna technology and sending power. Therefore,  $A_{uv} = A_{vu}$ . From above, it is clear that interference cannot be eliminated completely even with directional antennas. In order to capture the realistic interference pattern between any two links operating on the same channel using an SINR model, we need to calculate the interfering power received at each link due to the simultaneous transmission of other links. To this end, we further define  $H_l^e$ , the maximum channel gain between any endpoint of  $l$  and any endpoint of  $e$  as the channel gain from link  $l$  to link  $e$ . The interfering power received at link  $e$  due to the transmission on the same channel of link  $l$  is then computed as  $PH_l^e$ .

The above can be illustrated by the following example, let  $e$  be a link between node  $u$  and  $v$  and  $l$  be a link between node  $y$  and  $z$ . Recall that  $A_{yv}$  is the channel gain at node  $v$  due to propagation attenuation between nodes  $y$  and  $v$ ,  $H_l^e$  is the channel gain at link  $e$  due to the transmission of link  $l$ . Note that even when there is only one directional flow on the links, the sender of link  $l$  is required by the 802.11 protocol to receive link layer control messages from the receiver. In other words, each endpoint of a link acts as a transceiver. Hence,  $H_l^e$  can be modeled as the maximum channel gain between any two nodes, i.e.,  $H_l^e = \max\{A_{yu}, A_{yv}, A_{zu}, A_{zv}\}$ . For the sake of convenience in notation, we define  $H_e^e = A_{uv} = A_{vu}$ .

### B. Problem Formulation

We model the DMesh as a undirected graph  $G(V, E)$ , where  $V$  is the set of mesh nodes (including gateways) and  $E$  is the edge set representing the neighbour relationship between mesh routers. If two nodes  $u$  and  $v$  are within the communication range  $R$ , there is an edge (link) between  $u$  and  $v$ . Traffic originates from source routers may be destined to any one of the gateways, or one of the routers in the network. Hence,

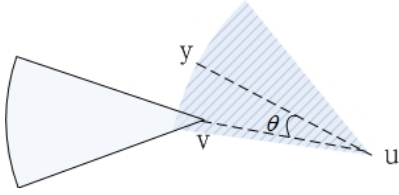


Fig. 1.  $\theta$  is the angle between beaming direction and antenna orientation.

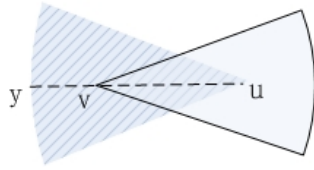


Fig. 2. Two antennas form a link by beaming to each other.

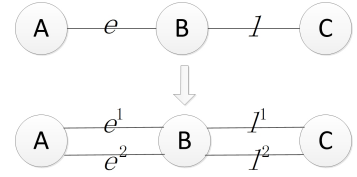


Fig. 3. An example of reachability graph with two nodes and corresponding auxiliary graph.

each traffic flow requirement can be defined by a triplet  $(i, s(i), t(i))$ . Where  $i$  is the flow index,  $s(i)$  is the source node and  $t(i)$  is the destination of the flow. For two different flow requirements  $i$  and  $j$ , they can have the same source node. Therefore, each node can have traffic both to gateway and some other nodes.

Given the reachability graph  $G = (V, E)$  and the set of orthogonal channels  $K = \{1, 2, \dots\}$ , we construct an auxiliary graph  $G' = (V', E')$  as follows. Let  $|K|$  be the cardinality of  $K$ . For any edge  $e$  in graph  $G$ , we expand it into  $|K|$  edges denoted as  $e^1, e^2, \dots, e^{|K|}$ .

Figure 3 illustrates an example of constructing an auxiliary graph  $G'$  from the reachability graph  $G$ . There are three nodes and two undirected edges, namely  $e$  and  $l$  between them in the original network graph. Suppose that there are two orthogonal channels available and each router has two antenna interfaces. An auxiliary graph as shown in 3(b) can be constructed, where  $e$  in original graph has been expanded into  $e^1$  and  $e^2$ . There may be multiple edges incident to a node, but the node can only connect to at most 2 edges. For example, although there are 4 edges incident to router  $B$ ,  $B$  can only choose no more than 2 edges to connect to.

For each edge  $e^k$  in the auxiliary graph, we associate it with an edge assignment boolean  $x_e^k \in \{0, 1\}$ . Suppose that edge  $e$  is the edge between the two nodes  $u$  and  $v$ . If Node  $u$  communicates with  $v$  with the  $k$ th frequency channel, then  $x_e^k = 1$ ; otherwise,  $x_e^k = 0$ . An edge assignment in an auxiliary graph is to assign each edge assignment variable  $x_e^k$  with value 0 or 1. It is clear that topology control and channel assignment problem in DMesh can be jointly solved by finding an edge assignment in the auxiliary graph. We denote  $O$  as the set of flow requirements,  $W$  as the set of gateways and  $L(j)$  as the set of links incident to router  $j$ . Router  $j$  must satisfy the following degree constraint:

$$\sum_{e \in L(j)} \sum_{k \in K} x_e^k \leq I_j, \quad \forall j \in V. \quad (1)$$

Due to narrow beamwidth of DMesh, one antennas is generally used to connect with one neighbor. The degree constraints corresponds to the fact that the number of neighbors to communicate should not exceed the number of antennas. Let  $e$  be a link incident to  $u$ . Traffic flow carried on  $e$  may originated from  $u$  or destined to  $u$ . We denote the amount of outgoing traffic from  $u$  as  $f_{e^+}$  and the amount of incoming traffic to  $u$  as  $f_{e^-}$ . As flow on  $e$  can be from different

sources and sent via different channels, we further use decision variable  $f_{e^+}^{i,k}$  to indicate the amount of traffic generated by source  $s(i)$  and transmitted from  $u$  to another node on channel  $k$ . Similarly, traffic flow transmitted towards the opposite direction is denoted by  $f_{e^-}^{i,k}$ . Denote the amount of traffic that can be delivered from source router  $s(i)$  to  $t(i)$  as  $D(i)$ , which is our optimizing parameter (to maximize throughput). Note that traffic may be destined to the Internet via any one of the gateways, or one of the routers in the mesh. We hence must have the following flow conservation constraints:

$$\sum_{e \in L(j)} \sum_{k \in K} f_{e^+}^{i,k} - \sum_{e \in L(j)} \sum_{k \in K} f_{e^-}^{i,k} = \begin{cases} D(i), & \text{if } j = s(i); \\ -D(i), & \text{if } j = t(i); \\ 0, & \text{otherwise,} \end{cases} \quad (2)$$

$\forall i \in O, j \in V \setminus W$ . The first term  $\sum_{e \in L(j)} \sum_{k \in K} f_{e^+}^{i,k}$  on the left-hand side of the Equation (2) is the total amount of outgoing traffic generated by source  $s(i)$  sent from router  $j$  to other routers. Similarly, the second term  $\sum_{e \in L(j)} \sum_{k \in K} f_{e^-}^{i,k}$  is the total amount of incoming traffic generated by source  $s(i)$  sent from other routers to router  $j$ . Note that there is no flow conservation constraints for a gateway node in set  $W$ . The reason is stated as follows. Let say source  $s(i)$  is destined to the Internet. The traffic generated by  $s(i)$  can go to any gateway. Due to the flow conservation constraints of non-gateway nodes, non-gateway nodes will not be the sinks of the traffic. All traffic must be aborted by gateways. To reflect better reality, we use the following interference model for DMesh. In wireless environment, the success of data transmission depends on the SINR at the receiver. We consider that all routers transmit signal at a homogeneous power level  $P$ . In order to correctly decode the signal, the SINR at the receiver must be higher than a certain threshold  $\gamma$ . Therefore, we must have the following link quality constraints:

$$\frac{PH_e^e x_e^k}{N + \sum_{l \in E} PH_l^e x_l^k} \geq \gamma x_e^k, \quad \forall e \in E, k \in K. \quad (3)$$

Note that both routers  $u$  and  $v$  of a communication link  $e$  act as receivers (even when there is only one directional flow on the link, sender of the link is required by the 802.11 protocol to receive link layer control messages from the receiver). On the right hand side of the inequality,  $\gamma x_e^k$  means that if link  $e$  is assigned channel  $k$ , the SINR on channel  $k$  must be greater than  $\gamma$ ; otherwise, there is no constraint imposed on link  $e$ .  $PH_e^e x_e^k$

is the received signal strength on channel  $k$  at receivers of link  $e$ , while  $\sum_{l \in E} PH_l^e x_l^k$  is the interfering power at link  $e$  due to the simultaneous communication of other links on the same channel  $k$ . Constraint (3) is a set of equations which are clearly non-linear. To linearize it, we introduce a constant  $M$ , such that  $M \geq \gamma(N + \sum_{l \in E} PH_l^e x_l^k), \forall e \in E, k \in K$ . Hence, the set of equations can be linearized to:

$$PH_e^e x_e^k + (1 - x_e^k)M \geq \gamma \left( N + \sum_{l \in E} PH_l^e x_l^k \right), \quad (4)$$

$\forall e \in E, k \in K$ .

The flow carried on a link cannot exceed its effective capacity, i.e, we need the link capacity constraint:

$$\sum_{i \in S} f_{e^+}^{i,k} + f_{e^-}^{i,k} \leq x_e^k C. \quad (5)$$

We also require the flow assigned to each link to be nonnegative, the variables  $x_s$  to be either 0 or 1, and the amount of traffic demand to be nonnegative, i.e,

$$f_{e^+}^{i,k}, f_{e^-}^{i,k} \geq 0, \quad \forall i \in S, k \in K \text{ and } e \in E; \quad (6)$$

$$x_e^k \in \{0, 1\}, \quad \forall i \in S, k \in K \text{ and } e \in E; \quad (7)$$

$$D(i) \geq 0, \quad \forall i \in S. \quad (8)$$

We model the joint topology control, channel assignment and routing problem as the degree-constrained multiple-source multiple-sink max-throughput problem. The objective of our problem can be maximizing any concave utilization function of  $U(D)$ .  $D$  is a vector where the  $i$ th component is  $D(i)$  i.e.  $D = (D(1), D(2), \dots)^T$ . Two most commonly used utilization function are:  $U(D) = \sum \log(D(i))$  and  $U(D) = \min D(i)$ . The first seeks to maximize the aggregate throughput while the other objective maximizes the throughput of the *worst-case* source.

$$\begin{aligned} & \max && U(D) \\ & \text{subject to} && \text{Constraints (1), (2), (4), (5), (6), (7) and (8).} \end{aligned}$$

For concreteness in our discussion, we will focus on the objective  $U(D) = \min D(i)$ . It is clear that our approach and algorithm are equally applicable to other concave objective function (such as maximizing throughput). As the well known NP-hard problem Max-k cut can be reduced to the above joint topology control, routing and channel assignment problem, it is clear a NP-hard problem.

### III. TORCA: JOINT OPTIMIZATION OF TOPOLOGY, ROUTING AND CHANNELS FOR DMESH

In this section, we present TORCA, an efficient joint optimization of topology, routing and channel assignment for DMesh with directional antenna. TORCA is based on LP relaxation. LP relaxation and rounding is an efficient way to find good solution for MILP. In order to address the degree constraints (Constraint (1)) which complicates the rounding

of a fractional solution to a feasible integer solution, TORCA employs an iterative rounding approach which guarantees to converge to tackle the problem. Since the hardness of the problem comes from the integer constraints, we first relax the integer constraints to get an LP problem. In the first iteration, TORCA solves the LP relaxation problem as follows:

$$\begin{aligned} & \text{LP-Relaxation:} && (9) \\ & \max && \min D(i) \\ & \text{subject to} && \text{Constraints (1), (2), (4), (5), (6), (8),} \\ & && 0 \leq x_e^k \leq 1, \forall x_e^k. \end{aligned}$$

TORCA then rounds the largest fractional variable  $x_e^k$  to 1 and keeps the other  $x_e^k$ s undecided (they will be assigned to 0 or 1 in the final integral solution). Let  $X'$  be the set of variables  $x_e^k$  that have been rounded to 1. Further let  $X$  be the remaining undecided variables in the current iteration. In the next iteration, we solve the residual LP (the LP with one fewer variables) again:

$$\begin{aligned} & \text{Residual-LP:} && (10) \\ & \max && \min D(i) \\ & \text{subject to} && \text{Constraints (1), (2), (4), (5), (6), (8),} \\ & && 0 \leq x_e^k \leq 1, \forall x_e^k \in X, \\ & && x_e^k = 1, \forall x_e^k \in X'. \end{aligned}$$

With this, TORCA finds another variable  $x_e^k$  to round to 1. It keeps iterating this procedure until the algorithm terminates at  $\max x_e^k = 0$ . The pseudo-code of TORCA algorithm is presented in Algorithm 1.

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#### Algorithm 1 TORCA

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 $X \leftarrow X \cup x_e^k \forall k \in K \text{ and } e \in E, X' = \emptyset;$ 
 $x_{e'}^{k'} = \inf;$ 
while  $x_{e'}^{k'} \neq 0$  do
  solve optimization problem given by (10);
   $e', k' \leftarrow \arg \max_{e,k} x_e^k;$ 
  round  $x_{e'}^{k'}$  to 1;
   $X' \leftarrow X' \cup x_{e'}^{k'}, X \leftarrow X \setminus x_{e'}^{k'};$ 
end while

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Initially, all the  $x_e^k$ s are undecided. They are put into the set  $X$ . Hence, the Residual-LP is the same as the LP-Relaxation. In each iteration, TORCA gets the optimal fractional solution to the Residual-LP. Though the  $x_e^k$ s in the solution are fractional, their values give us guidance about which edges and which channels are more preferable. TORCA rounds the  $x_e^k$  with the largest value following the greedy heuristics and moves it to  $X'$  from the set of undecided variables. Note that rounding  $x_e^k$  to 1 means picking edge  $e$  and assigning channel  $k$  to it. The iterative procedure terminates when there is no more  $x_e^k$  can be rounded to 1 due to limited number of antennas at each node.

We analyze the complexities of TORCA as follows. Each iteration of TORCA involves solving a Residual-LP given by problem (10). It is well known that linear programming can be solved within polynomial time. Therefore, the time complexity of each TORCA iteration is bounded. Since there are  $I_j$  antennas at mesh router  $j$ , there are a total of  $\sum_{j \in N} I_j$  antennas in the network. Note that rounding a variable  $x_e^k$  to 1 is equivalent to assign channel  $k$  to two antennas of link  $e$ . Any antenna can only be assigned once. In each iteration, we assign a channel to two antennas. Clearly, it needs no more than  $\lceil (\sum_{j \in N} I_j)/2 \rceil$  iterations to assign channels to all antennas. Hence, TORCA runs for at most  $\lceil (\sum_{j \in N} I_j)/2 \rceil$  iterations and its overall run-time complexity is polynomial.

#### IV. ILLUSTRATIVE SIMULATION RESULTS

##### A. Simulation Environment and Metrics

We use the latest NS-3 in our simulation environment. Because the Mesh module of NS-3 does not support directional antenna, we add an antenna module to NS3 and integrate it with the YansWifiphy model of NS-3. In our simulation, mesh nodes are randomly placed in an area (of size  $2000\text{m} \times 2000\text{m}$ ). Gateway nodes and sources (nodes generating traffic) are all randomly chosen from the deployed nodes. Unless otherwise stated, we use the following baseline parameters: max antenna gain as 10db, transmission range as 700 meters, beamwidth as  $40^\circ$ , traffic demand per flow as 1.5 Mbps, number of gateways as 1. We are interested in the following performance metrics: *Loss rate (UDP)*, *Delay*, *Throughput* and *Jain's fairness index*. *Jain's fairness index* measures the fairness of UDP throughput given by a set of  $n$  source nodes. It is calculated as  $(\sum_{i=1}^n x_i)^2 / (n \sum_{i=1}^n x_i^2)$ , where  $x_i$  is the throughput of the  $i$ th source nodes.

As there has been no other similar previous work on joint optimization, we compare TORCA with a scheme with state-of-the-art channel assignment and topology control. For channel assignment, the scheme begins with a random assignment and tries to minimize the traffic-weighted interference using Tabu search [2] (labeled as Tabu). For topology control, we use the minimum-hop tree topology construction protocol [9] (labeled as Minhop). The Minhop scheme constructs a degree-constrained minimum-hop tree using a modified version of Dijkstra's shortest path algorithm. Initially, the tree topology contains only the gateway node. In each iteration, an arbitrary node that is not in the tree will join the tree by connecting to the closet node that is in the tree. This joint operation should not violate the interface constraint at each node. We label the integration of these two schemes as Minhop+Tabu in our study.

##### B. Illustrative Results

We show in Figure 4 the capacity (objective value) achieved by TORCA and the "Super optimal" case (given by LP relaxation without integral consideration) versus number of channels. The achieved capacity increases with the number of available channels. As the number of channels reaches to some point, TORCA achieves closely optimum. In other words, LP relaxation provides a tight upper bound that can be achieved by

our scheme. As the number of orthogonal channels increases, interference due to simultaneous transmissions decreases. The network capacity is hence bounded by the number of antennas at each mesh router (which is 3 in our default). This result suggests that network capacity is bounded by both number of available channels and the number of antennas.

We plot the average loss rate versus the traffic load per flow in Figure 5. The loss rate increases with the traffic rate because higher traffic load leads to higher interference and congestion. The loss rate of TORCA, however, increases less sharply. It substantially outperforms the state-of-the-art Minhop+Tabu scheme because it jointly optimizes routing, channel assignment and topology. On the other hand, Minhop+Tabu optimizes the topology too aggressively by building a rather shallow tree in order to minimize hop counts by trying to meet the degree constraints of nodes. As is clear from the figure, its loss rate increases drastically beyond a certain point because each node connects to many links, leading to higher interference between links. The figure shows that the interference model used in our TORCA works well by capturing network interference in the optimization steps.

The Jain's fairness index for UDP throughput is shown in Figure 6. Jain's fairness index decreases with traffic rate, because flows are competing for the limited resources in the network. As the traffic rate increases, some flows start to receive unfair allocation. TORCA achieves much better and more stable fairness than the other scheme, because it considers fairness by maximizing the worst-case per-node throughput while Minhop+Tabu only minimizes the hop count of routing paths.

We study UDP loss rate with respect to the number of orthogonal channels in Figure 7. The loss rate drops as number of orthogonal channels increases due to lower interference. The substantially lower loss rate of TORCA means that it is able to make better use of channels. As the loss rate flats off when the number of channels is similar to the number of antennas, the figure shows that there is little incremental benefit in having channels much more than the number of antennas in DMesh.

Figure 8 plots average UDP loss rate versus average inter-node distance in a grid topology. With the transmission range remains the same, we vary the inter-node distance. As the inter-node distance increases, the loss rate first decreases and then increases. This is because loss rate depends on two factors: interference and signal strengths. As the inter-node distance increases, the network becomes sparser, leading to lower interference and hence lower loss rate. However, as the inter-node distance further increases, signal strengths between nodes decrease. Such fading leads to higher loss. When the internode distance increases further beyond the transmission range, loss rate drastically increases. The figure shows that, given transmission power, there is an optimal network density to achieve minimum loss rate. The rather flat U-shape curve also means that such optimality is not a shape one; a wide range of inter-node distance performs similarly well.

We finally show in Figure 9 TCP performance versus

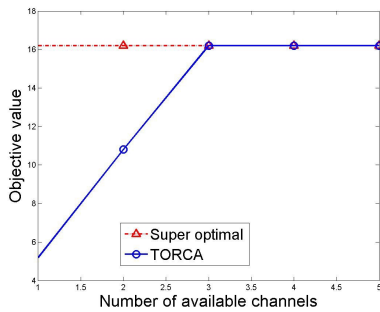


Fig. 4. Capacity achieved by LP and TORCA.

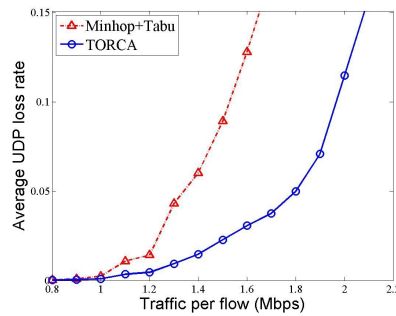


Fig. 5. Loss rate comparison.

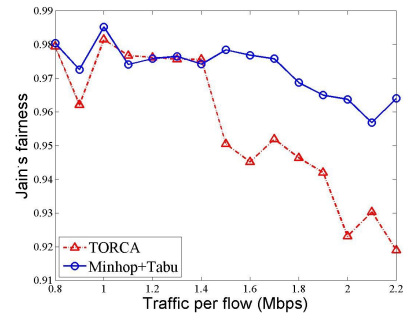


Fig. 6. Jain's fairness comparison.

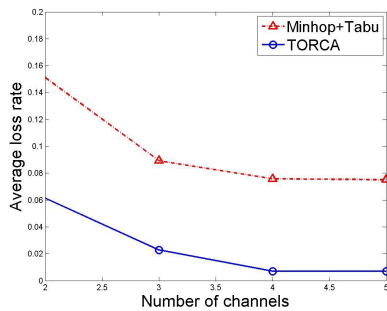


Fig. 7. Loss rate versus number of channels.

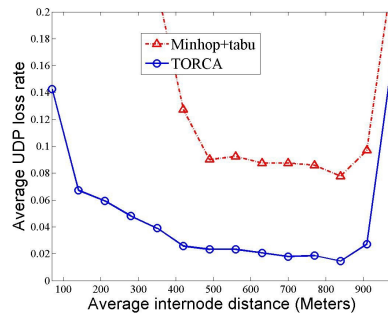


Fig. 8. Loss rate versus inter-node distance.

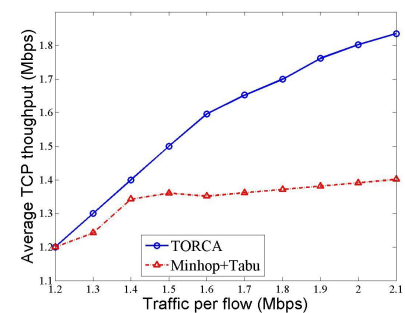


Fig. 9. TCP throughput versus traffic load.

traffic flow for TORCA and Minhop+Tabu. While the network throughput increases quite linearly with TORCA, the throughput of Minhop+Tabu flats off much earlier with much lower network capacity. This shows that TORCA makes much better joint optimization on topology, routing and channel assignment with the interference model used. We have done many other simulations on TCP performance. As the results and conclusions are qualitatively the same as the UDP case, we will not show them here due to brevity.

## V. CONCLUSION

In this paper, we have addressed the throughput maximization problem in a wireless mesh with directional antennas (DMesh). In the network, there are multiple gateways, routers with possibly heterogeneous number of antennas, and users attached to any subset of the routers with traffic to any of the gateways or other routers. Using an SINR interference model which better captures the reality than the traditional approach based on conflict graph, we have presented a novel formulation that incorporates topology control (which interface beams to which neighbor), routing and channel decisions. We then present an efficient joint algorithm called TORCA (topology control, routing and channel assignment), which is based on iterative LP rounding to achieve closely optimal performance. Using NS3 simulation, we have shown the effectiveness of TORCA in terms of loss rate, delay, fairness and throughput. A joint design substantially outperforms sequential optimization, and DMesh performs much better than a mesh employing omni-directional antennas.

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