# HKUST Local Programming Contest 2019 

Venue: Lab 4213
Time: $2 \mathrm{pm}-5 \mathrm{pm}$ (3 hours)

- There are seven problems in the contest
- The compilation configuration is " $\mathrm{g}++-\mathrm{O} 2$ "
- You can use Java or C++
- The time limit for each problem is 3 seconds


## Problem A

## Which Base is it Anyway?

Programming languages such as C++ and Java can prefix characters to denote the base of constant integer values. For example, hexadecimal (base 16) constants are preceded by the string " $0 x$ ". Octal (base 8 ) values are preceded by the character "0" (zero). Decimal (base 10) values do not have a prefix. For example, all the following represent the same integer constant, albeit in different bases.
$0 \times 1234$
011064
4660
The prefix makes it clear to the compiler what base the value is in. Without the " $0 x$ " prefix, for example, it would be impossible for the compiler to determine if 1234 was hexadecimal. It could be octal or decimal. For this problem, you will write a program that interprets a string of decimal digits as if it were an octal value, a decimal value or a hexadecimal value.

## Input

The first line of input contains a single decimal integer $P$, ( $1<=P<=10000$ ), which is the number of data sets that follow. Each data set should be processed identically and independently. Each data set consists of a single line of input. It contains the data set number, K , followed by a single space, followed by a string of at most 7 decimal digits.

## Output

For each data set there is one line of output. The single output line consists of the data set number, K, followed by a space followed by 3 space separated decimal integers which are the value of the input as if it were interpreted to as octal, decimal and hexadecimal respectively. If the input value cannot be interpreted as an octal value, use the value 0 .

## Sample Input and Output

| Sample Input | Sample Output |
| :--- | :--- |
| 4 | 1 |
| 1 | 668 |
| 1234 | 1234 |
|  | 9 |
| 2 | 9 |
| 3 | 1023 |
| 3 | 1777 |
| 4 | 129 |$|$| 4 | 0 | 129 | 297 |
| :--- | :--- | :--- | :--- |

## Problem B <br> FBI Universal Control Numbers

The FBI has recently changed its Universal Control Numbers (UCN) for identifying individuals who are in the FBl's fingerprint database to an eight digit base 27 value with a ninth check digit. The digits used are:

## 0123456789 ACDEFHJKLMNPRTVWX

Some letters are not used because of possible confusion with other digits:

$$
B->8, G->C, I->1, O->0, Q->0, S->5, U->V, Y->V, Z->2
$$

The check digit is computed as:

```
(2*D1 + 4*D2 + 5*D3 + 7*D4 + 8*D5 + 10*D6 + 11*D7 + 13*D8) mod 27
```

Where $D n$ is the $n^{\text {th }}$ digit from the left.
This choice of check digit detects any single digit error and any error transposing an adjacent pair of the original eight digits. For this problem, you will write a program to parse a UCN input by a user. Your program should accept decimal digits and any capital letter as digits. If any of the confusing letters appear in the input, you should replace them with the corresponding valid digit as listed above. Your program should compute the correct check digit and compare it to the entered check digit. The input is rejected if they do not match otherwise the decimal (base 10) value corresponding to the first eight digits is returned.

## Input

The first line of input contains a single decimal integer $P$, ( $1<=P<=10000$ ), which is the number of data sets that follow. Each data set should be processed identically and independently. Each data set consists of a single line of input. It contains the data set number, K , followed by a single space, followed by 9 decimal digits or capital (alphabetic) characters.

## Output

For each data set there is one line of output. The single output line consists of the data set number, K, followed by a single space followed by the string "Invalid" (without the quotes) or the decimal value corresponding to the first eight digits.

## Sample Input and Output

| Sample Input | Sample Output |
| :--- | :--- |
| 3 | 1 11280469652 |
| 1 | 12345678 A |
| 2 | 12435678 A |
| 3 | 12355678A |

## Problem C m-ary Partitions

A partition of an integer $n$ is a set of positive integers which sum to $n$, typically written in descending order. For example:

```
10=4+3+2+1
```

A partition is $m$-ary if each term in the partition is a power of $m$. For example, the 3 -ary partitions of 9 are:

```
9
3+3+3
3+3+1+1+1
3+1+1+1+1+1+1
1+1+1+1+1+1+1+1+1
```


## Input

The first line of input contains a single decimal integer $P$, ( $1<=P<=1000$ ), which is the number of data sets that follow. Each data set should be processed identically and independently. Each data set consists of a single line of input. The line contains the data set number, K, followed by the base of powers, $m,(3<=m<=100)$, followed by a space, followed by the integer, $n,(3<=n$ $<=10000$ ), for which the number of m-ary partitions is to be found.

## Output

For each data set there is one line of output. The output line contains the data set number, K, a space, and the number of m-ary partitions of $n$. The result should fit in a 32-bit unsigned integer

## Sample Input and Output

| Sample Input | Sample Output |  |
| :--- | :--- | :--- |
| 5 | 1 | 5 |
| 1 | 3 | 9 |
| 2 | 3 | 47 |
| 3 | 5 | 123 |
| 4 | 7 | 4321 |
| 5 | 97 | 9999 |
|  | 75 |  |

## Problem D

## A Rational Sequence

An infinite full binary tree labeled by positive rational numbers is defined by:
$>$ The label of the root is $1 / 1$.
$>$ The left child of label $p / q$ is $p /(p+q)$.
$>$ The right child of label $p / q$ is $(p+q) / q$.
The top of the tree is shown in the following figure:


A rational sequence is defined by doing a level order (breadth first) traversal of the tree (indicated by the light dashed line). So that:

```
F(1) = 1/1, F(2) = 1/2, F(3) = 2/1, F(4) = 1/3, F(5) = 3/2, F(6) = 2/3, ...
```

Write a program to compute the $n^{\text {th }}$ element of the sequence, $F(\boldsymbol{n})$. Does this problem sound familiar? Well it should! We had variations of this problem at the 2014 and 2015 Greater NY Regionals.

## Input

The first line of input contains a single integer $P$, (1 <=P <= 1000), which is the number of data sets that follow. Each data set should be processed identically and independently. Each data set consists of a single line of input. It contains the data set number, $K$, and the index, $N$, of the sequence element to compute ( $1<=\mathrm{N}<=2147483647$ ).

## Output

For each data set there is a single line of output. It contains the data set number, K , followed by a single space which is then followed by the numerator of the fraction, followed immediately by
a forward slash (' $/ 1$ ) followed immediately by the denominator of the fraction. Inputs will be chosen so neither the numerator nor the denominator will overflow a 32-bit unsigned integer.

## Sample Input and Output

| Sample Input | Sample Output |
| :--- | :--- |
| 4 | 1 |
| 1 | 1 |
| 2 | 4 |
| 3 | 11 |
| 4 | 1431655765 |

## Problem E

## Permutation Descent Counts

Given a positive integer, $\boldsymbol{N}$, a permutation of order $\boldsymbol{N}$ is a one-to-one (and thus onto) function from the set of integers from $\mathbf{1}$ to $\boldsymbol{N}$ to itself. If $\boldsymbol{p}$ is such a function, we represent the function by a list of its values:

$$
[p(\mathbf{1}) p(\mathbf{2}) \ldots \mathbf{p}(N)]
$$

For example,
[562471 3] represents the function from $\{\mathbf{1} \ldots \mathbf{7}\}$ to itself which takes $\mathbf{1}$ to $\mathbf{5 , 2}$ to 6, .., $\mathbf{7}$ to $\mathbf{3}$.
For any permutation $\boldsymbol{p}$, a descent of $\boldsymbol{p}$ is an integer $\boldsymbol{k}$ for which $\boldsymbol{p}(\boldsymbol{k})>\boldsymbol{p}(\boldsymbol{k}+\mathbf{1})$. For example, the permutation [5624713] has a descent at $2(6>2)$ and $5(7>1)$.

For permutation $\boldsymbol{p}, \operatorname{des}(\boldsymbol{p})$ is the number of descents in $\boldsymbol{p}$. For example, $\boldsymbol{\operatorname { d e s }}\left(\left[\begin{array}{l}5 \\ 6 \\ 2\end{array} \mathbf{4} 1\right.\right.$ 3]) = 2. The identity permutation is the only permutation with $\operatorname{des}(\boldsymbol{p})=0$. The reversing permutation with $p(\boldsymbol{k})=\mathbf{N + 1 - k}$ is the only permutation with $\operatorname{des}(\boldsymbol{p})=\mathrm{N}-\mathbf{1}$.

The permutation descent count (PDC) for given order $\boldsymbol{N}$ and value $\boldsymbol{v}$ is the number of permutations $\boldsymbol{p}$ of order $\boldsymbol{N}$ with $\operatorname{des}(\boldsymbol{p})=\boldsymbol{v}$. For example:

```
PDC(3, 0) = 1 {[\begin{array}{lllll}{1}&{2}&{3}\end{array}}
PDC(3, 1) = 4{[[1 3 2 ], [ 2 1 3 ], [ 2 3 1 ], 3 1 2 ]}
PDC(3, 2) = 1 {[ [ 3 2 1 ] }
```

Write a program to compute the $P D C$ for inputs $\boldsymbol{N}$ and $\boldsymbol{v}$. To avoid having to deal with very large numbers, your answer (and your intermediate calculations) will be computed modulo 1001113.

## Input

The first line of input contains a single integer $P$, ( $1<=P<=1000$ ), which is the number of data sets that follow. Each data set should be processed identically and independently. Each data set consists of a single line of input. It contains the data set number, K , followed by the integer order, $\mathrm{N}(2<=\mathrm{N}<=100$ ), followed by an integer value, v ( $0<=\mathrm{v}<=\mathrm{N}-1$ ).

## Output

For each data set there is a single line of output. The single output line consists of the data set number, K, followed by a single space followed by the PDC of N and v modulo 1001113 as a decimal integer.

## Sample Input and Output

| Sample Input | Sample Output |  |
| :--- | :--- | :--- |
| 4 | 1 | 4 |
| 1 | 3 | 1 |
| 2 | 5 | 2 |
| 3 | 8 | 3 |
| 4 | 99 | 50 |

## Problem F

## Tight-Fit Sudoku

At some point or another, most computer science students have written a standard Sudoku solving program. A slight twist has been added to standard Sudoku to make it a bit more challenging. Digits from 1 to 9 are entered in a $6 \times 6$ grid so that no number is repeated in any row, column or $3 \times 2$ outlined region as shown below. Some squares in the grid are split by a slash and need 2 digits entered in them. The smaller number always goes above the slash.


For this problem, you will write a program that takes as input an incomplete puzzle grid and outputs the puzzle solution grid.

## Input

The first line of input contains a single decimal integer $P$, ( $1<=P<=100$ ), which is the number of data sets that follow. Each data set should be processed identically and independently. Each data set consists of 7 lines of input. The first line of the data set contains the data set number, K. The remaining 6 lines represent an incomplete Tight-Fit Sudoku grid, each line has 6 data elements, separated by spaces. A data element can be a digit (1-9), a dash ('-') for a blank square or two of these separated by a slash ( $/ \mathrm{l}$ ).

## Output

For each data set there are 7 lines of output. The first output line consists of the data set number, K. The following 6 lines of output show the solution grid for the corresponding input data set. Each line will have 6 data elements, separated by spaces. A data element can be a digit (1-9), or 2 digits separated by a slash ('/').

| Sample Input | Sample Output |
| :---: | :---: |
| 1 | 1 |
| 1 |  |
| -/--/5 432 -/- | $\begin{array}{lllll}3 & 6 & 2 / 8 & 1 / 9 & 7 / 5\end{array}$ |
| - 6 -/- -/- - -/- | $\begin{array}{llllll}1 & 7 / 9 & 3 / 5 & 6 / 8 & 2\end{array}$ |
| - 7/- - -/- -/-2 | $82 / 45 / 671 / 39$ |
| $8-/-1 /--13-$ |  |
| -/- - -/- -/- 4 - | $\begin{array}{llllllll}2 / 4 & 8 & 7 & 6 & 5 / 9 & 1 / 3\end{array}$ |
| -/-8765/--1- |  |

## Problem G

## DA-Sort

You recently learned a new way to sort an array of numbers in your algorithms course. The algorithm sorts an array of numbers by repeatedly performing the Delete-and-Append operation. The Deleteand-Append operation consists of three steps: 1) Choose an element from the array. 2) Delete the chosen element from the array. 3) Append the chosen element to the end of the array. Being a curious student, you wonder what is the minimum number of Delete-and-Append operations required to sort a given array.

## Input

The first line of input contains a single decimal integer $P,(1<=P<=100)$, which is the number of data sets that follow. Each data set should be processed identically and independently. Each data set consists of two or more lines of input. The first line contains the data set number, K, followed by a single space, followed by an integer $N,(1<=N<=1000)$, which is the length of the array to sort. The remaining lines in the dataset contains N positive integers that comprise the array to be sorted, 10 values per line, except for the last line which may have less than 10 values. All the array elements are no larger than $10^{\wedge} 9$. The same value may appear more than once in the array to be sorted.

## Output

For each data set there is one line of output. The single output line consists of the data set number, K , followed by a single space followed by an integer which is the minimum number of Delete-andAppend operations required to sort the array.

## Sample Input and Output

| Sample Input | Sample Output |
| :---: | :---: |
| 3 | 11 |
| 13 | 23 |
| 132 | 315 |
| 26 |  |
| $\begin{array}{llllll}1 & 5 & 2 & 4 & 3 & 6\end{array}$ |  |
| 323 |  |
| 67890563129999999991234523456 |  |
| 38927456321003459876523456 |  |
| 876544327823456117654321899 |  |
| 25432543262174352684531782 |  |
| 334564123456213 |  |

