Indexing Uncertain Data

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PODS '09

Motivation

- Two sessions devoted to uncertain/probabilistic data management in each of SIGMOD'08, VLDB'08, and SIGMOD'09

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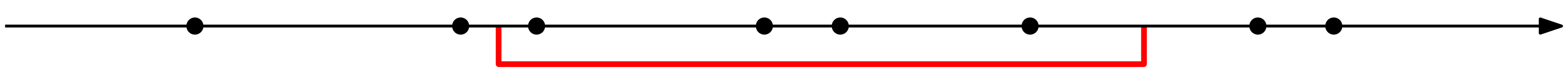
- So, let's skip the cliché and just get to the problem

The Problem: Range Searching

- One of the very first and fundamental problems studied in querying uncertain data

The certain case:



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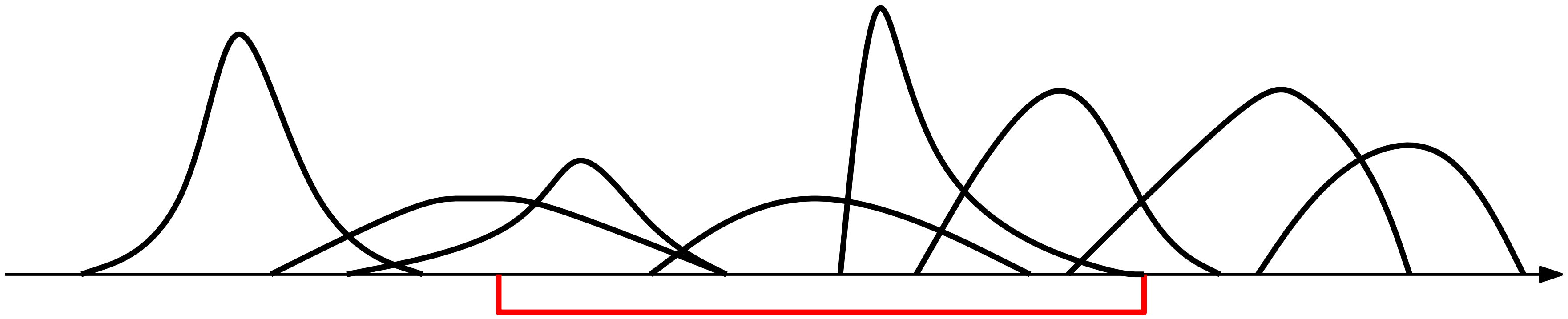

Report all points that fall inside the range with probability $\geq \tau$

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SELECT * FROM sensorreadings
WHERE Prob[20 <= temp <= 25] >= 0.5
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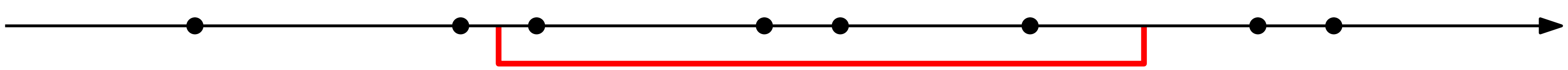
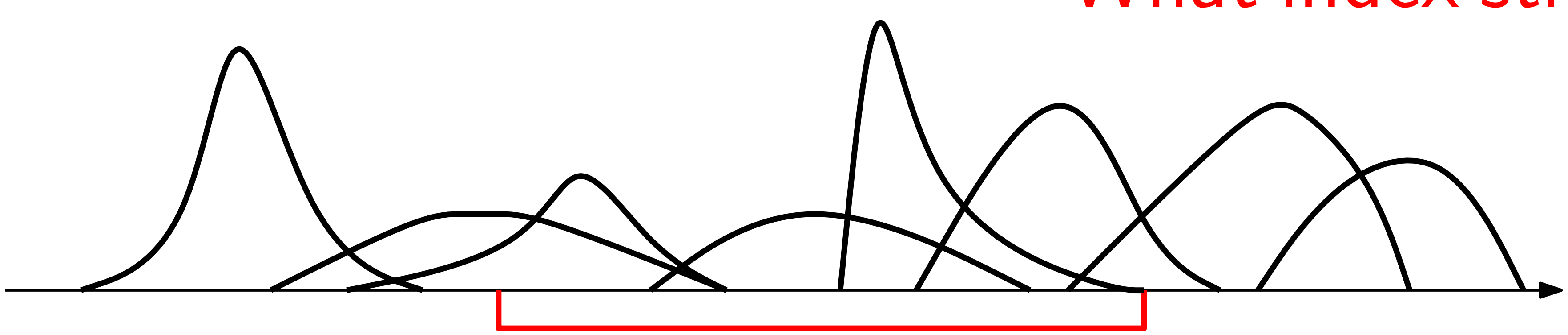


The uncertain case:

What index structure?



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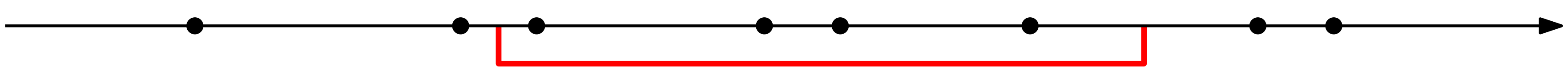

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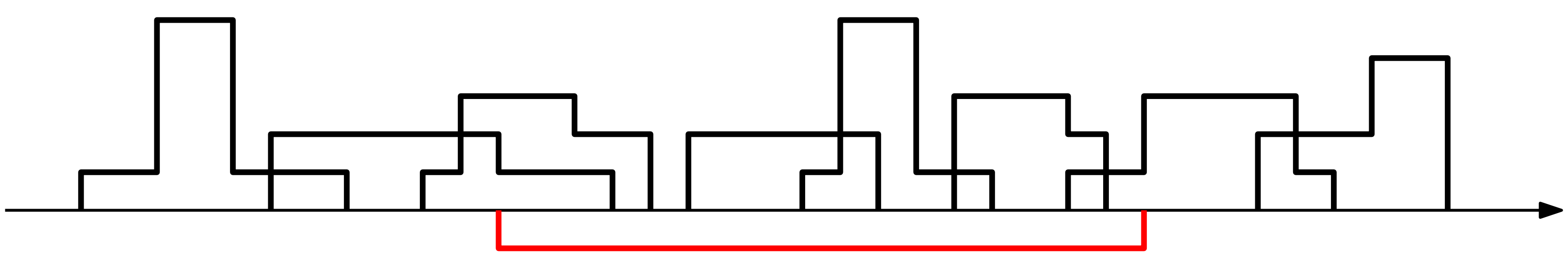

Report all points that fall inside the range with probability $\geq \tau$

Assume each pdf is piecewise constant (histogram)

Will also talk about Gaussian and piecewise algebraic

Queries with a Fixed τ

- Previous results: [Cheng, Xia, Prabhakar, Shah, Vitter '04]
  - When each pdf is uniform (i.e., histogram with one piece)
  - An index with size $O(n\tau^{-1})$ and query time $O(\tau^{-1} \log n + k)$
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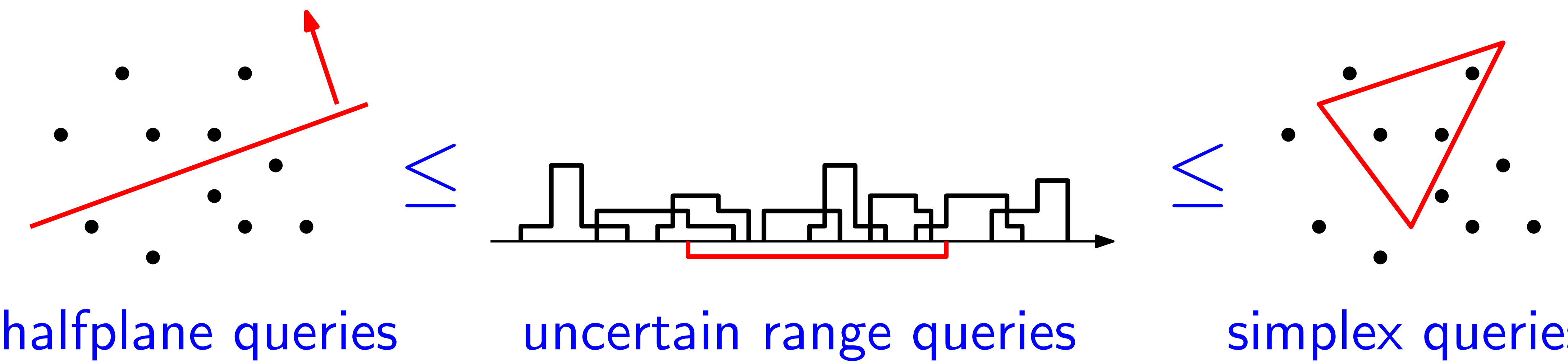

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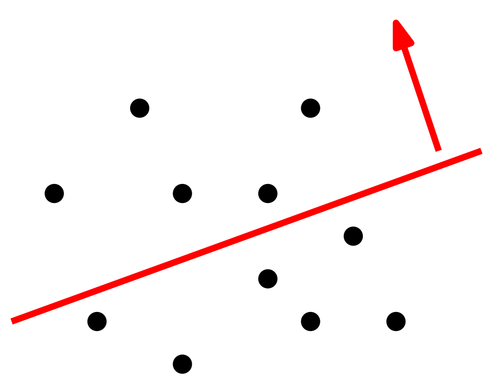

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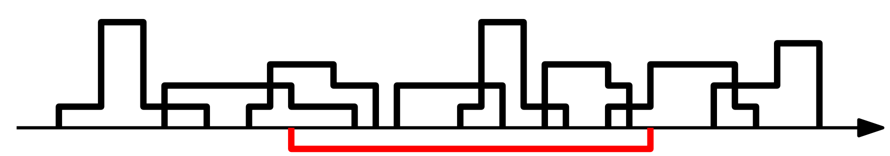

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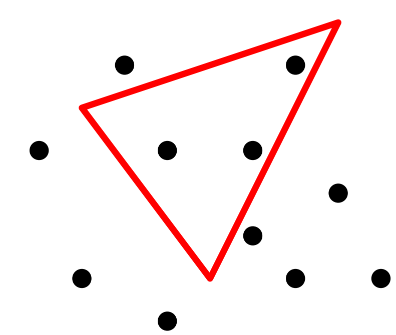

halfplane queries	uncertain range queries	simplex queries
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- Our results
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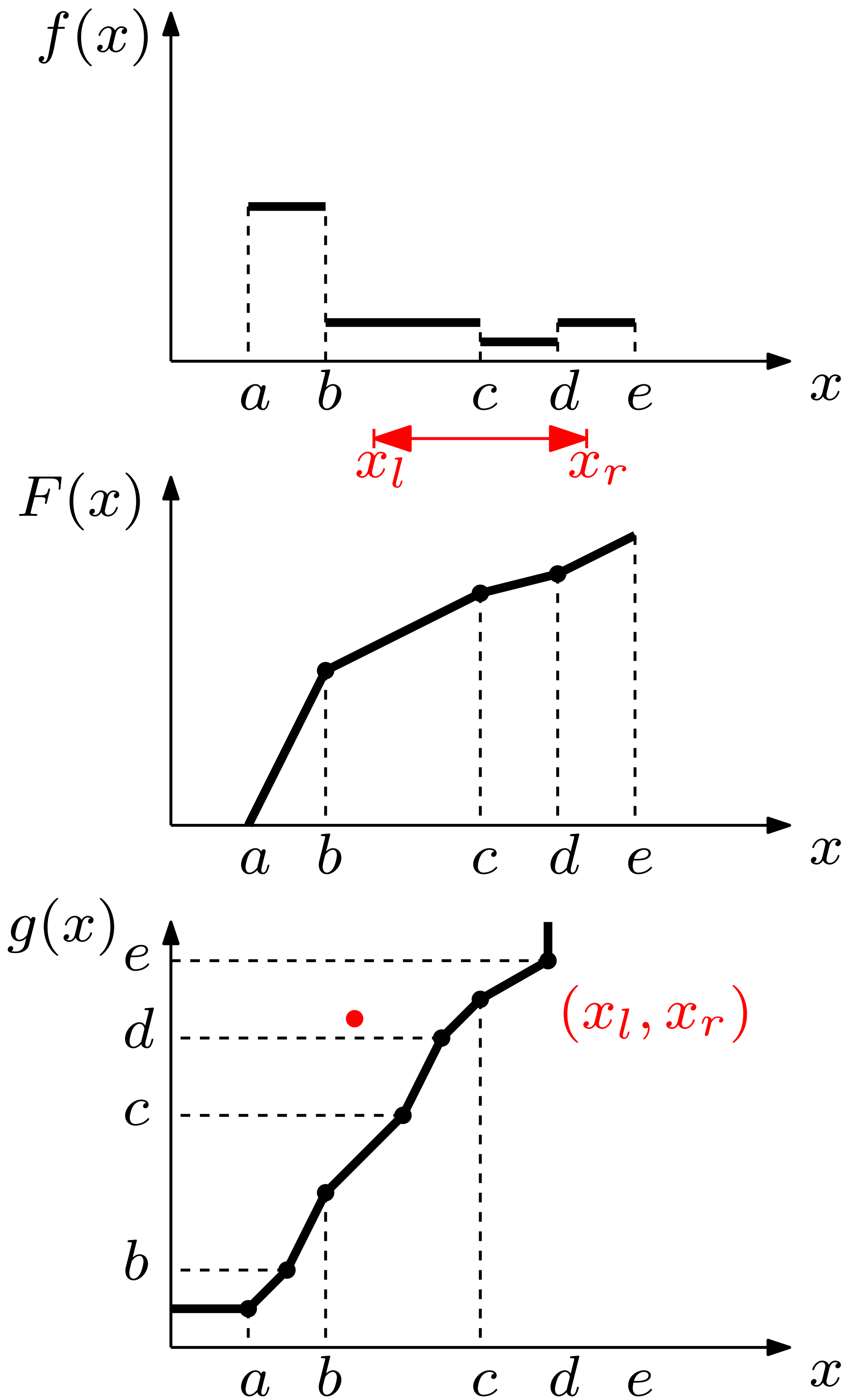
  - Can be made dynamic
  - Can be made I/O-efficient

Queries with a Fixed τ: A Geometric Reduction



pdf

cdf

the threshold function

For any a, $g(a) :=$ the minimum b s.t. $F(b) - F(a) \geq \tau$;
If no such b exists, $g(a) := \infty$.

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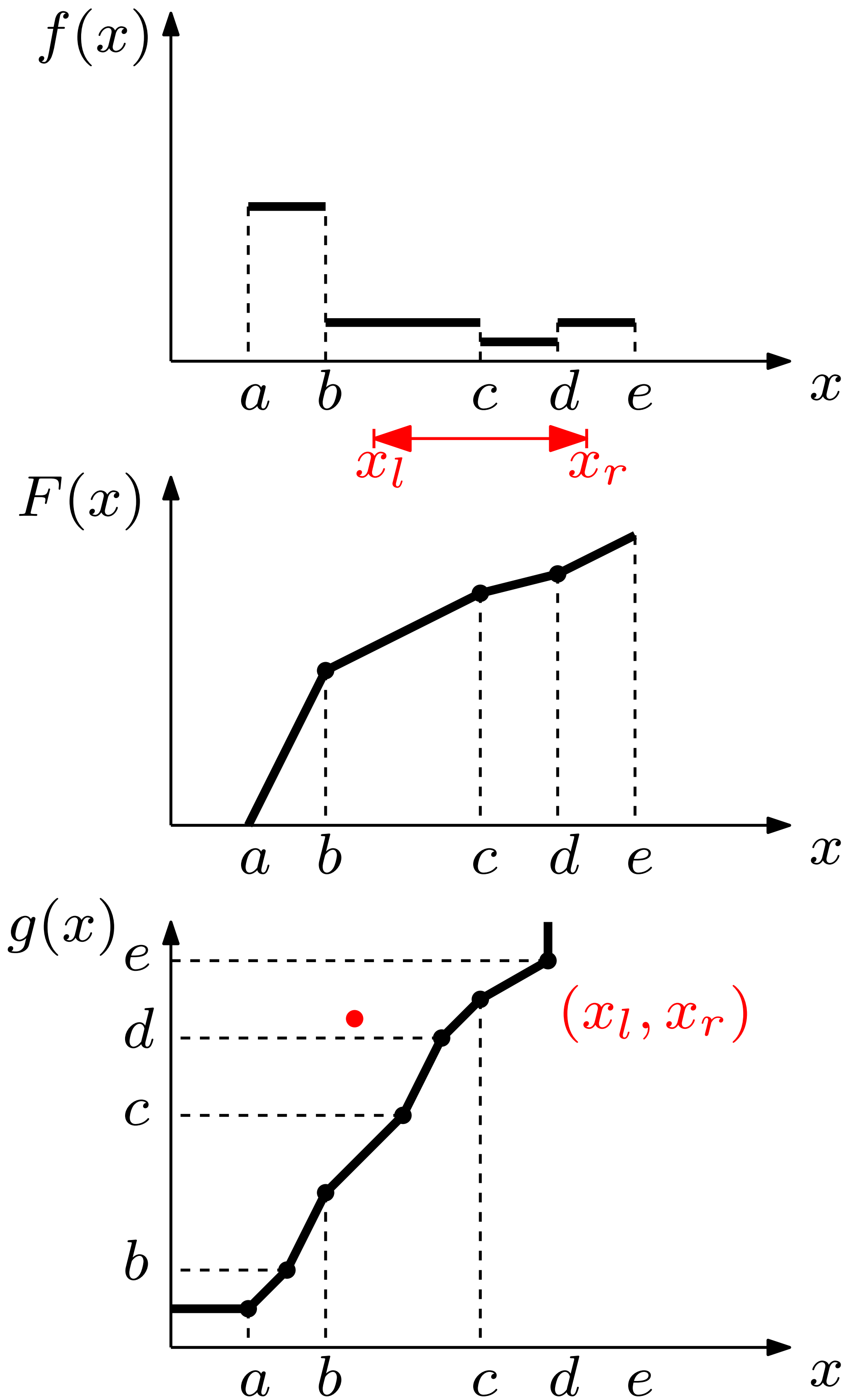

pdf

Properties of $g(x)$:

- $g(x)$ has complexity linear in the size of pdf
- $\Pr[p \in (x_l, x_r)] \geq \tau \Leftrightarrow x_r \geq g(x_l)$

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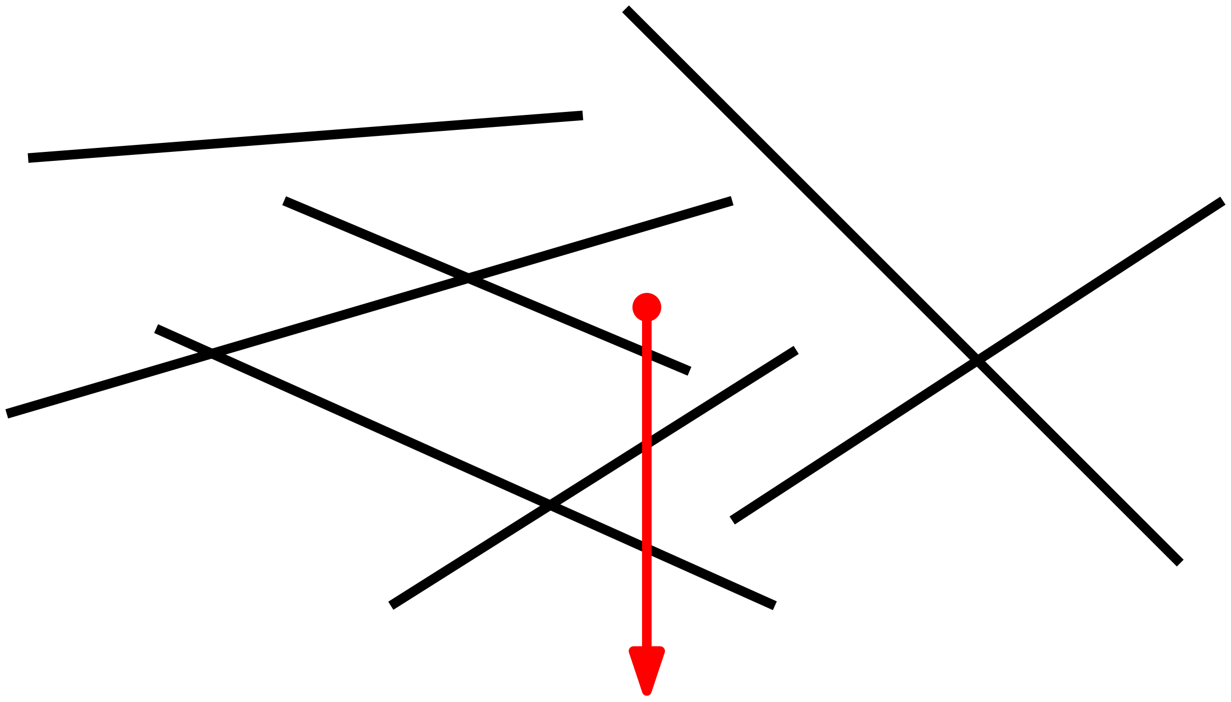

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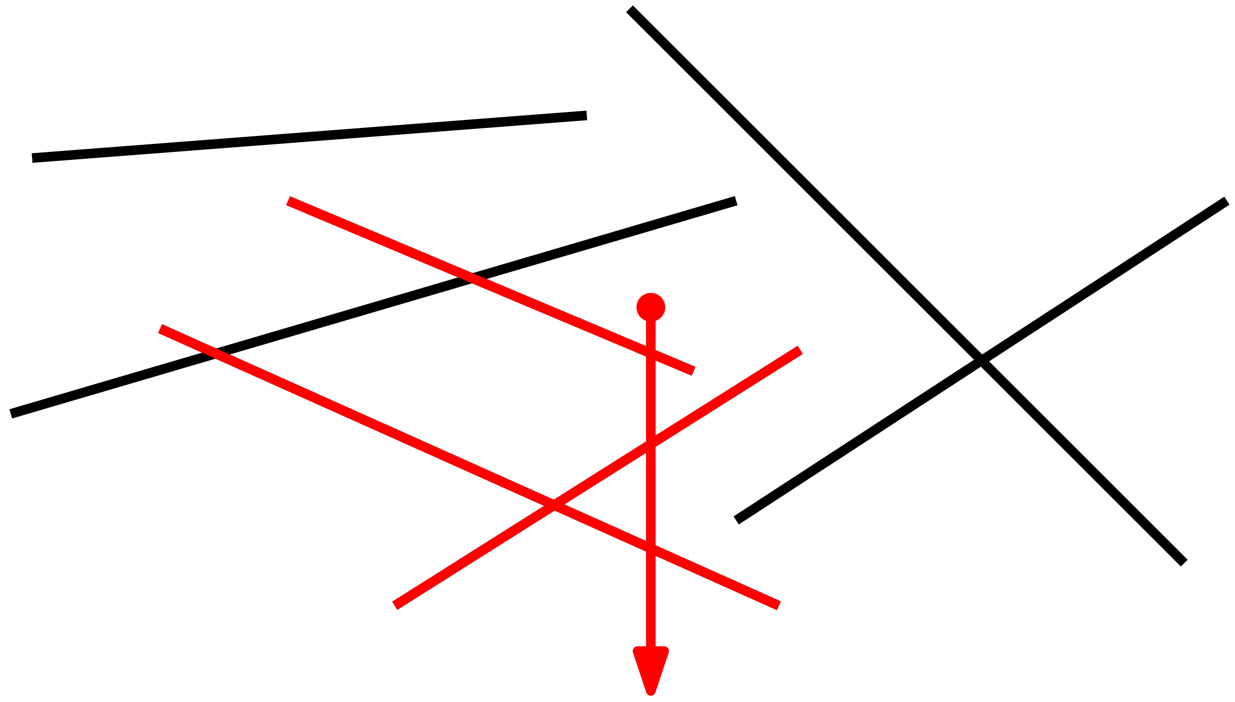

The segment-below-point problem

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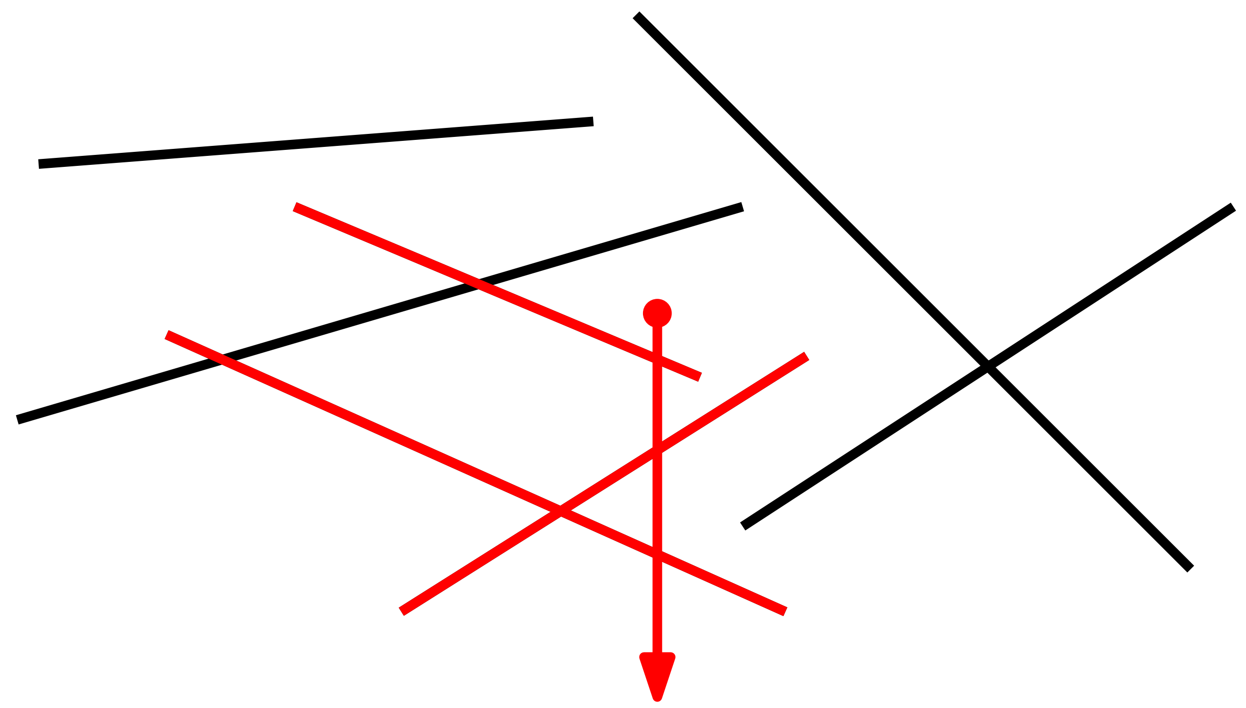
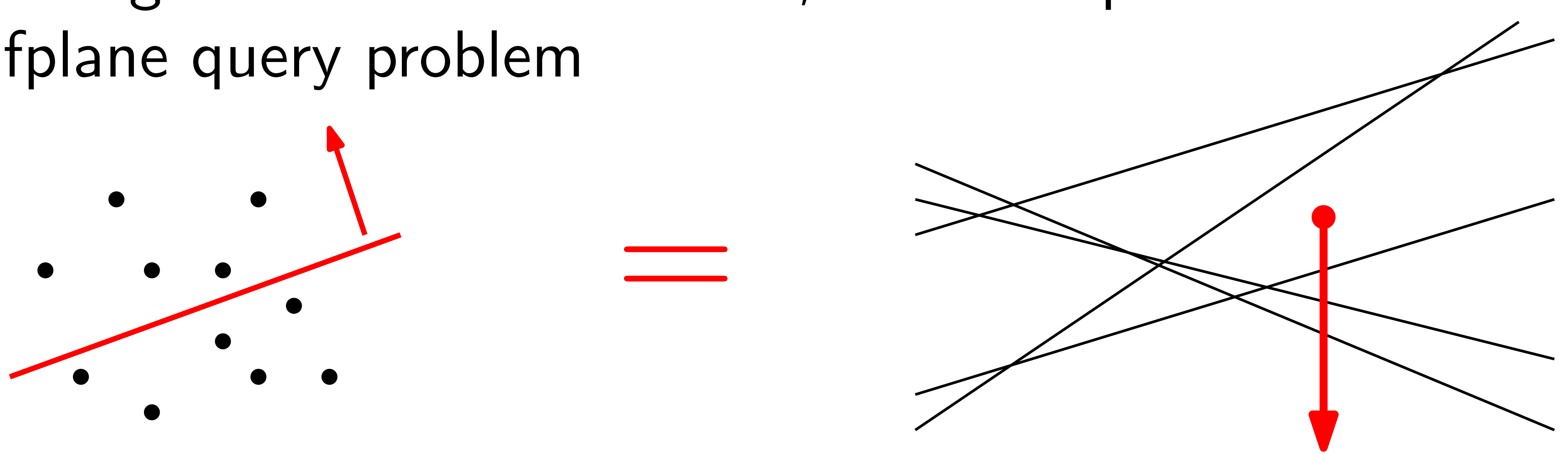


The segment-below-point problem

If all segments are infinite lines, then the problem becomes the halfplane query problem



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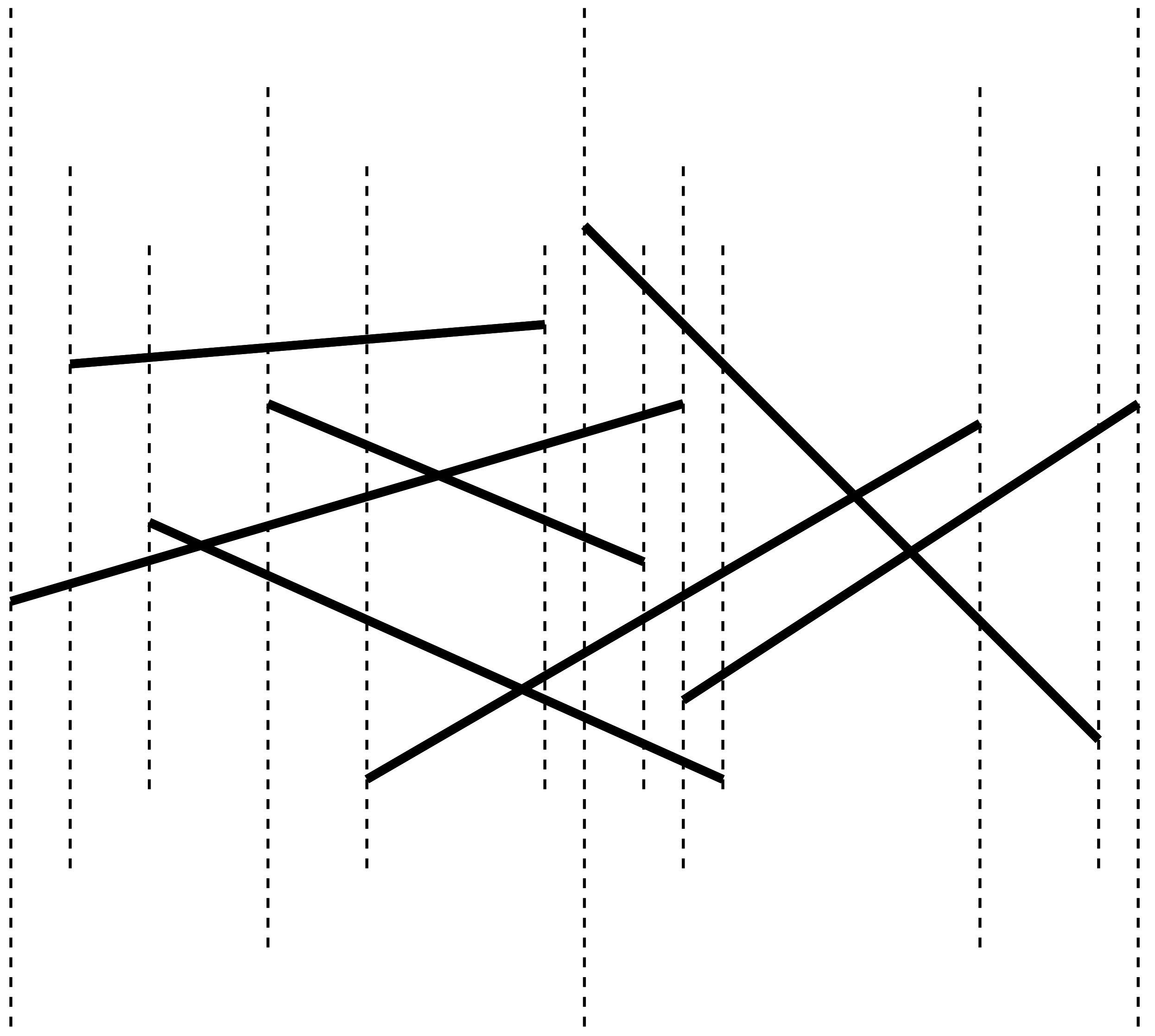

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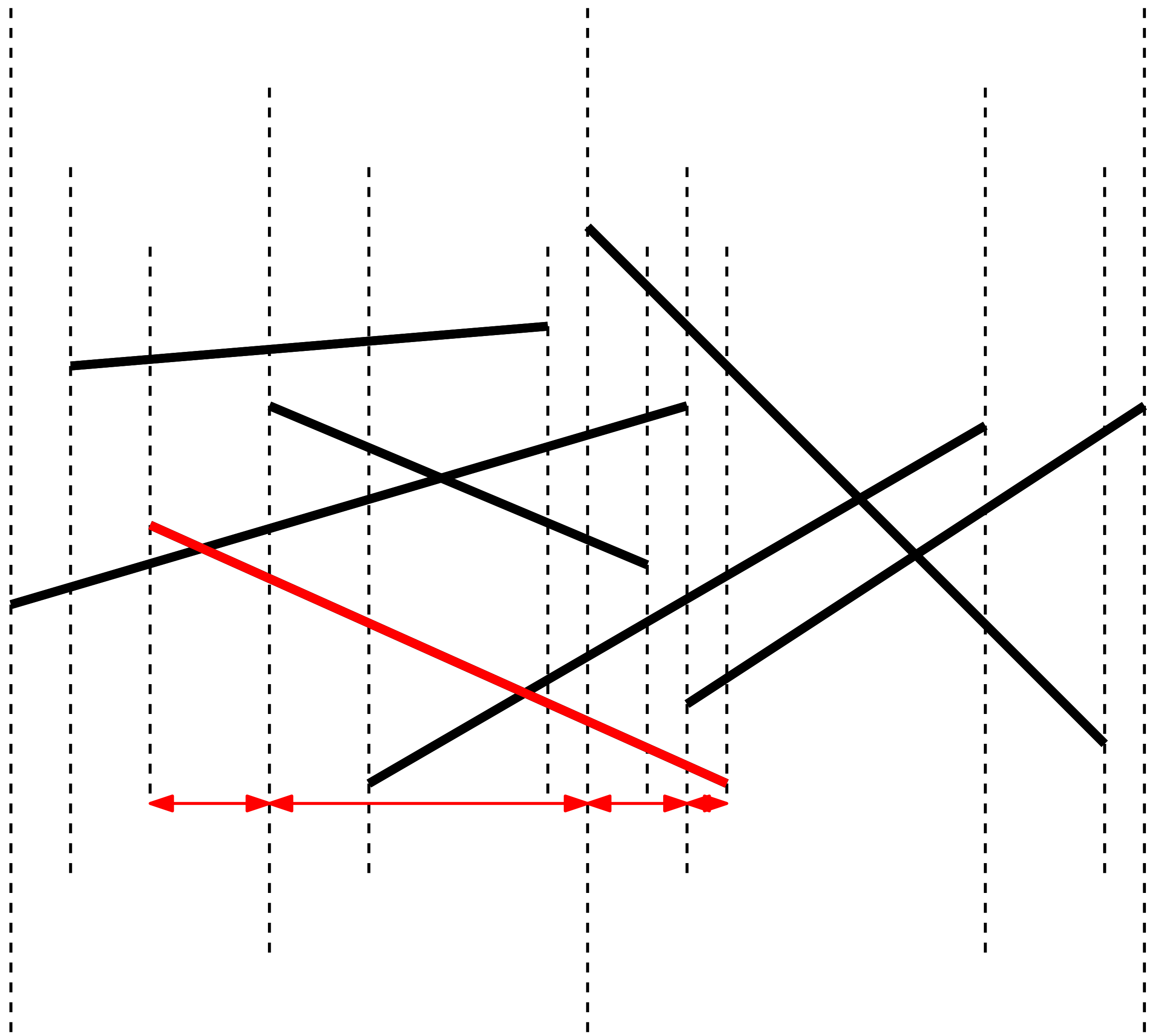

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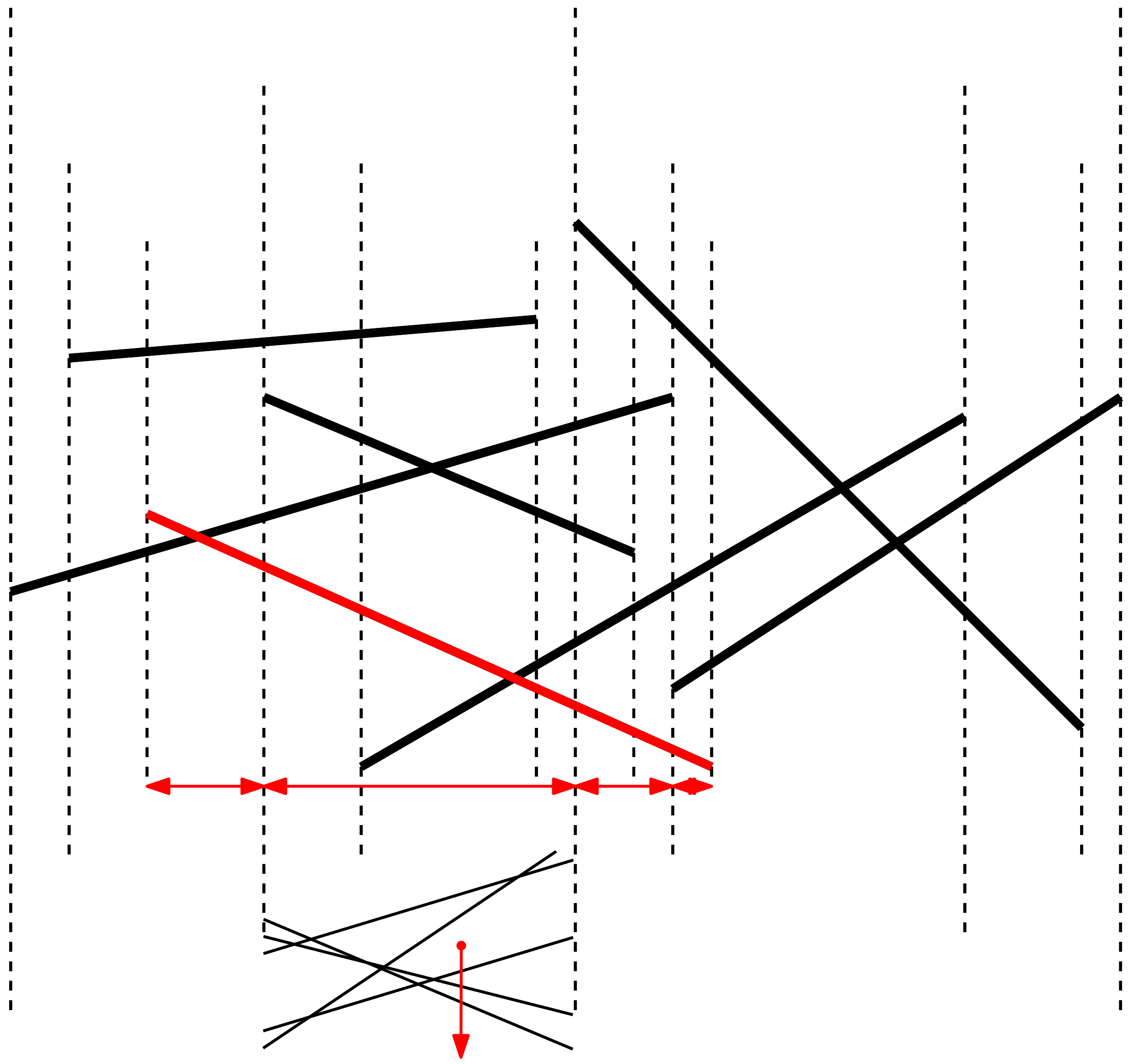

Segment tree approach:
$O(n)$ canonical slabs;
Each segment decomposed into $O(\log n)$ slabs;
Build a halfplance structure for each slab

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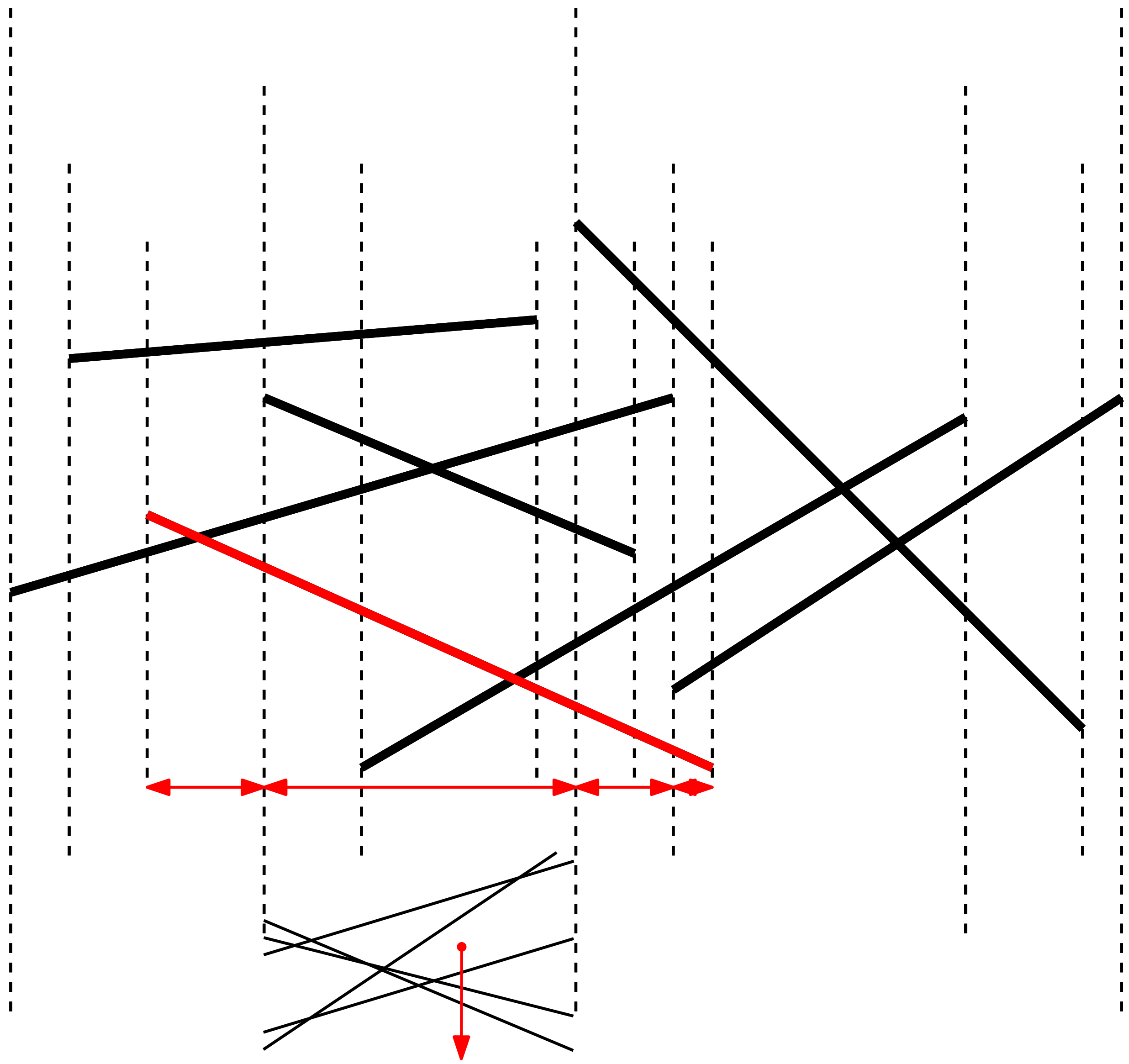

Obtain a structure:
size $O(n \log n)$,
query $O(\log^2 n + k)$.

Query can be improved to $O(\log n + k)$ using fractional cascading

But hard to reduce size to linear

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Queries with a Fixed τ: Achieving Optimal

- Clever ideas and complicated techniques
  - Segment tree
  - Interval tree
  - Sampling
  - Compression
  - Fractional cascading
  - Boostrapping

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- Clever ideas and complicated techniques
  - Segment tree
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  - Sampling
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  - Fractional cascading
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- Nice theoretical result, but too complicated to implement

A Simpler and More General Structure

(although not optimal)

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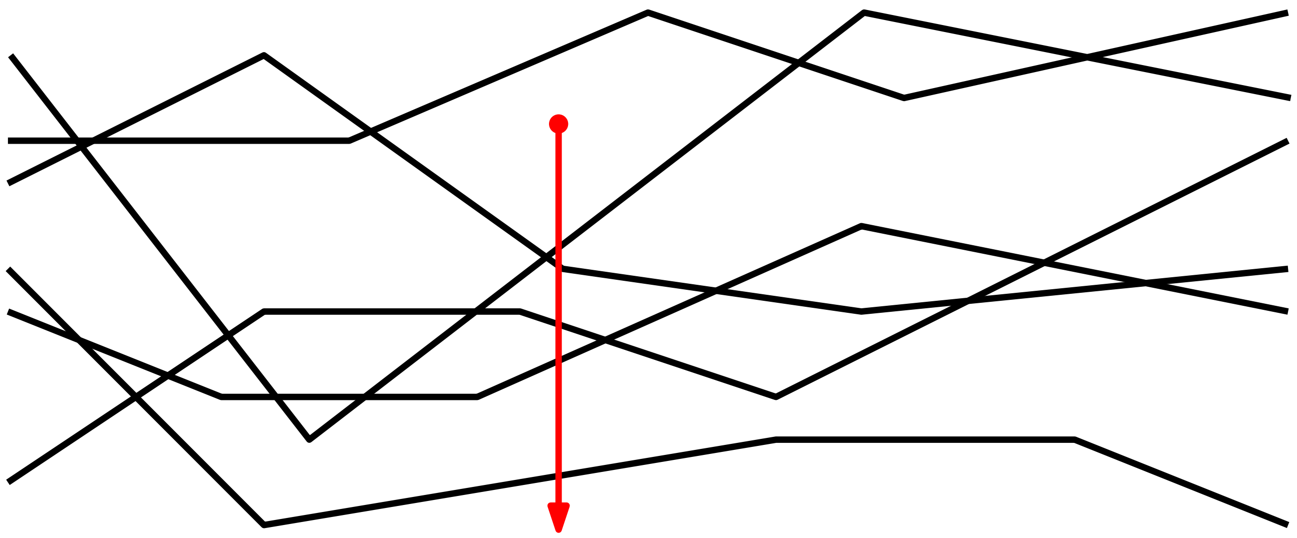
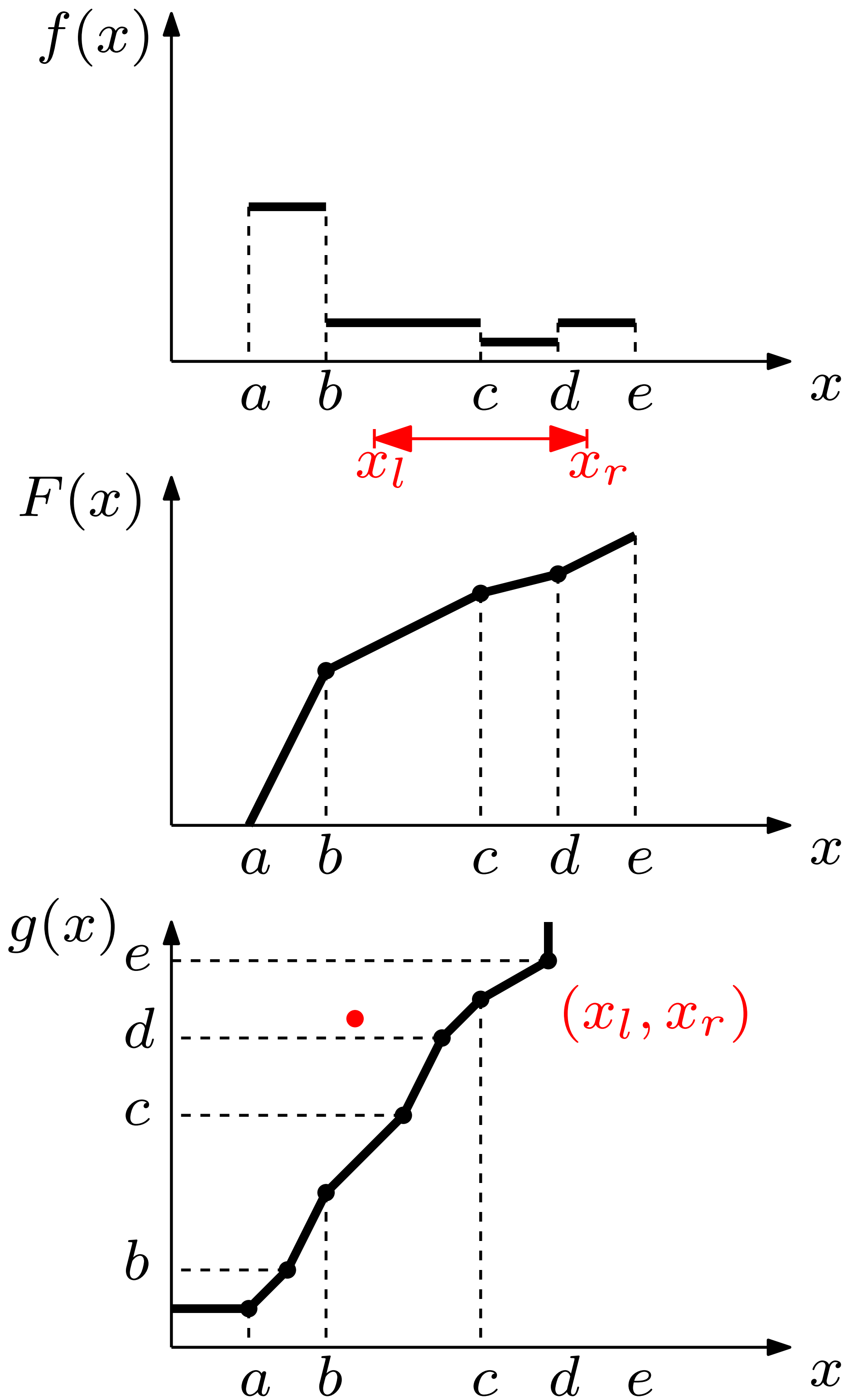
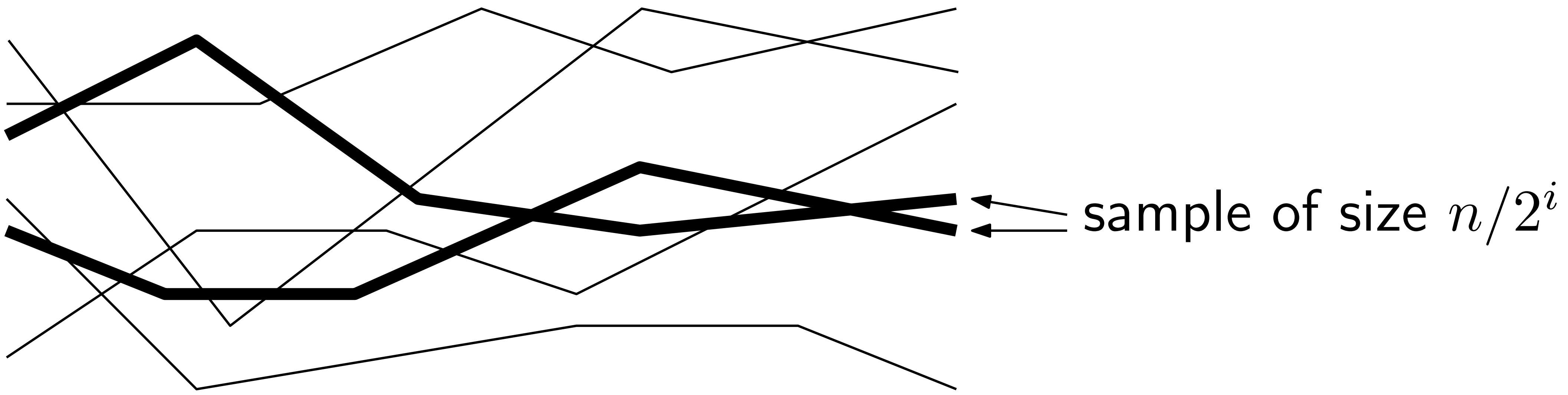




Report all polygonal chains below a query point

A Simpler and More General Structure



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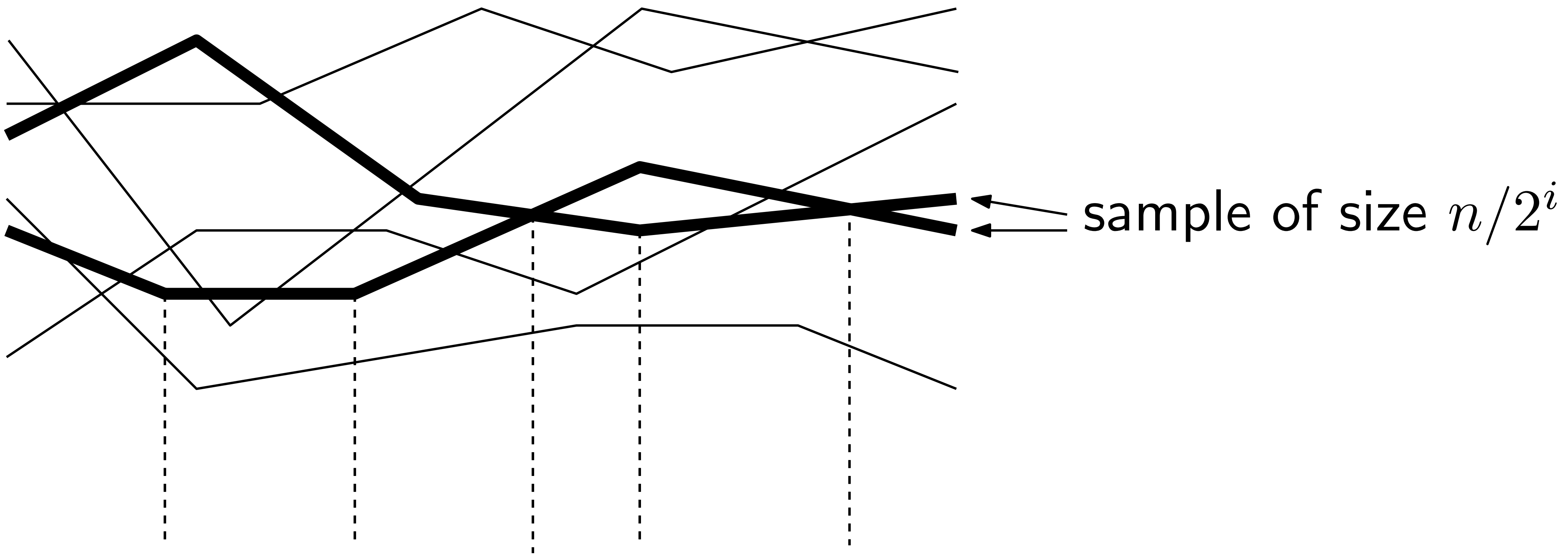

Compute the trapezoidal decomposition of the area bounded by the lower envelope of the sampled chains

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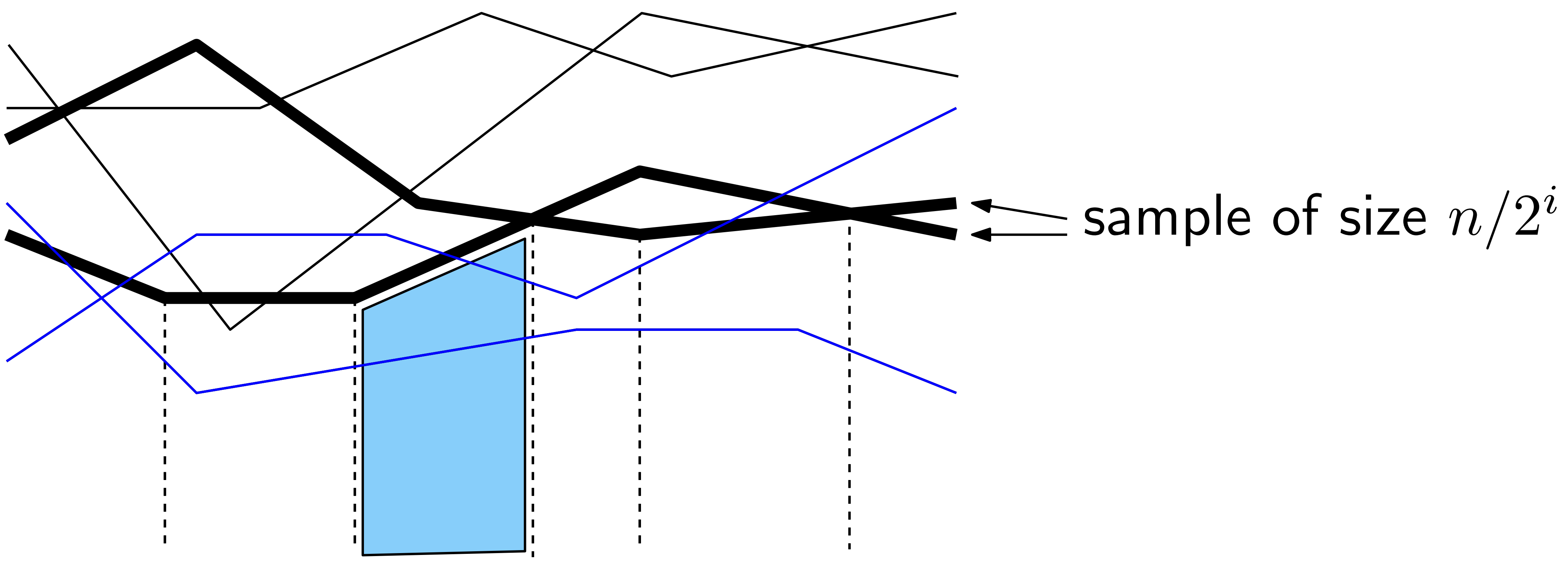

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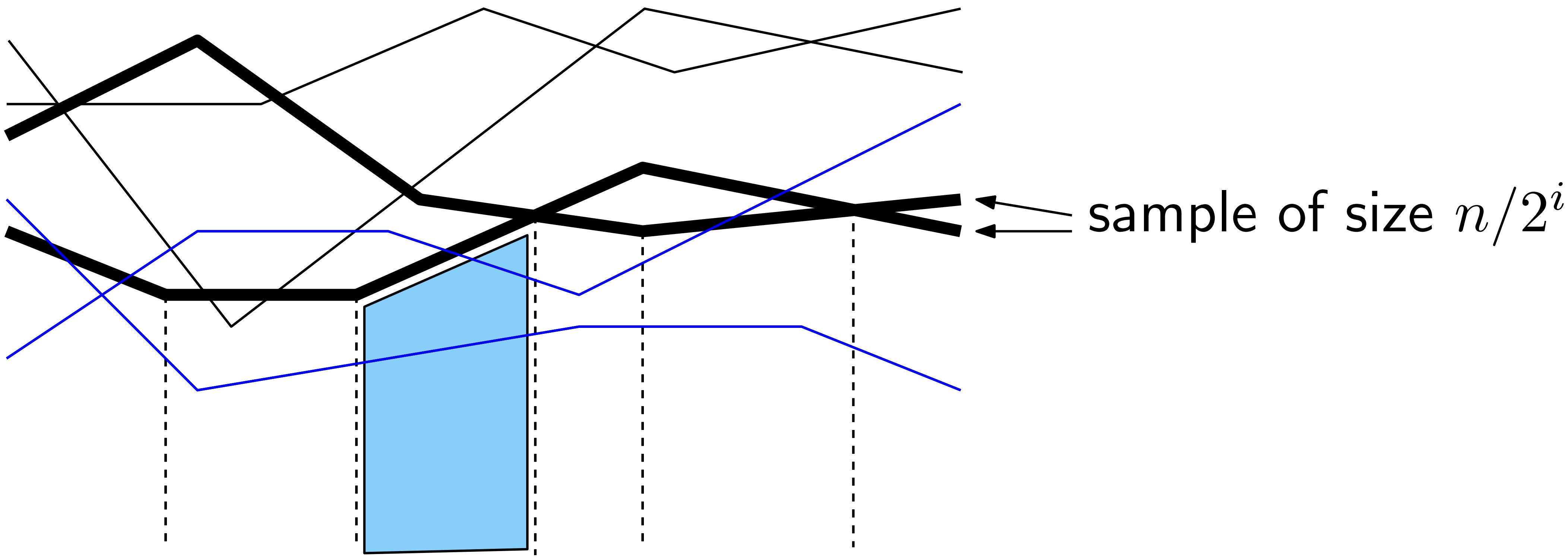

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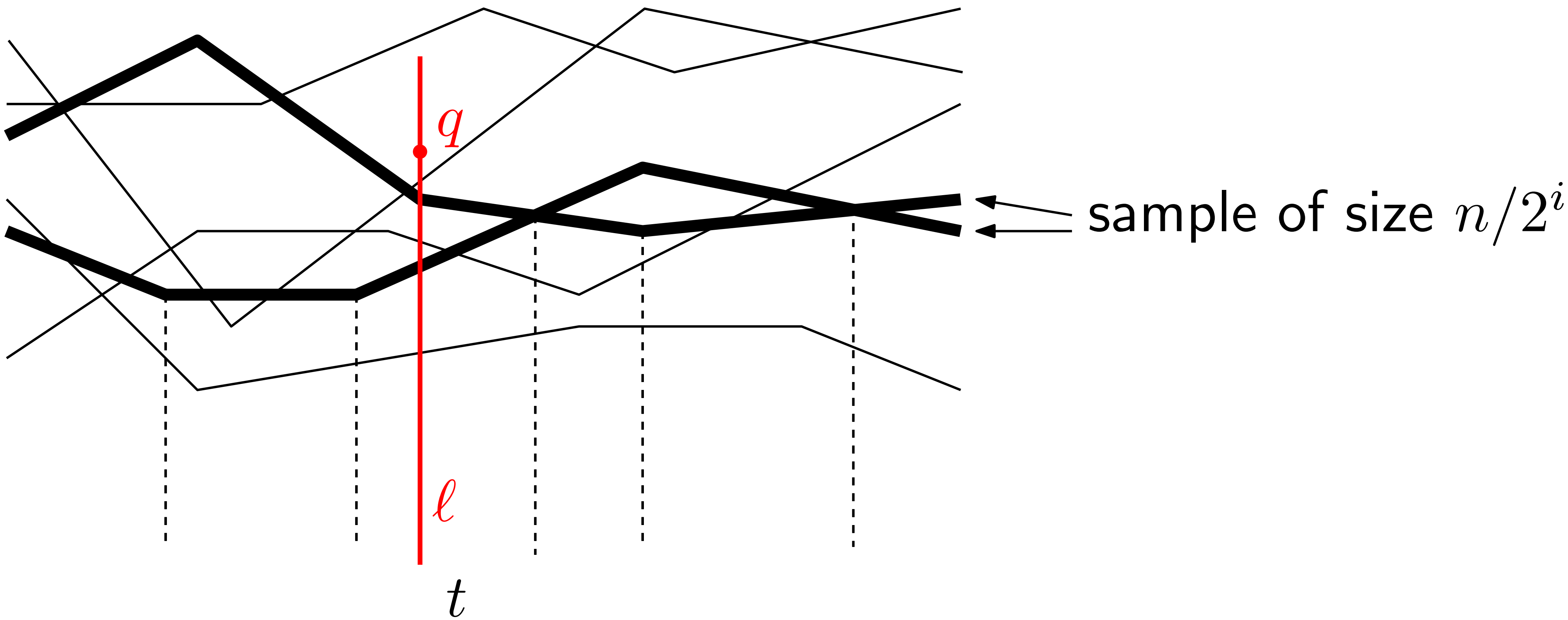
Do the above for $i = 1, 2, \ldots, \log n$

Querying the Structure



Try successive structures for $i = 1, 2, \ldots$:

locate the trapezoid t that ℓ intersects;

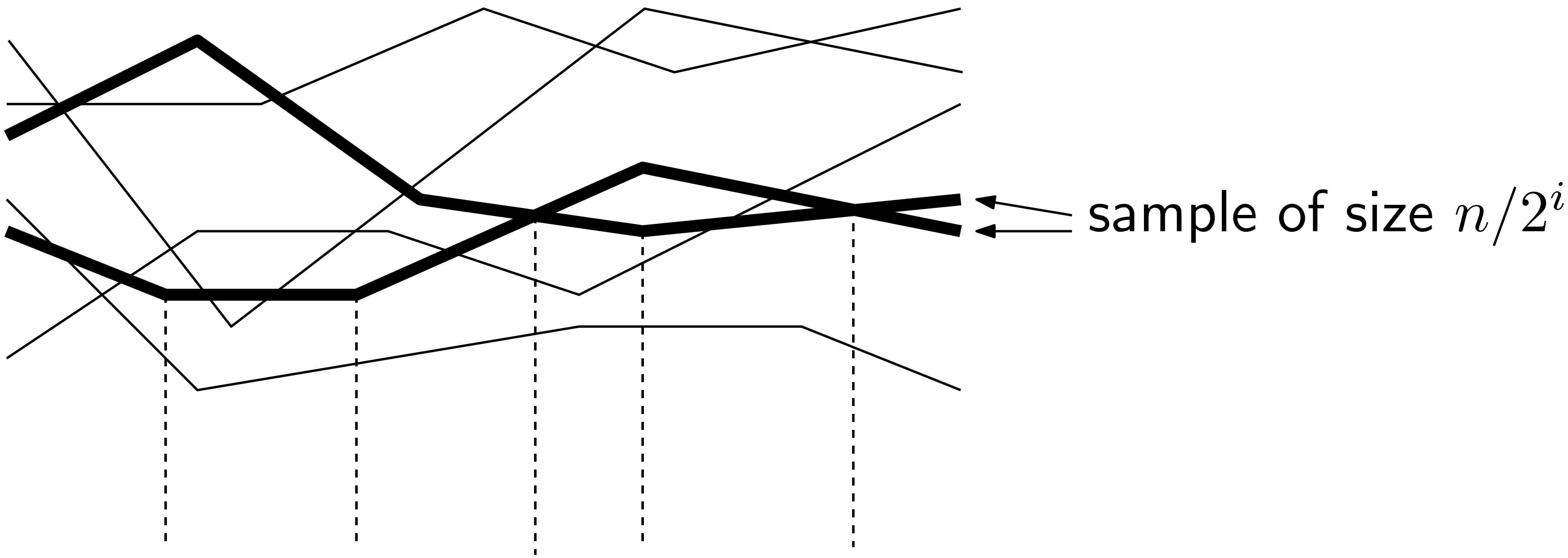
If $q \in t$

scan t's conflict list and report all chains below q;

stop;

Expected time: $O(\log n + k)$

Size of the Structure



size of each conflict list $= O(2^i)$
$\#$ trapezoids $=$ complexity of the lower envelope $= O(\frac{n}{2^i}\alpha(\frac{n}{2^i}))$
size of structure for each i: $O(n\alpha(n))$
total size: $O(n\alpha(n)\log n)$

$\alpha(n)$: inverse Ackermann function — extremely slow-growing

Supporting Other pdf's

- Suppose each pdf is piecewise algebraic
  - The threshold function $g(n)$ is also piecewise algebraic
- The structure and analysis remain the same, only the complexity of the lower envelope could change
- One can write out the piecewise form of $g(x)$, and determine the maximum number of intersections between any two different pieces, say c
- Query remains optimal $O(\log n + k)$
- Size becomes $O(\lambda_{c+2}(n) \log n)$ [Davenport, Schinzel '65]

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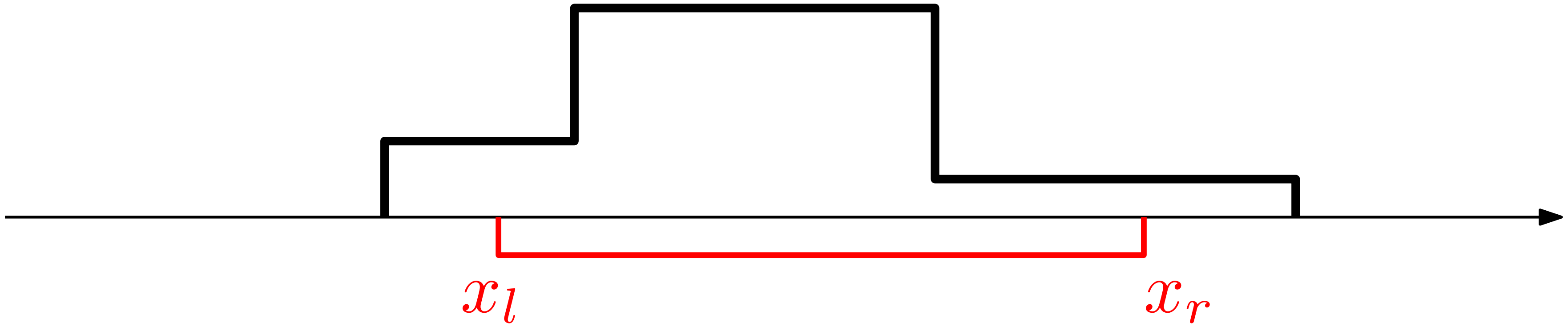
  - $\lambda_c(n)$: the maximum length of (n, c) *Davenport-Schinzel sequences*
  - $\lambda_2(n) = \Theta(n), \lambda_3(n) = \Theta(n\alpha(n))$, $\lambda_4(n) = \Theta(n2^{\alpha(n)})$, $\lambda_{2t+2}(n) = n2^{(1/t!)\alpha^t(n)+\Theta(\alpha^{t-1}(n))}$ [Agarwal, Sharir '00]

Queries with Variable τ



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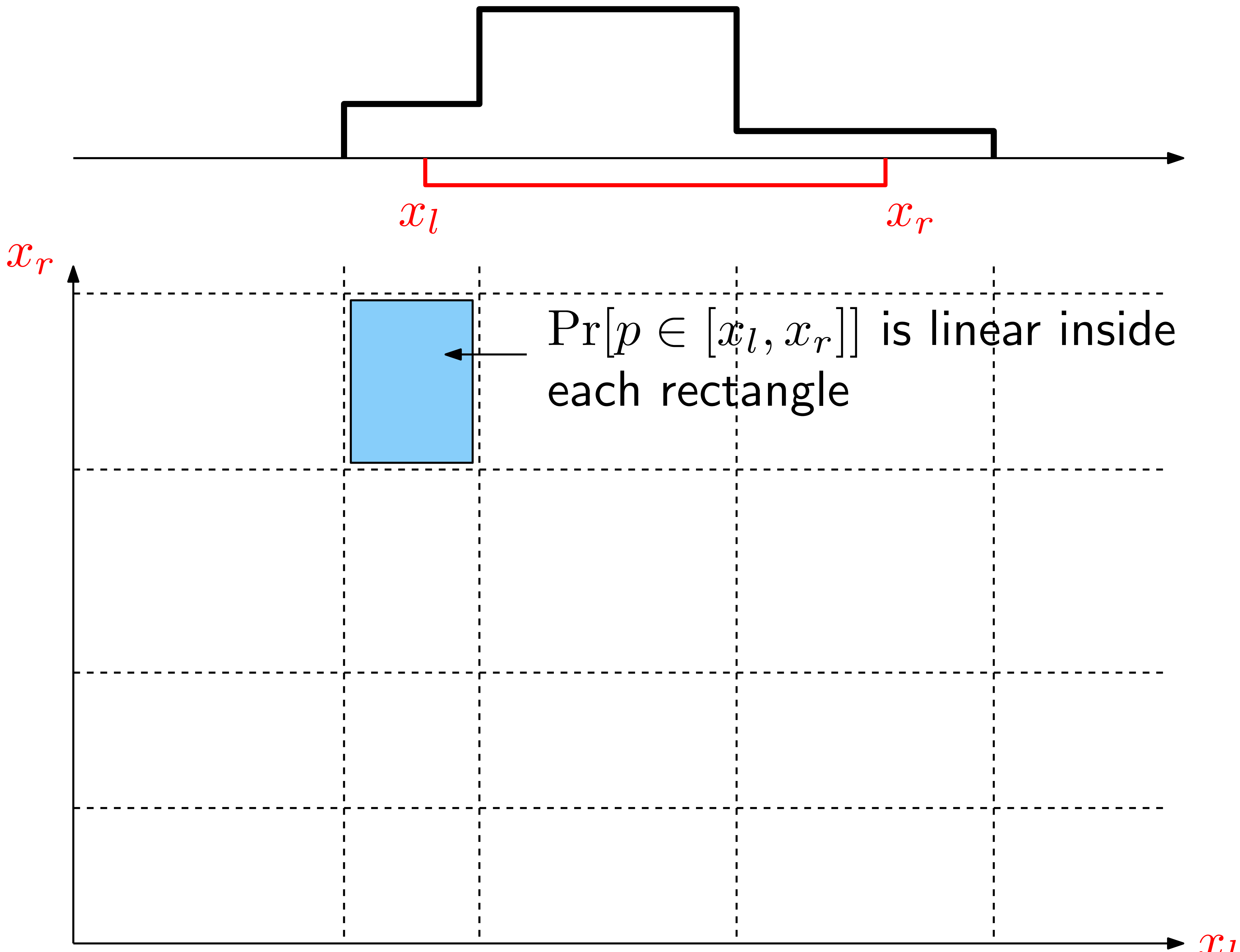

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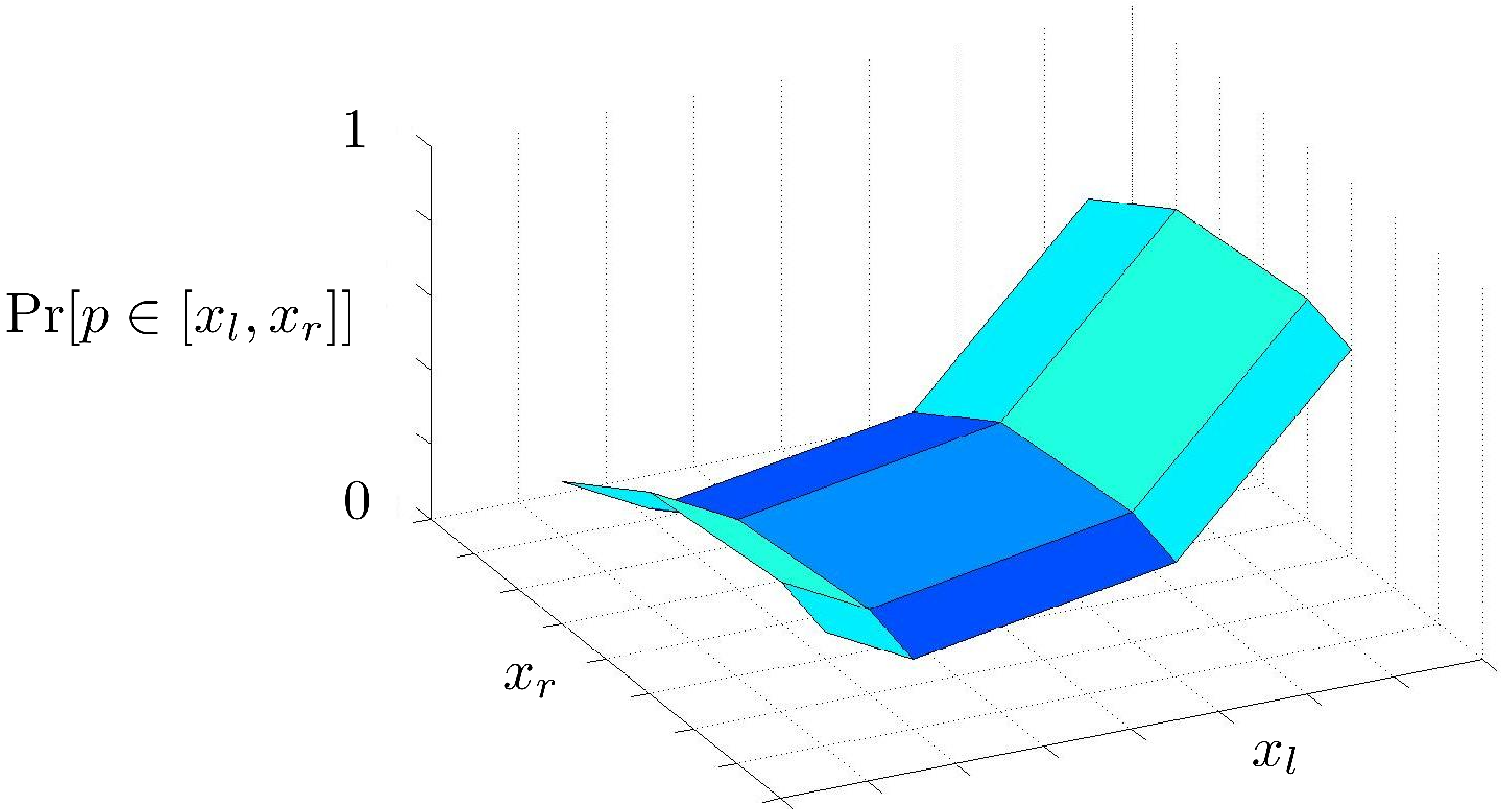

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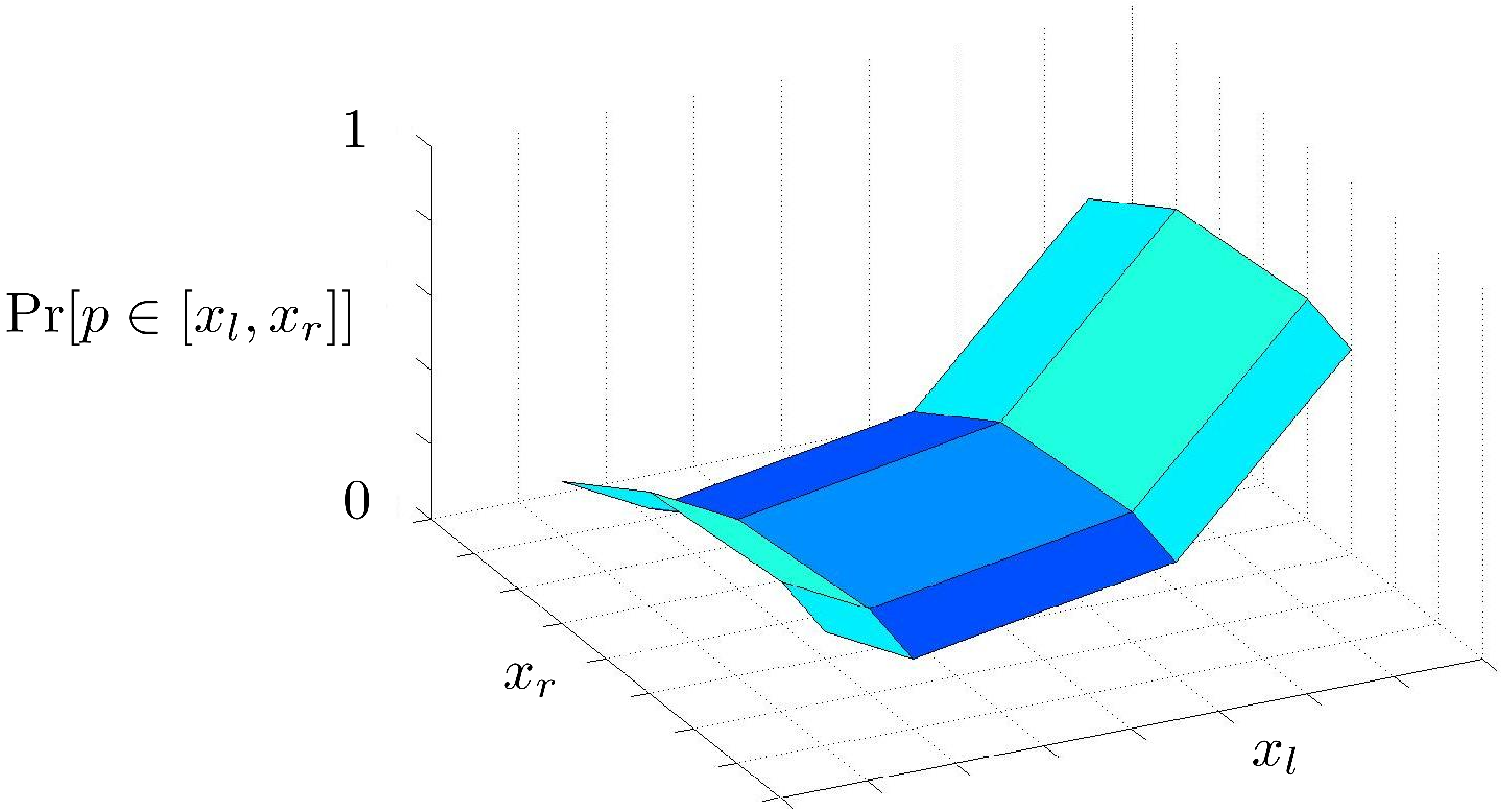
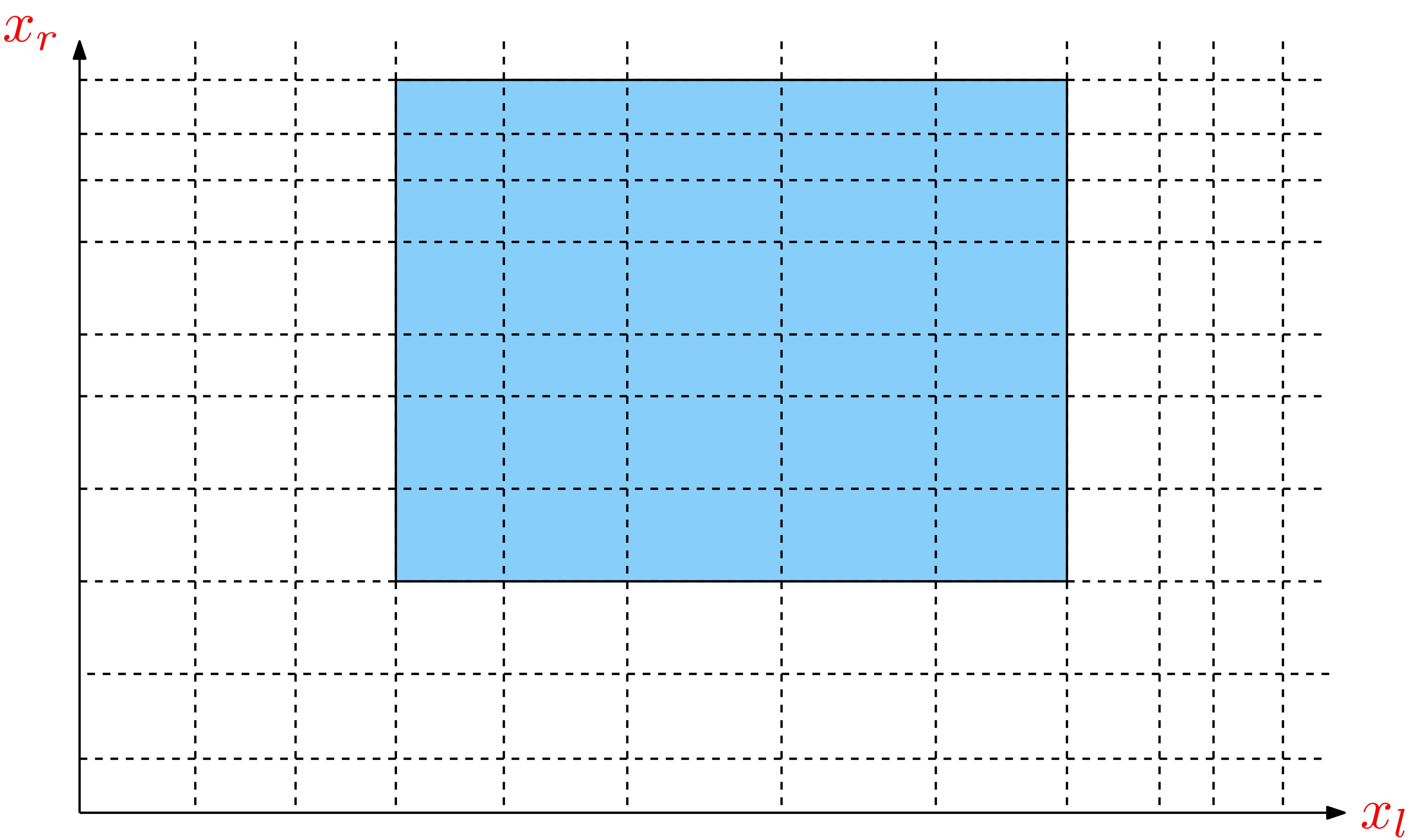


Problem: Indexing a collection of bivariate piecewise-linear functions where each piece spans an orthogonal rectangle, such that for a given query point q in 3D, we can report all functions below q

Queries with Variable τ



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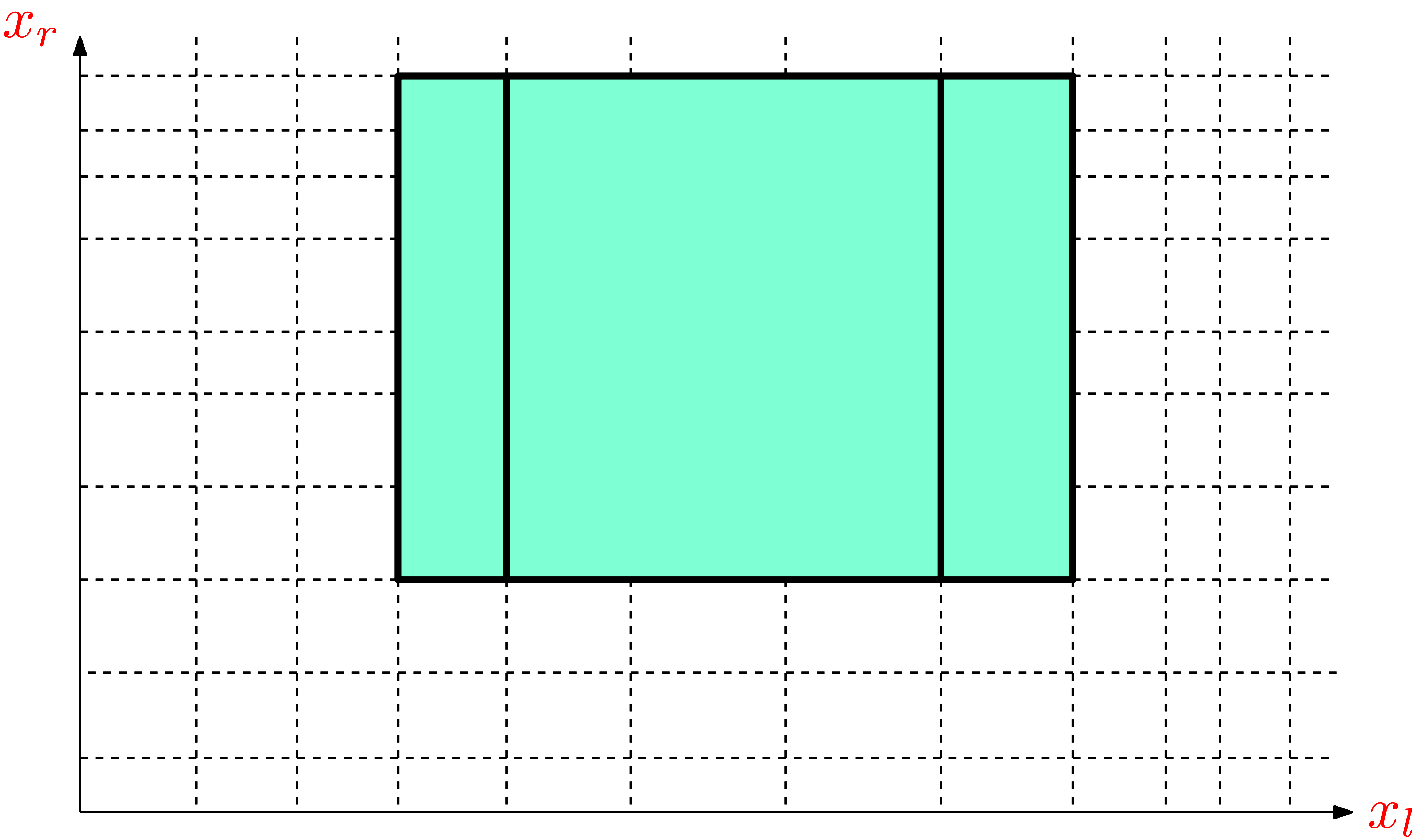

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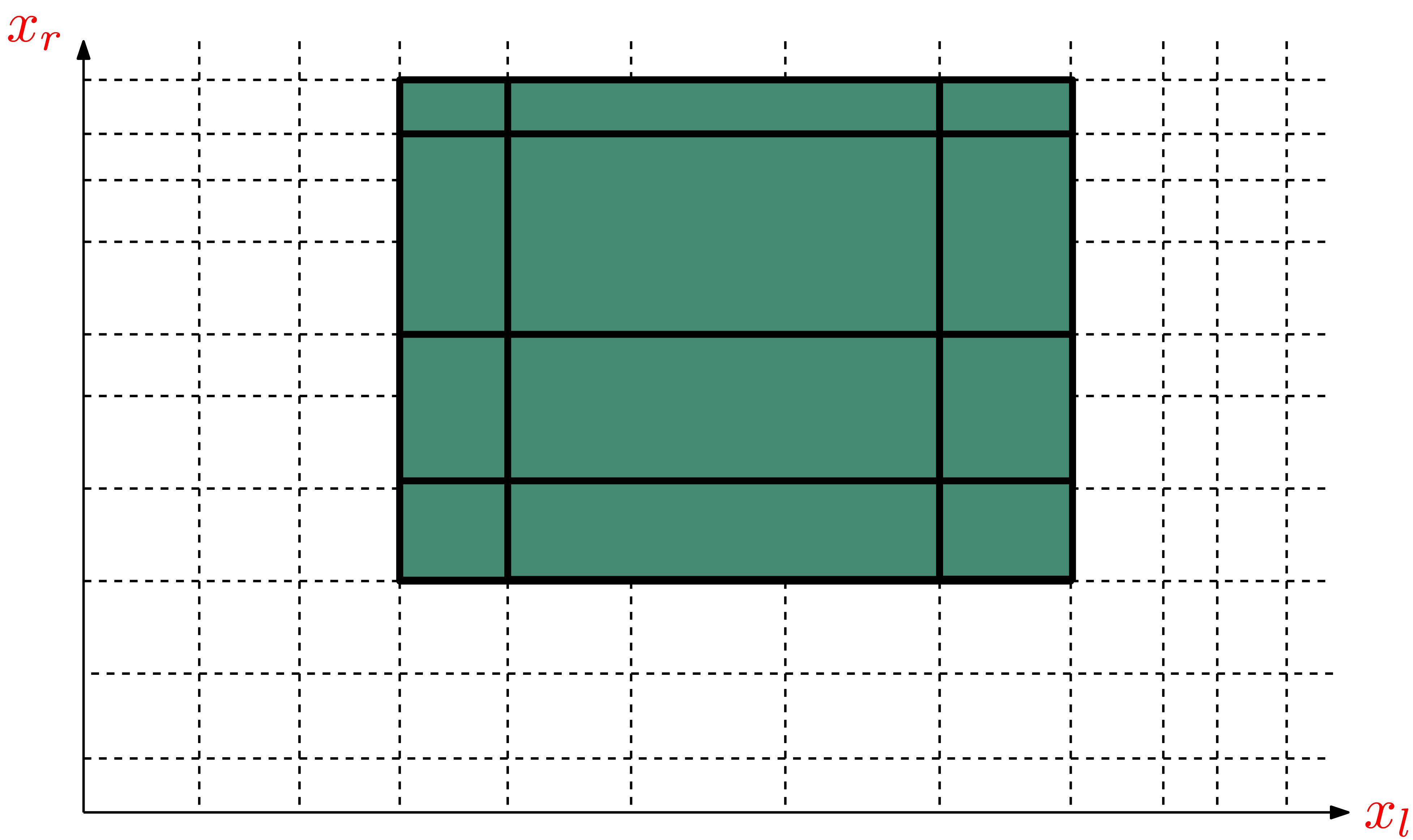

Each piece is decomposed into $O(\log^2 n)$ canonical rectangles

Queries with Variable τ

- For each canonical rectangle, build a 3D halfspace structure
  - Linear size, $O(\log n + k)$ query [Afshani, Chan '09]
- Total size of the index: $O(n \log^2 n)$

Queries with Variable τ

- For each canonical rectangle, build a 3D halfspace structure
  - Linear size, $O(\log n + k)$ query [Afshani, Chan '09]
- Total size of the index: $O(n \log^2 n)$
- Answering a query
  - The query point is covered by $O(\log n)$ canonical vertical slabs
  - Inside each canonical horizontal slabs, there are $O(\log n)$ canonical vertical slabs (rectangles) covering q
  - Need to query $O(\log^2 n)$ 3D halfspace structures
  - Total query time: $O(\log^3 n + k)$

Conclusions

- Queries with a fixed threshold
  - Can solve in linear space and optimal query time
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- Problems to consider
  - Higher dimensions?
  - Range counting in uncertain db?
  - Nearest neighbors in uncertain db?