Semantics of Ranking Queries for Probabilistic Data and Expected Ranks

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- Many types of real-world data are uncertain: sensor readings, multimedia, data integration, ...

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- An uncertain database is a probability distribution of certain (deterministic) databases

$\{t_1, t_2, t_3\}$, prob $= 0.1$

$\{t_1, t_3, t_4\}$, prob $= 0.2$

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(e.g. an x-relation)

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tuples	$p(t)$
t_1	0.4
t_2	0.5
t_3	1
t_4	0.5

rules	
τ_1	$\{t_1\}$
τ_2	$\{t_2, t_4\}$
τ_3	$\{t_3\}$

Query algorithm

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Query semantics
(definition)

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  - Or, rank all the tuples by this probability [Re, Dalvi, Suciu, 07]
- Has been applied to a variety of queries
  - Selection, projection, join [Dalvi, Suciu, 04]
  - Range queries [Tao, Cheng, Xiao, Ngai, Kao, Prabhakar, 05]
  - Frequent items [Zhang, Li, Yi, 08]

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  - Expected ranks: [this work]

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t_3	80	1
t_4	70	0.5

rules	
τ_1	$\{t_1\}$
τ_2	$\{t_2, t_4\}$
τ_3	$\{t_3\}$

world W	$\Pr[W]$
$\{t_1, t_2, t_3\}$	$p(t_1)p(t_2)p(t_3) = 0.2$
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tuple-level uncertainty

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t_3	$\{(85, 1)\}$

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$\{t_1 = 100, t_2 = 92, t_3 = 85\}$	$0.4 \times 0.6 \times 1 = 0.24$
$\{t_1 = 100, t_3 = 85, t_2 = 80\}$	$0.4 \times 0.4 \times 1 = 0.16$
$\{t_2 = 92, t_3 = 85, t_1 = 70\}$	$0.6 \times 0.6 \times 1 = 0.36$
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attribute-level (score-level) uncertainty

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Suppose $k = 2, p = 0.5$

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At rank $i = 1$

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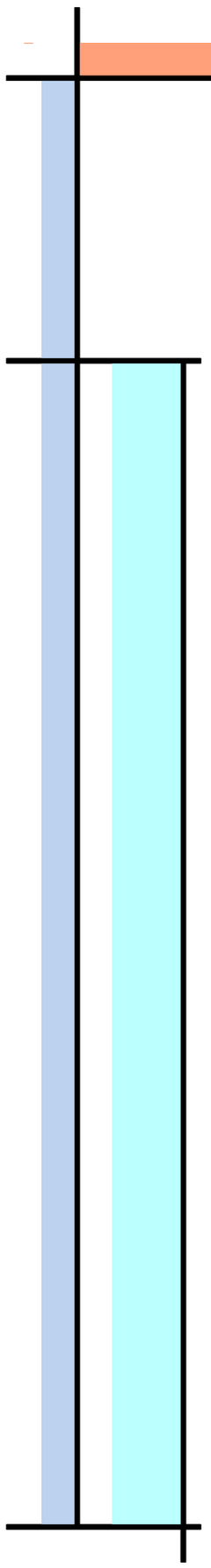

In stead of jumping forward and formulate yet another definition, we step back and first think about what properties that a ranking query should satisfy.

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U-top$k$	✓
U-kRanks	✓
PT-k	✓
Global-top$k$	✓

Property two: *Exact-k*

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At rank 1:

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At rank 3:

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U-top$k$ with $k = 1$

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Global-top$k$ with $k = 1$

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Global-top$k$ with $k = 2$

$\Pr[t_1] = 0.4$

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Property five: *Stability*

Making an item in the top-k list more likely (higher probability) or more important (higher score) should not remove it from the list.
Also in [Zhang, Chomicki, 08]

In the score-level uncertainty model, replace "higher probability or higher score" with "stochastically greater than"

Ranking method	Unique-rank	
U-top$k$	✓	
U-kRanks	×	[Zhang, Chomicki, 08]
PT-k	✓	
Global-top$k$	✓	

Properties: Summary

Ranking method	*Exact-k*	*Containment*	*Unique-Rank*	*Value-Invariant*	*Stability*
U-top$k$	weak	$\times$	✓	✓	✓
U-kRanks	✓	✓	$\times$	✓	$\times$
PT-k	$\times$	weak	✓	✓	✓
Global-top$k$	✓	$\times$	✓	✓	✓

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t_1	$\{(100, 0.4), (70, 0.6)\}$
t_2	$\{(92, 0.6), (80, 0.4)\}$
t_3	$\{(85, 1)\}$

$$E[r(t2)] = 0.24 \times 2 + 0.16 \times 3 + 0.36 \times 1 + 0.24 \times 2 = 1.8$$

world W	$\Pr[W]$
$\{t_1 = 100, t_2 = 92, t_3 = 85\}$	$0.4 \times 0.6 \times 1 = 0.24$
$\{t_1 = 100, t_3 = 85, t_2 = 80\}$	$0.4 \times 0.4 \times 1 = 0.16$
$\{t_2 = 92, t_3 = 85, t_1 = 70\}$	$0.6 \times 0.6 \times 1 = 0.36$
$\{t_3 = 85, t_2 = 80, t_1 = 70\}$	$0.6 \times 0.4 \times 1 = 0.24$

Our definition: Expected Ranks

- Consider $E[t\text{'s rank}]$, and return the k tuples having the highest expected ranks

tuples	score
t_1	$\{(100, 0.4), (70, 0.6)\}$
t_2	$\{(92, 0.6), (80, 0.4)\}$
t_3	$\{(85, 1)\}$

$$E[r(t1)] = 2.2$$

$$E[r(t2)] = 0.24 \times 2 + 0.16 \times 3 + 0.36 \times 1 + 0.24 \times 2 = 1.8$$

$$E[r(t3)] = 2$$

world W	$\Pr[W]$
$\{t_1 = 100, t_2 = 92, t_3 = 85\}$	$0.4 \times 0.6 \times 1 = 0.24$
$\{t_1 = 100, t_3 = 85, t_2 = 80\}$	$0.4 \times 0.4 \times 1 = 0.16$
$\{t_2 = 92, t_3 = 85, t_1 = 70\}$	$0.6 \times 0.6 \times 1 = 0.36$
$\{t_3 = 85, t_2 = 80, t_1 = 70\}$	$0.6 \times 0.4 \times 1 = 0.24$

Our definition: Expected Ranks

- If a tuple doesn't appear in a world, its rank is considered to be the last one

Our definition: Expected Ranks

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tuples	score	$p(t)$
t_1	100	0.4
t_2	92	0.5
t_3	80	1
t_4	70	0.5

rules	
τ_1	$\{t_1\}$
τ_2	$\{t_2, t_4\}$
τ_3	$\{t_3\}$

$$E[r(t2)] = 0.2\times 2 + 0.2\times 4 + 0.3\times 1 + 0.3\times 3 = 2.4$$

world W	$\Pr[W]$
$\{t_1, t_2, t_3\}$	$p(t_1)p(t_2)p(t_3) = 0.2$
$\{t_1, t_3, t_4\}$	$p(t_1)p(t_3)p(t_4) = 0.2$
$\{t_2, t_3\}$	$(1-p(t_1))p(t_2)p(t_3) = 0.3$
$\{t_3, t_4\}$	$(1-p(t_1))p(t_3)p(t_4) = 0.3$

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$\{t_2, t_3\}$	$(1 - p(t_1))p(t_2)p(t_3) = 0.3$
$\{t_3, t_4\}$	$(1 - p(t_1))p(t_3)p(t_4) = 0.3$

$$E[r(t1)] = 2.2$$

$$E[r(t2)] = 0.2 \times 2 + 0.2 \times 4 + 0.3 \times 1 + 0.3 \times 3 = 2.4$$

$$E[r(t3)] = 1.9$$

$$E[r(t3)] = 2.9$$

Properties: Summary

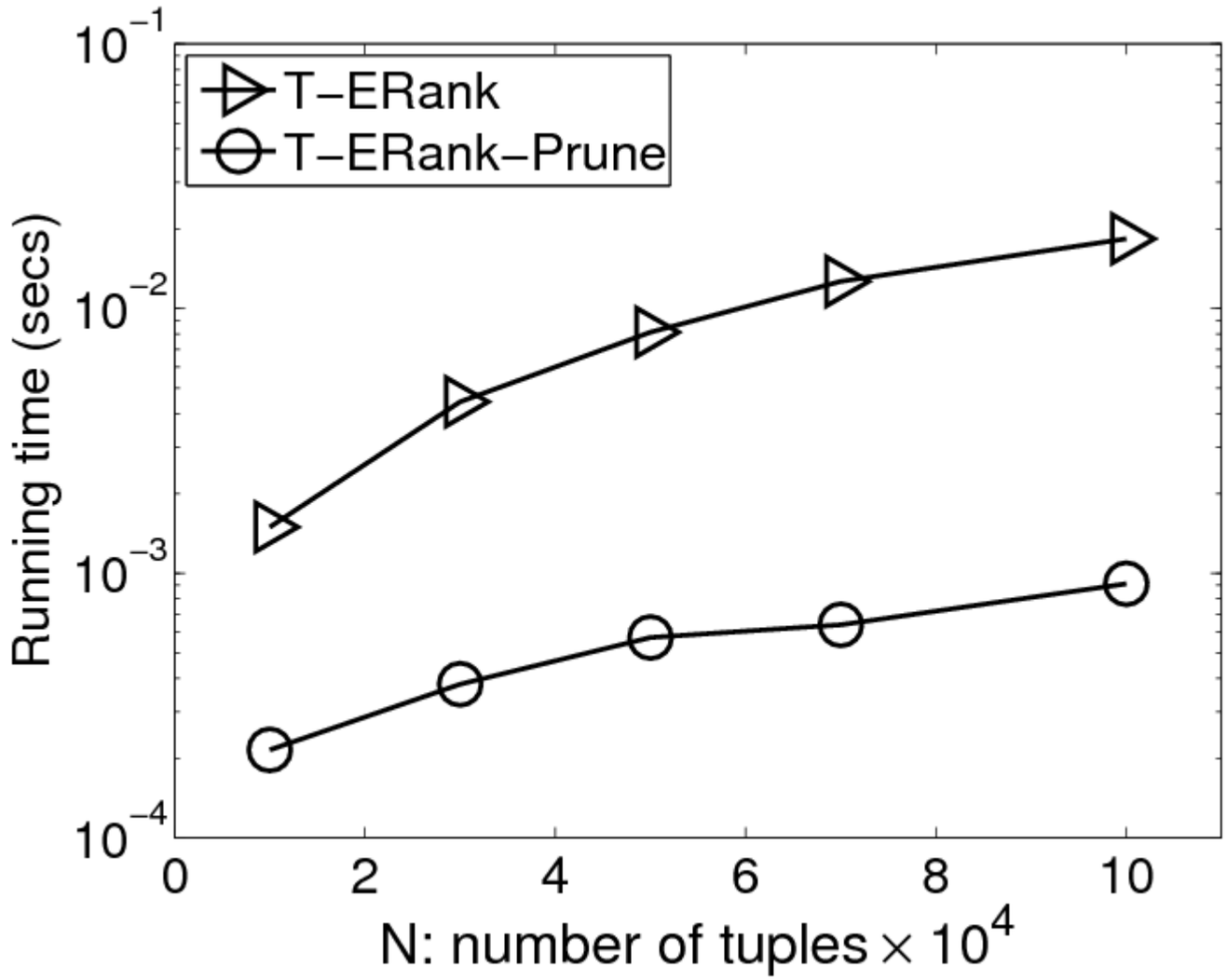
Ranking method	*Exact-k*	*Containment*	*Unique-Rank*	*Value-Invariant*	*Stability*
U-top$k$	weak	$\times$	✓	✓	✓
U-kRanks	✓	✓	$\times$	✓	$\times$
PT-k	$\times$	weak	✓	✓	✓
Global-top$k$	✓	$\times$	✓	✓	✓
Expected rank	✓	✓	✓	✓	✓

Computation

Ranking method	*Exact-k*	*Containment*	*Unique-Rank*	*Value-Invariant*	*Stability*	*Computation*
U-top$k$	weak	$\times$	✓	✓	✓	$n \log n$
U-kRanks	✓	✓	$\times$	✓	$\times$	kn^2
PT-k	$\times$	weak	✓	✓	✓	kn^2
Global-top$k$	✓	$\times$	✓	✓	✓	kn^2
Expected rank	✓	✓	✓	✓	✓	$n \log n$

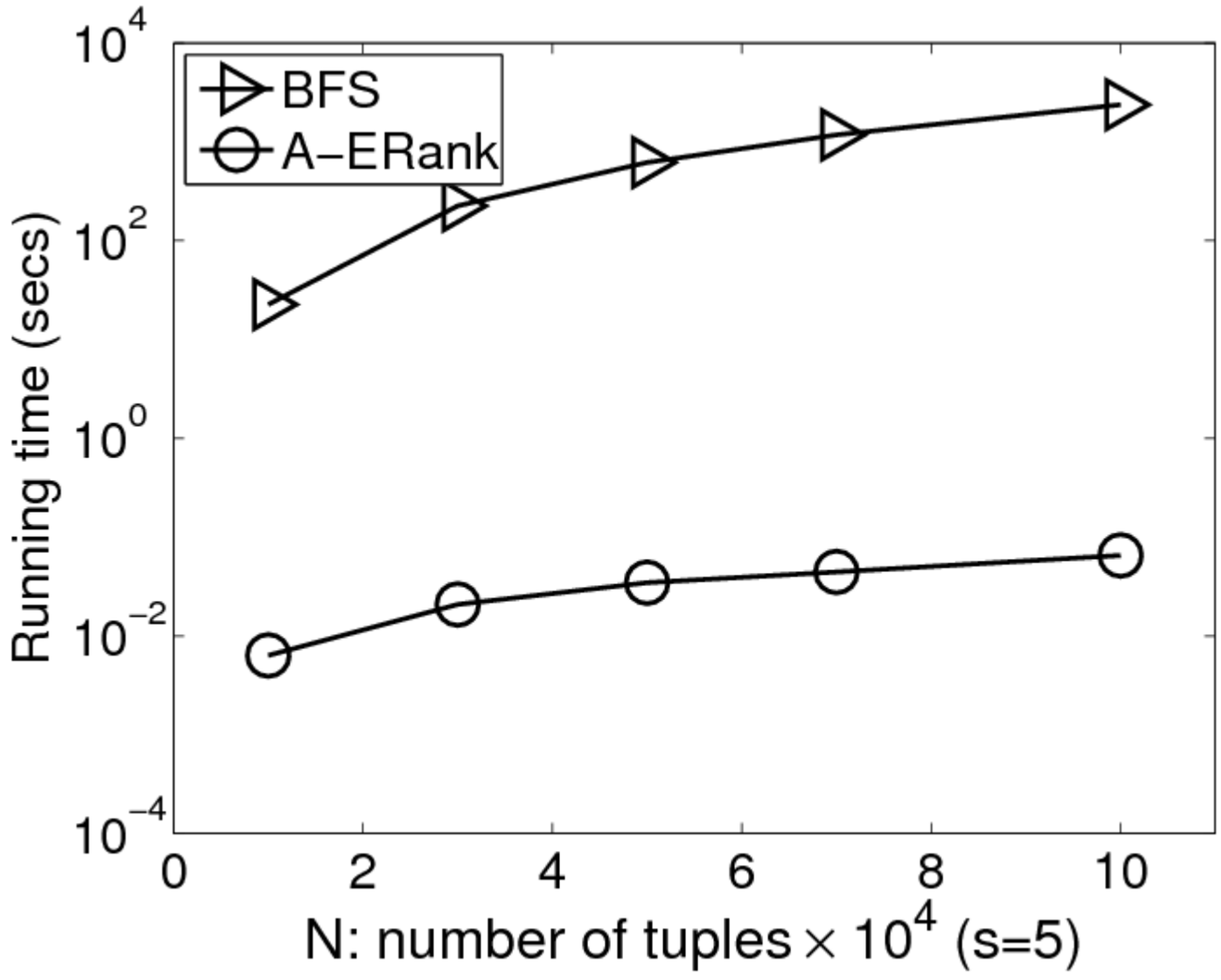
There are also pruning techniques that prune tuples that are not in the top-k

Experimental results: Tuple-level uncertainty



$$k = 100$$

Experimental results: Score-level uncertainty



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  - "Faithfulness" [Zhang, Chomicki, 08]
- Why expectation of the rank?
  - How about "median rank"?

The End

THANK YOU

Q and A